Quality Management Systems and Statistical Quality Control

2

Contents

Organizations depend on their customers and therefore should understand current and future customer needs, should meet customer requirements and strive to exceed customer expectations... Identifying, understanding and managing interrelated processes as a system contributes to the organization's effectiveness and efficiency in achieving its objectives (EN ISO 9000:2006 Quality management systems – fundamentals and vocabulary).

Nowadays, user and consumer assume their own choices regarding very important competitive factors such as quality of product, production process, and production system. Users and consumers start making their choices when they feel they are able to value and compare firms with high quality standards by themselves.

This chapter introduces the reader to the main problems concerning management and control of a quality system and also the main supporting decision measures and tools for so-called statistical quality control (SQC) and Six Sigma.

2.1 Introduction to Quality Management Systems

The standard EN ISO 8402:1995, replaced by EN ISO 9000:2005, defines "quality" as "the totality of characteristics of an entity that bear on its ability to satisfy stated and implied needs," and "product" as

R. Manzini, A. Regattieri, H. Pham, E. Ferrari, *Maintenance for Industrial Systems*

"the result of activities or processes and can be tangible or intangible, or a combination thereof." Consequently, these definitions refer to production systems both in industrial sectors, such as insurance, banking, and transport, and service sectors, in accordance with the conceptual framework introduced in Chap. 1. Another synthetic definition of quality is the "degree to which a set of inherent characteristics fulfills requirements" (ISO 9000:2005).

A *requirement* is an expectation; it is generally related to the organization, customers, or other interested, or involved, parties. We choose to name all these entities, i. e., the stakeholders of the organization, as *customers* and, consequently, the basic keyword in quality management is *customer satisfaction*. Another basic term is *capability* as the ability of the organization, system, or process to realize a product fulfilling the requirements.

A *quality management system* is a particular management system driving the organization with regard to quality. In other words, it assists companies and organizations in enhancing customer satisfaction. This is the result of products capable of satisfying the ever-changing customer needs and expectations that consequently require the continuous improvement of products, processes, and production systems.

Quality management is a responsibility at all levels of management and involves all members of an organization. For this reason, in the 1980s *total quality management (TQM)* as a business management strategy aimed at embedding awareness of quality in all organizational processes found very great success. According to the International Organization for Standardization (ISO) standards (ISO 9000:2006), the basic steps for developing and implementing a quality management system are:

- determination of needs and expectations of customers and other involved parties;
- definition of the organization's quality policy and quality objectives;
- determination of processes and responsibilities for quality assessment;
- identification and choice of production resources necessary to attain the quality objectives;
- determination and application of methods to measure the effectiveness and efficiency of each process within the production system;
- prevention of nonconformities and deletion of the related causes;
- definition and application of a process for continuous improvement of the quality management system.

Figure 2.1 presents the model of a process-based quality management system, as proposed by the ISO standards.

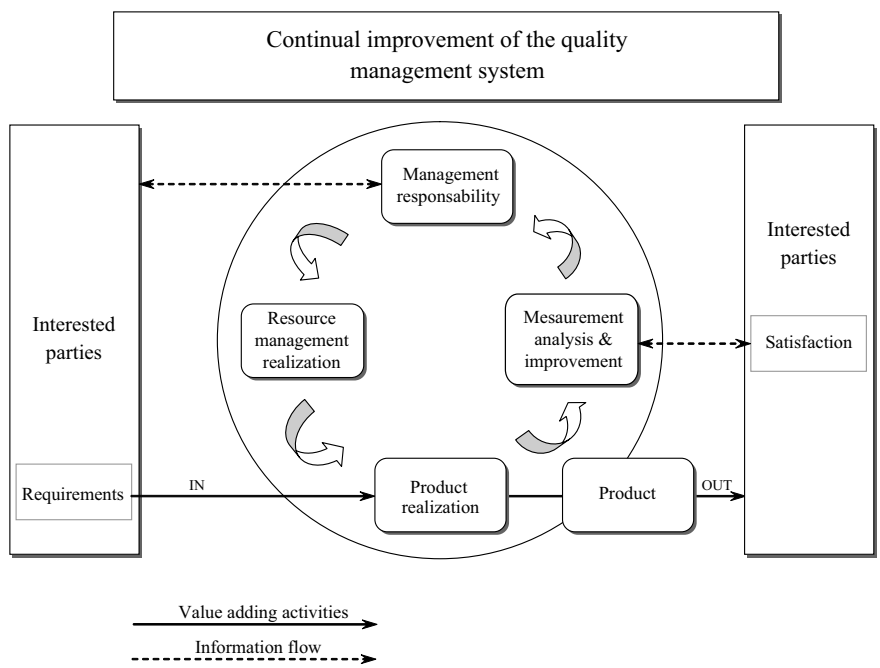


Fig. 2.1 Process-based quality management system (ISO 9000:2005)

2.2 International Standards and Specifications

According to European Directive 98/34/EC of 22 June 1998, a "standard" is a technical specification for repeated or continuous application approved, without a compulsory compliance, by one of the following recognized standardization bodies:

- ISO;
- European standard (EN);
- national standard (e. g., in Italy UNI).

Standards are therefore documents defining the characteristics (dimensional, performance, environmental, safety, organizational, etc.) of a product, process, or service, in accordance with the state of the art, and they are the result of input received from thousands of experts working in the European Union and elsewhere in the world. Standards have the following distinctive characteristics:

- *Consensuality:* They must be approved with the consensus of the participants in the works of preparation and confirmed by the result of a public enquiry.
- *Democracy:* All the interested economic/social parties can participate in the works and, above all, have the opportunity to make observations during the procedure prior to final and public approval.
- *Transparency:* UNI specifies the basic milestones of the approval procedure for a draft standard, placing the draft documents at the disposal of the interested parties for consultation.
- *Voluntary nature:* Standards are a source of reference that the interested parties agree to apply freely on a noncompulsory basis.

In particular CEN, the European Committee for Standardization founded in 1961 by the national standards bodies in the European Economic Community and EFTA countries, is contributing to the objectives of the European Union and European Economic Area with voluntary technical standards promoting free trade, safety of workers and consumers, interoperability of networks, environmental protection, exploitation of research and development programs, and public procurement.

CEN works closely with the European Committee for Electrotechnical Standardization (CENELEC), the European Telecommunications Standards Institute (ETSI), and the ISO. CEN is a multisectorial organization serving several sectors in different ways, as illustrated in the next sections and chapters dealing with safety assessment.

2.3 ISO Standards for Quality Management and Assessment

The main issues developed by the technical committee for the area of quality are:

1. CEN/CLC/TC 1 – criteria for conformity assessment bodies;
2. CEN/SS F20 – quality assurance.

Table 2.1 reports the list of standards belonging to the first technical committee since 2008.

Similarly, Table 2.2 reports the list of standards belonging to the technical committee CEN/SS F20 since 2008, while Table 2.3 shows the list of standards currently under development.

Quality issues are discussed in several standards that belong to other technical groups. For example, there is a list of standards of the aerospace series dealing with quality, as reported in Table 2.4. Table 2.5 presents a list of standards for quality management systems in health care services. Similarly, there are other sets of standards for specific sectors, businesses, or products.

2.3.1 Quality Audit, Conformity, and Certification

Quality audit is the systematic examination of a quality system carried out by an internal or external quality auditor, or an audit team. It is an independent and documented process to obtain audit evidence and to allow its objective evaluation, in order to verify the extent of the fulfillment of the audit criteria. In particular, third-party audits are conducted by external organizations providing certification/registration of conformity to a standard or a set of standards, e. g., ISO 9001 or ISO 14001. The audit process is the basis for the declaration of conformity.

The audit process is conducted by an auditor, or an audit team, i. e., a person or a team, with competence

Table 2.1 CEN/CLC/TC 1 criteria for conformity assessment bodies, standards published since 2008

Standard	Title
EN 45011:1998	General requirements for bodies operating product certification systems (ISO/IEC Guide 65:1996)
EN 45503:1996	Attestation Standard for the assessment of contract award procedures of entities operating in the water, energy, transport and telecommunications sectors
EN ISO/IEC 17000:2004	Conformity assessment – Vocabulary and general principles (ISO/IEC 17000:2004)
EN ISO/IEC 17011:2004	Conformity assessment – General requirements for accreditation bodies accrediting conformity assessment bodies (ISO/IEC 17011:2004)
EN ISO/IEC 17020:2004	General criteria for the operation of various types of bodies performing inspection (ISO/IEC 17020:1998)
EN ISO/IEC 17021:2006	Conformity assessment – Requirements for bodies providing audit and certification of management systems (ISO/IEC 17021:2006)
EN ISO/IEC 17024:2003	Conformity assessment – General requirements for bodies operating certification of persons (ISO/IEC 17024:2003)
EN ISO/IEC 17025:2005	General requirements for the competence of testing and calibration laboratories (ISO/IEC 17025:2005)
EN ISO/IEC 17025:2005/AC:2006	General requirements for the competence of testing and calibration laboratories (ISO/IEC 17025:2005/Cor.1:2006)
EN ISO/IEC 17040:2005	Conformity assessment – General requirements for peer assessment of conformity assessment bodies and accreditation bodies (ISO/IEC 17040:2005)
EN ISO/IEC 17050-1:2004	Conformity assessment – Supplier's declaration of conformity – Part 1: General requirements (ISO/IEC 17050-1:2004)
EN ISO/IEC 17050-2:2004	Conformity assessment – Supplier's declaration of conformity – Part 2: Supporting documentation (ISO/IEC 17050-2:2004)

Table 2.2 CEN/SS F20 quality assurance, standards published since 2008

Standard	Title
EN 45020:2006	Standardization and related activities – General vocabulary (ISO/IEC Guide 2:2004)
EN ISO 10012:2003	Measurement management systems – Requirements for measurement processes and measuring equipment (ISO 10012:2003)
EN ISO 15378:2007	Primary packaging materials for medicinal products – Particular requirements for the application of ISO 9001:2000, with reference to good manufacturing practice (GMP) (ISO 15378:2006)
EN ISO 19011:2002	Guidelines for quality and/or environmental management systems auditing (ISO 19011:2002)
EN ISO 9000:2005	Quality management systems – Fundamentals and vocabulary (ISO 9000:2005)
EN ISO 9001:2000	Quality management systems – Requirements (ISO 9001:2000)
EN ISO 9004:2000	Quality management systems – Guidelines for performance improvements (ISO 9004:2000)

Table 2.3 CEN/SS F20 quality assurance, standards under development as of October 2008

Standard	Title
ISO 15161:2001	Guidelines on the application of ISO 9001:2000 for the food and drink industry (ISO 15161:2001)
prEN ISO 9001	Quality management systems – Requirements (ISO/FDIS 9001:2008)
prEN ISO 19011 rev	Guidelines for auditing management systems
prEN ISO 9004	Managing for the sustained success of an organization – A quality management approach (ISO/DIS 9004:2008)

Table 2.4 Aerospace series, quality standards

Standard	Title
EN 9102:2006	Aerospace series – Quality systems – First article inspection
EN 9103:2005	Aerospace series – Quality management systems – Variation management of key characteristics
EN 9110:2005	Aerospace series – Quality systems – Model for quality assurance applicable to maintenance organizations
EN 9120:2005	Aerospace series – Quality management systems –Requirements for stockist distributors (based on ISO 9001:2000)
EN 9104:2006	Aerospace series – Quality management systems –Requirements for Aerospace Quality Management System Certification/Registrations Programs
EN 9111:2005	Aerospace series – Quality management systems – Assessment applicable to maintenance organizations (based on ISO 9001:2000)
EN 9121:2005	Aerospace series – Quality management systems – Assessment applicable to stockist distributors (based on ISO 9001:2000)
EN 9132:2006	Aerospace series – Quality management systems – Data Matrix Quality Requirements for Parts Marking
EN 4179:2005	Aerospace series – Qualification and approval of personnel for nondestructive testing
EN 4617:2006	Aerospace series – Recommended practices for standardizing company standards
EN 9101:2008	Aerospace series – Quality management systems – Assessment (based on ISO 9001:2000)
EN 9104-002:2008	Aerospace series – Quality management systems – Part 002: Requirements for Oversight of Aerospace Quality Management System Certification/Registrations Programs

Table 2.5 CEN/TC 362, health care services, quality management systems

Standard	Title
CEN/TR 15592:200	Health services – Quality management systems – Guide for the use of EN ISO 9004:2000 in health services for performance improvement
CEN/TS 15224:2005	Health services – Quality management systems – Guide for the use of EN ISO 9001:2000

to conduct an audit, in accordance with an audit program consisting of a set of one or more audits planned for a specific time frame. Audit findings are used to assess the effectiveness of the quality management system and to identify opportunities for improvement. Guidance on auditing is provided by ISO 19011:2002 (Guidelines for quality and/or environmental management systems auditing).

The main advantages arising from certification are:

- improvement of the company image;
- increase of productivity and company profit;
- rise of contractual power;
- quality guarantee of the product for the client.

In the process of auditing and certification, the documentation plays a very important role, enabling communication of intent and consistency of action. Several types of documents are generated in quality management systems.

2.3.2 Environmental Standards

Every standard, even if related to product, service, or process, has an environmental impact. For a product this can vary according to the different stages of the product life cycle, such as production, distribution, use, and end-of-life. To this purpose, CEN has recently been playing a major role in reducing environmental impacts by influencing the choices that are made in connection with the design of products and processes. CEN has in place an organizational structure to respond to the challenges posed by the developments within the various sectors, as well as by the evolution of the legislation within the European Community. The main bodies within CEN are:

1. The Strategic Advisory Body on the Environment (SABE) – an advisory body for the CEN Technical Board on issues related to environment. Stakeholders identify environmental issues of importance

to the standardization system and suggest corresponding solutions.
2. The CEN Environmental Helpdesk provides support and services to CEN Technical Bodies on how to address environmental aspects in standards.
3. Sectors – some sectors established a dedicated body to address environmental matters associated with their specific needs, such as the Construction Sector Network Project for the Environment (CSNPE).
4. Associates – two CEN associate members provide a particular focus on the environment within standardization:

   - European Environmental Citizens Organization for Standardization (ECOS);
   - European Association for the Coordination of Consumer Representation in Standardization (ANEC).

Table 2.6 reports the list of technical committees on the environment.

There are several standards on environmental management. To exemplify this, Table 2.7 reports the list of standards grouped in accordance with the committee CEN/SS S26 – environmental management.

ISO 14000 is a family of standards supporting the organizations on the containment of the polluting effects on air, water, or land derived by their operations, in compliance with applicable laws and regulations. In particular, ISO 14001 is the international specification for an environmental management system (EMS). It specifies requirements for establishing an environmental policy, determining environmental aspects and impacts of products/activities/services, planning environmental objectives and measurable targets, implementation and operation of programs to meet objectives and targets, checking and corrective action, and management review.

Table 2.6 Technical committees on the environment

Technical commitee	Title
CEN/TC 223	Soil improvers and growing media
CEN/TC 230	Water analysis
CEN/TC 264	Air quality
CEN/TC 292	Characterization of waste
CEN/TC 308	Characterization of sludges
CEN/TC 345	Characterization of soils
CEN/TC 351	Construction Products – Assessment of release of dangerous substances

Table 2.7 Committee CEN/SS S26 – environmental management

Standard	Title
EN ISO 14031:1999	Environmental management – Environmental performance evaluation – Guidelines (ISO 14031:1999)
EN ISO 14001:2004	Environmental management systems – Requirements with guidance for use (ISO 14001:2004)
EN ISO 14024:2000	Environmental labels and declarations – Type I environmental labeling – Principles and procedures (ISO 14024:1999)
EN ISO 14021:2001	Environmental labels and declarations – Self-declared environmental claims (Type II environmental labelling) (ISO 14021:1999)
EN ISO 14020:2001	Environmental labels and declarations – General principles (ISO 14020:2000)
EN ISO 14040:2006	Environmental management – Life cycle assessment – Principles and framework (ISO 14040:2006)
EN ISO 14044:2006	Environmental management – Life cycle assessment – Requirements and guidelines (ISO 14044:2006)
prEN ISO 14005	Environmental management systems – Guidelines for a staged implementation of an environmental management system, including the use of environmental performance evaluation

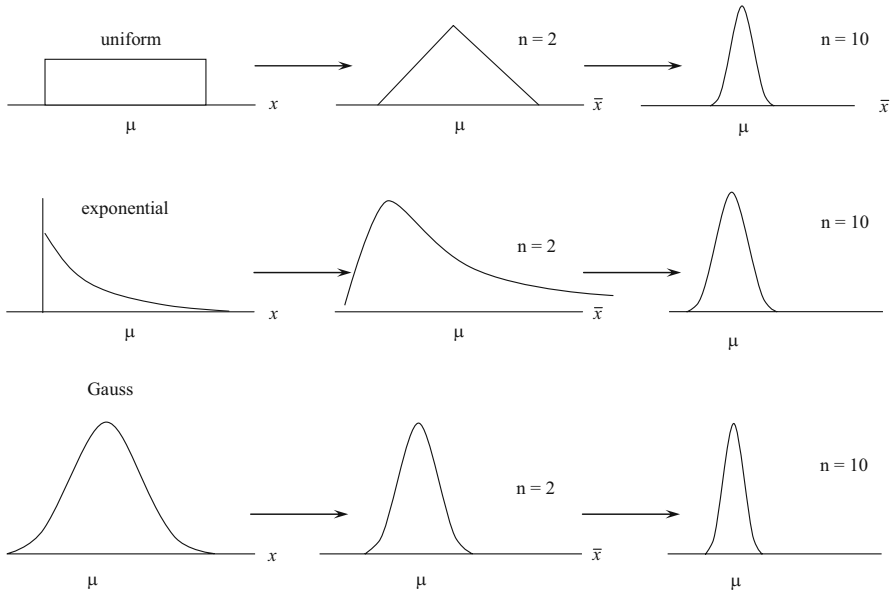


Fig. 2.2 Central limit theorem, examples

2.4 Introduction to Statistical Methods for Quality Control

The aim of the remainder of this chapter is the introduction and exemplification of effective models and methods for statistical quality control. These tools are very diffuse and can be used to guarantee also the reliability,[1] productivity and safety of a generic production system in accordance with the purpose of this book, as illustrated in Chap. 1.

2.4.1 The Central Limit Theorem

This section briefly summarizes the basic result obtained by this famous theorem. Given a population or process, a random variable x, with mean μ and standard deviation σ, let $\bar{x}$ be the mean of a random sample made of n elements $x_1, x_2, \ldots, x_n$ extracted from this population: when the sample size n is sufficiently large, the sampling distribution of the random variable $\bar{x}$ can be approximated by a normal distribution. The larger the value of n, the better the approximation.

This theorem holds irrespective of the shape of the population, i. e., of the density function of the variable x.

The analytic translation of the theorem is given by the following equations:

$$M(\bar{x}) = \bar{\bar{x}} = \hat{\mu}, \tag{2.1}$$

$$\sigma(\bar{x}) = \frac{\hat{\sigma}}{\sqrt{n}}, \tag{2.2}$$

where $\hat{\mu}$ is the estimation of μ and $\hat{\sigma}$ is the estimation of σ.

Figure 2.2 graphically and qualitatively demonstrates these results representing the basis for the development and discussion of the methods illustrated and applied below. In the presence of a normal distribution of population, the variable $\bar{x}$ is normal too for each value of size n.

Figure 2.3 quantitatively demonstrates the central limit theorem starting from a set of random values distributed in accordance with a uniform distribution $[0, 10]$: the variable $\bar{x}$ is a normally distributed variable when the number of items used for the calculus of mean $\bar{x}_i$ is sufficiently large. In detail, in Fig. 2.3 the size n is assumed be 2, 5, and 20.

[1] Reliability, properly defined in Chap. 5, can be also defined as "quality in use."

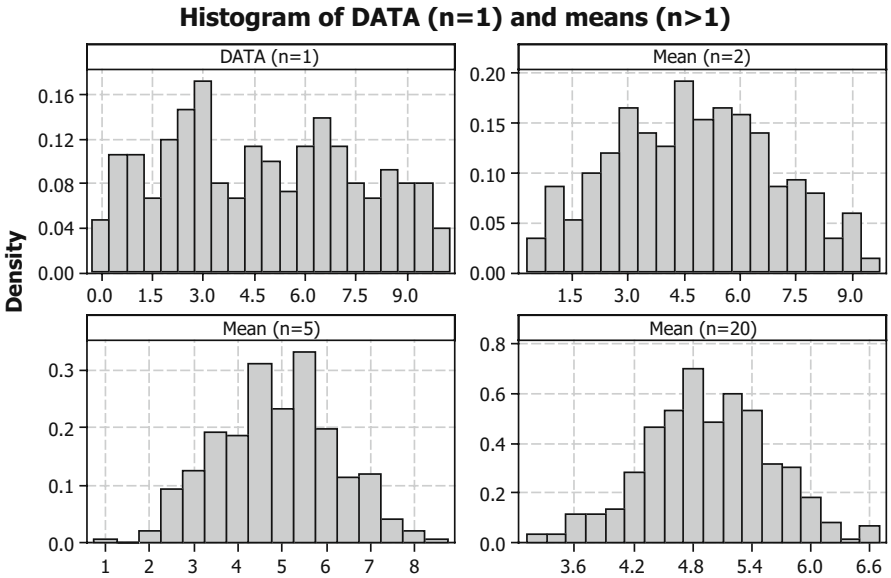


Fig. 2.3 Central limit theorem, histogram of $\bar{x}$ for $n = \{1, 2, 5, 20\}$. Uniform distribution of variable x

2.4.2 Terms and Definition in Statistical Quality Control

Quality control is a part of quality management (ISO 9000:2005) focused on the fulfillment of quality requirements. It is a systematic process to monitor and improve the quality of a product, e. g., a manufactured article, or service by achieving the quality of the production process and the production plant. A list of basic terms and definitions in accordance with the ISO standards follows:

- *Process*, set of interrelated activities turning input into output. It is a sequence of steps that results in an outcome.
- *Product*, result of a process.
- *Defect*, not fulfillment of a requirement related to an intended or specified use.
- *Measurement process*, set of operations to determine the value of a quantity.
- *Key characteristic*, a quality characteristic the product or service should have to fulfill customer requirements and expectations.
- *Value* of a key characteristic. For several products a single value is the desired quality level for a characteristic.
- *Nominal* or *target value*. It is the expected value for the key characteristic. It is almost impossible to make each unit of product or service identical to the next; consequently it is nonsense to ask for identical items having a key characteristic value exactly equal to the target value. This need for flexibility is supported by the introduction of limits and tolerances.
- *Specification limit*, or tolerance, conformance boundary, range, specified for a characteristic. The *lower specification limit* (LSL) is the lower conformance boundary, the *upper specification limit* (USL) is the upper conformance boundary.
  The following equation summarizes the relationship among these terms:

$$\textit{Specification limits} = \textit{(nominal value)} \pm \textit{tolerance}. \tag{2.3}$$

- *One-sided tolerance*. It relates to characteristics with only one specification limit.
- *Two-sided tolerance.* It refers to characteristics with both USLs and LSLs.
- *Nonconformity*. It is a nonfulfillment of a requirement. It is generally associated with a unit: a nonconformity unit, i. e., a unit that does not meet the specifications.
- *Nonconforming product or service*. A product or service with one or more nonconformities. A nonconforming product is not necessary defective, i. e., no longer fit for use.

2.5 Histograms

Histograms are effective and simple graphic tools for the comprehension and analysis of a process behavior with regards to the target value and the specification limits. The histograms illustrate the frequency distribution of variable data: the values assumed by the variable are reported on the abscissa, while the vertical axis reports the absolute or relative frequency values. The specification limits are generally included in the graph and give warnings of possible process problems. Figure 2.4 exemplifies a few histogram shapes. The control charts illustrated in the next section represent a more effective tool for the analysis of a production process.

2.6 Control Charts

Control charts, introduced by W.A. Shewhart in 1924, are effective tools for the analysis of the variation of repetitive processes. They are able to identify possible sources of process variation in order to control and eventually eliminate them. In a generic process, two different kinds of variations can be distinguished:

1. *Common causes variations*. They are the noise of a production system and are uncontrollable variations.
2. *Assignable (or special) causes variations*. They can be properly identified and controlled. Some examples are turnover in workman load, breakdowns, machine or tool wear out, and tool change.

Control charts are a family of tools for detecting the existence of special causes variations in order to avoid them, i. e., eliminate all anomalous controllable patterns, and bring the process into a state called "of statistical control," or simply "in control," whose random behavior is justified by the existence of common causes variations. The "in control" state is necessary to obtain conforming products, as properly discussed in the following sections on *capability analysis* and *Six Sigma*.

Control charts can be constructed by extracting successive *samples* from the variable output of the process. These samples, also called "subgroups," all have size n and have to be taken at regular intervals of time. For each group a summary statistic is calculated and plotted as illustrated in Fig. 2.5.

Typical statistical measures calculated for each subgroup are reported in Table 2.8, where the related statistical distribution is cited together with the values of

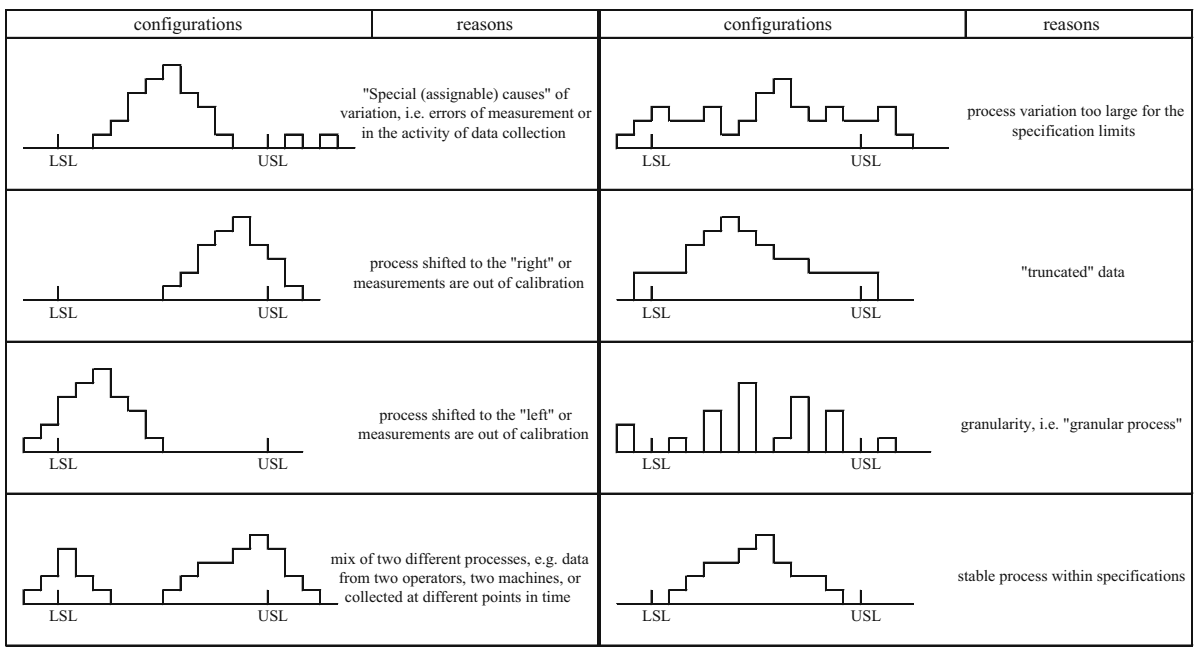


Fig. 2.4 Exemplifying histograms shapes. *LSL* lower specification limit, *USL* upper specification limit

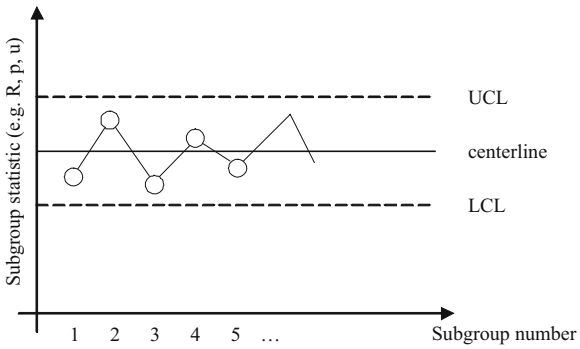


Fig. 2.5 Control chart

the centerline and control limits, as properly defined in the next subsections.

A control chart is made of three basic lines as illustrated in Fig. 2.5:

1. *Centerline*. It is the mean of the statistic quantified for each subgroup, the so-called subgroup statistic (e. g., mean, range, standard deviation).
2. *Control limits*. These limits on a control chart delimit that region where a data point falls outside, thus alerting one to special causes of variation. This region is normally extended three standard deviations on either side of the mean. The control limits are:
   - *upper control limit* (UCL), above the mean;
   - *lower control limit* (LCL), below the mean.

The generic point of the chart in Fig. 2.5 may represent a subgroup, a sample, or a statistic. k different samples are associated with k different points whose temporal sequence is reported on the chart. Control limits are conventionally set at a distance of three standards errors, i. e., three deviations of the subgroup statistic, from the centerline, because the distribution of samples closely approximates a normal statistical distribution by the central limit theorem. Consequently, the analyst expects that about 99.73% of samples lie within three standard deviations of the mean. This corresponds to a probability of 0.27% that a control chart point falls outside one of the previously defined control limits when no assignable causes are present.

In some countries, such as in the UK, the adopted convention of $\pm$ three standard deviations is different.

Figure 2.6 presents eight different anomalous patterns of statistic subgroups tested by Minitab® Statistical Software to find reliable conditions for the in, or out, control state of the process.

A process is said to be "in control" when all subgroups on a control chart lie within the control limits and no anomalous patterns are in the sequence of points representing the subgroups. Otherwise, the process is said to be "out of control," i. e., it is not random because there are special causes variations affecting the output obtained.

What happens in the presence of special causes? It is necessary to identify and eliminate them. Consequently, if a chart shows the possible existence of special causes by one of the anomalous behaviors illustrated in Fig. 2.6, the analyst and the person responsible for the process have to repeat the analysis by eliminating the anomalous subgroups. Now, if all the tests are not verified, the process has been conducted to the state of statistical control.

2.7 Control Charts for Means

These charts refer to continuous measurement data, also called "variable data" (see Table 2.8), because there are an infinite number of data between two generic ones.

2.7.1 The R-Chart

This is a chart for subgroup ranges. The range is the difference between the maximum and the minimum values within a sample of size n:

$$R_i = \max_{j=1,\ldots,n} \{x_{ij}\} - \min_{j=1,\ldots,n} \{x_{ij}\}, \tag{2.4}$$

where i is a generic sample and x_{ij} is the jth value in the sample i.

Consequently, the centerline is

$$\mu_R = \bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i. \tag{2.5}$$

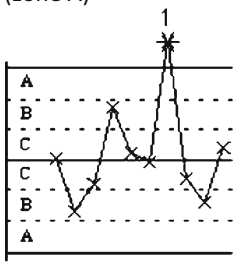
This value is a good estimation of the mean value of variable R_i, called "μ_R." We also define the statistic measure of variability of the variable R_i, the standard deviation σ_R. By the central limit theorem, the distribution of values R_i is normal. As a consequence, the

Table 2.8 Statistical measures and control chart classification

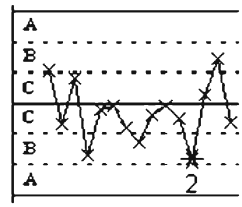
Name	Type of data	Statistical distribution	Statistic measure	Centerline	Control limits	Standard deviation
R-chart	Variable	Continuous – normal distribution	Range R	$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i$	$\text{UCL} = \mu_R + 3\sigma_R \cong D_4\bar{R}$ $\text{LCL} = \mu_R - 3\sigma_R \cong D_3\bar{R}$	
s-chart	Variable	Continuous – normal distribution	Standard deviation s	$\hat{\mu}_S = \bar{s} = \frac{1}{k}\sum_{i=1}^{k} s_i$	$\text{UCL} = B_4\bar{s}$ $\text{LCL} = B_3\bar{s}$	
$\bar{x}$-chart	Variable	Continuous – normal distribution	Mean $\bar{x}$	$\hat{\mu} = \bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k} \bar{X}_i$	$\text{UCL} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{X}} + 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{X}} + A_2\bar{R}$ $\text{LCL} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{X}} - 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{X}} - A_2\bar{R}$	$\hat{\sigma} = \frac{\bar{R}}{d_2}$
					$\text{UCL} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{X}} + 3\frac{\bar{s}/c_4}{\sqrt{n}} = \bar{\bar{X}} + A_3\bar{s}$ $\text{LCL} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{X}} - 3\frac{\bar{s}/c_4}{\sqrt{n}} = \bar{\bar{X}} - A_3\bar{s}$	$\hat{\sigma} = \frac{\bar{s}}{c_4}$
p-chart	Attribute	Discrete – binomial distribution	Nonconforming proportion p	$\bar{p} = \frac{1}{k}\sum_{i=1}^{k} p_i$	$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ $\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	
				$\bar{p} = \frac{x_1 + x_2 + \cdots + x_{k-1} + x_k}{n_1 + n_2 + \cdots + n_{k-1} + n_k}$	$\text{UCL}_i = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$ $\text{LCL}_i = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$	
np-chart	Attribute	Discrete – binomial distribution	Number of nonconformities np	$n\bar{p}$	$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ $\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	
c-chart	Attribute	Discrete – Poisson distribution	Number of nonconformities	$\bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$	$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$ $\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$	
u-chart	Attribute	Discrete – Poisson distribution	Nonconformities per unit u	$\bar{u} = \frac{1}{k}\sum_{i=1}^{k} u_i$	$\text{UCL}_i = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$ $\text{LCL}_i = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$	

UCL upper control limit, *LCL* lower control limit, UCL_i upper control limit for sample i, LCL_i lower control limit for sample i

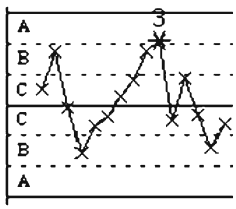
Test 1 – 1 point beyond 3 std.dev. (zone A)



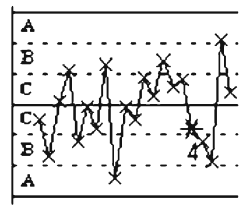
Test 2 – 9 points in a row on same side of the center line



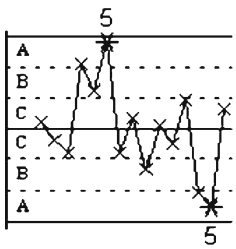
Test 3 – 6 points in a row all increasing (or decreasing)



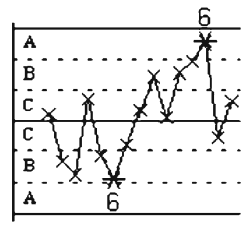
Test 4 – 14 points in a row alternating up and down



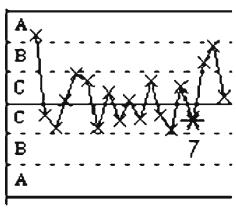
Test 5 – 2/3 points in a row more than 2 std.dev.



Test 6 – 4 out of 5 points more than 1 std.dev.



Test 7 – 15 points in a row within 1 std.dev. (either side)



Test 8 – 8 points in a row more than 1 std.dev. (either side)

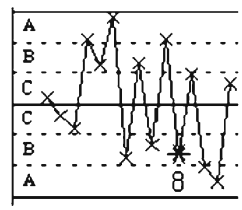


Fig. 2.6 Eight tests for special causes investigation, Minitab® Statistical Software

control limits are defined as

$$\begin{aligned} \mathrm{UCL}_R &= \mu_R + 3\sigma_R \cong D_4 \bar{R}, \\ \mathrm{LCL}_R &= \mu_R - 3\sigma_R \cong D_3 \bar{R}, \end{aligned} \tag{2.6}$$

where σ_R is the standard deviation of the variable R and D_4 is a constant value depending on the size of the generic subgroup. The values are reported in Appendix A.2.

The following equation represents an estimation of the standard deviation of the variable and continuous data x_{ij}:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \tag{2.7}$$

2.7.2 Numerical Example, R-Chart

This application refers to the assembly process of an automotive engine. The process variable is a distance, D, measured in tenths of millimeters, between two characteristic axes in the drive shafts and heads. Table 2.9 presents the data collected over 25 days of observation and grouped in samples of size $n = 5$.

By the application of Eqs. 2.5 and 2.6, we have

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i = \frac{1}{25}(R_1 + \cdots + R_{25}) \cong 6.50,$$

$$\text{UCL} = \mu_R + 3\sigma_R \cong D_4\bar{R} \underset{D_4(n=5)=2.114}{=} 13.74,$$

$$\text{LCL} = \mu_R - 3\sigma_R \cong D_3\bar{R} \underset{D_4(n=5)=0}{=} 0.$$

The R-chart obtained is reported in Fig. 2.7. Previously introduced tests for anomalous behaviors are not verified. As a consequence, the process seems to be random and "coherent with itself" and its characteristic noise and variance. There are no special causes of variation.

2.7.3 The $\bar{x}$-Chart

This is a chart for subgroup means. In the $\bar{x}$-chart, also called "x-chart," the problem is the estimation of the standard deviation of the population of values. In Sect. 2.7.1, Eq. 2.7 is an effective estimation. Consequently, this chart is generally constructed after the creation of the R-chart and reveals the process to be in the state of statistical control. The centerline of the statistic variable $\bar{x}$ is the average of the subgroup means:

$$\hat{\mu} = \hat{\mu}(\bar{x}) = \bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k}\bar{x}_i = \sum_{i=1}^{k}\sum_{j=1}^{n} x_{ij}. \quad (2.8)$$

The control limits are

$$\text{UCL}_{\bar{x}} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} + 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R},$$
$$\text{LCL}_{\bar{x}} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} - 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}, \quad (2.9)$$

where d_2 and A_2 are constant values as reported in Appendix A.2.

Table 2.9 Data – 25 subgroups, numerical example

Sample	Month	Day	D (mm/10)				
1	7	25	−0.387	5.192	1.839	0.088	1.774
2	7	26	4.251	3.333	4.398	6.082	1.706
3	7	27	−2.727	−2.806	4.655	0.494	−2.807
4	7	28	6.980	3.280	3.372	−1.914	2.478
5	7	29	3.978	3.479	7.034	4.388	−1.790
6	7	30	3.424	1.758	0.009	−0.216	1.832
7	7	31	−4.285	−2.369	−2.666	2.639	3.081
8	8	1	−1.756	−1.434	1.887	−1.678	7.060
9	8	2	4.184	1.005	0.825	−6.427	−4.598
10	8	3	−3.577	−1.684	1.800	4.339	0.027
11	8	4	−2.467	−2.752	−4.029	−2.798	−2.152
12	8	5	1.199	0.817	−0.213	−0.737	−1.757
13	8	6	4.312	1.127	2.534	1.618	−0.665
14	8	7	3.282	3.319	−3.564	3.430	1.556
15	8	8	2.000	−3.364	−1.996	−1.830	0.015
16	8	9	3.268	1.519	2.704	0.138	−0.050
17	8	10	3.356	−3.335	−3.358	−4.302	−2.856
18	8	11	−0.240	−3.811	−1.615	−3.510	−4.377
19	8	12	−4.524	−0.091	1.945	4.515	−1.667
20	8	13	0.837	−4.536	4.249	0.114	−0.087
21	8	14	−1.016	2.023	4.539	0.075	−2.724
22	8	15	4.547	0.262	−4.108	−1.881	−0.004
23	8	16	0.159	3.786	−1.951	6.315	5.161
24	8	17	0.842	−3.550	−1.805	−2.731	−1.610
25	8	18	4.435	1.730	−0.185	0.242	−4.689

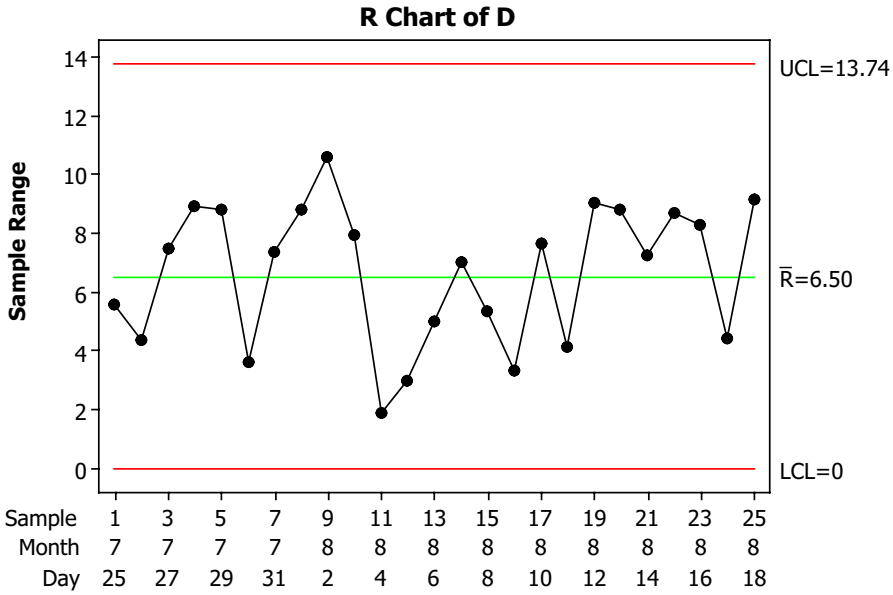


Fig. 2.7 *R*-chart, numerical example. Minitab® Statistical Software. *UCL* upper control limit, *LCL* lower control limit

2.7.4 Numerical Example, $\bar{x}$-Chart

Consider the application introduced in Sect. 2.7.2. By Eqs. 2.8 and 2.9,

$$\hat{\mu} = \bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k}\bar{x}_i = \frac{1}{25}(\bar{x}_1 + \cdots + \bar{x}_{25}) = 0.389,$$

$$\text{UCL} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} + 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$$
$$\underset{A_2=0.577}{=} 0.389 + 0.577 \cdot 6.5 \cong 4.139,$$

$$\text{LCL} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} - 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$$
$$\underset{A_2=0.577}{=} 0.389 - 0.577 \cdot 6.5 \cong -3.361.$$

The chart obtained is reported in Fig. 2.8. Test 6 for anomalous behaviors is verified in sample 5, month 7, and day 29, i. e., there are four of five points in zone B or beyond. As a consequence, the process seems to be "out of control." There is in fact a very scarce probability of having a sample in those points when the process is "in control." We assume we are able to properly identify this special cause of variation and to eliminate it. Figure 2.9 presents the charts obtained from the pool of samples without the anomalous subgroup 5. The chart shows another potential anomalous behavior regarding subgroup 4. In this way, assuming we identify and eliminate new special causes, we obtain Figs. 2.10 and 2.11. In particular, Fig. 2.11 presents a process in the state of statistical control: subgroups 2, 4, and 5 have been eliminated.

2.7.5 The s-Chart

This chart for subgroup standard deviation can be used to support the construction of the x-chart by the estimation of the standard deviation of the continuous variable x_{ij}. In particular, the control limits of the x-chart use the centerline of the s-chart.

The average of standard deviation of subgroups, $\hat{s}$, is the centerline of the s-chart:

$$\hat{\mu}_S = \hat{\mu}(s_i) = \bar{s} = \frac{1}{k}\sum_{i=1}^{k} s_i, \qquad (2.10)$$

where $\hat{\mu}(s_i)$ is the estimation of the mean of the variable s_i, the standard deviation of a subgroup.

The control limits are

$$\begin{aligned} \text{UCL}_s &= \hat{\mu}(s_i) + 3\frac{\hat{\sigma}(s_i)}{\sqrt{n}} = B_4\bar{s}, \\ \text{LCL}_s &= \hat{\mu}(s_i) - 3\frac{\hat{\sigma}(s_i)}{\sqrt{n}} = B_3\bar{s}, \end{aligned} \qquad (2.11)$$

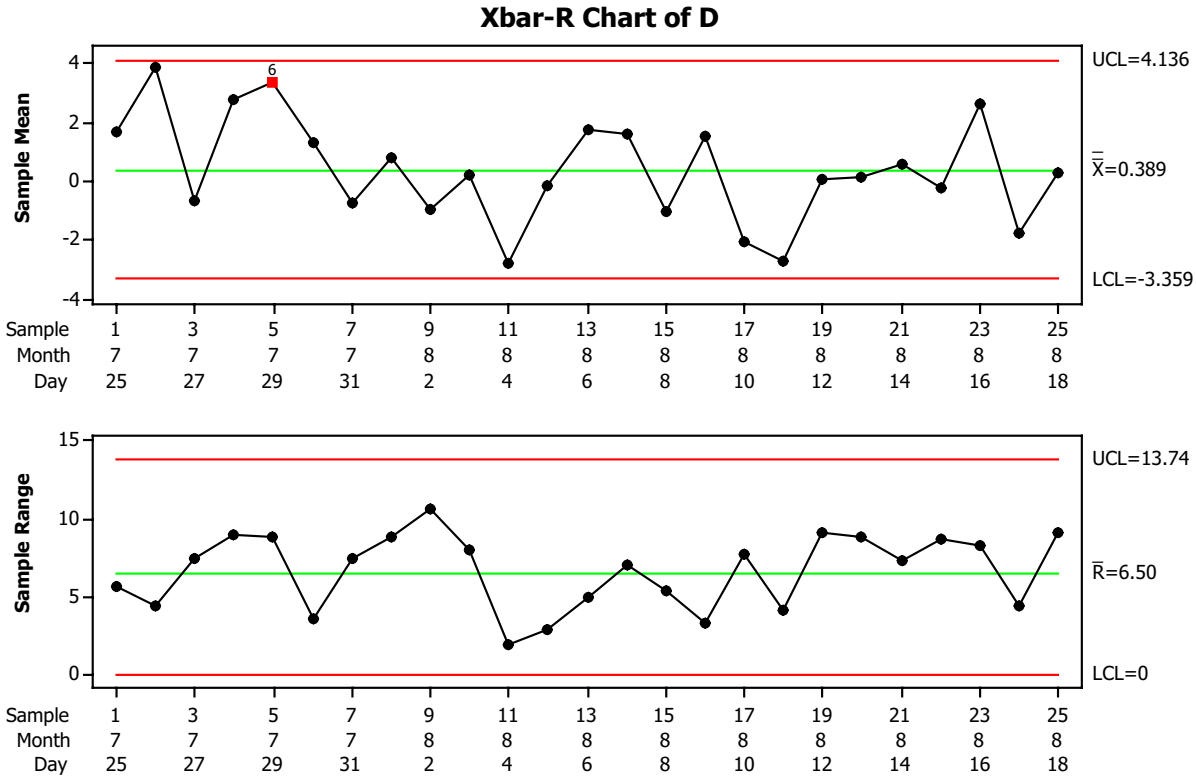


Fig. 2.8 x-chart from R. Numerical example (25 samples). Minitab® Statistical Software

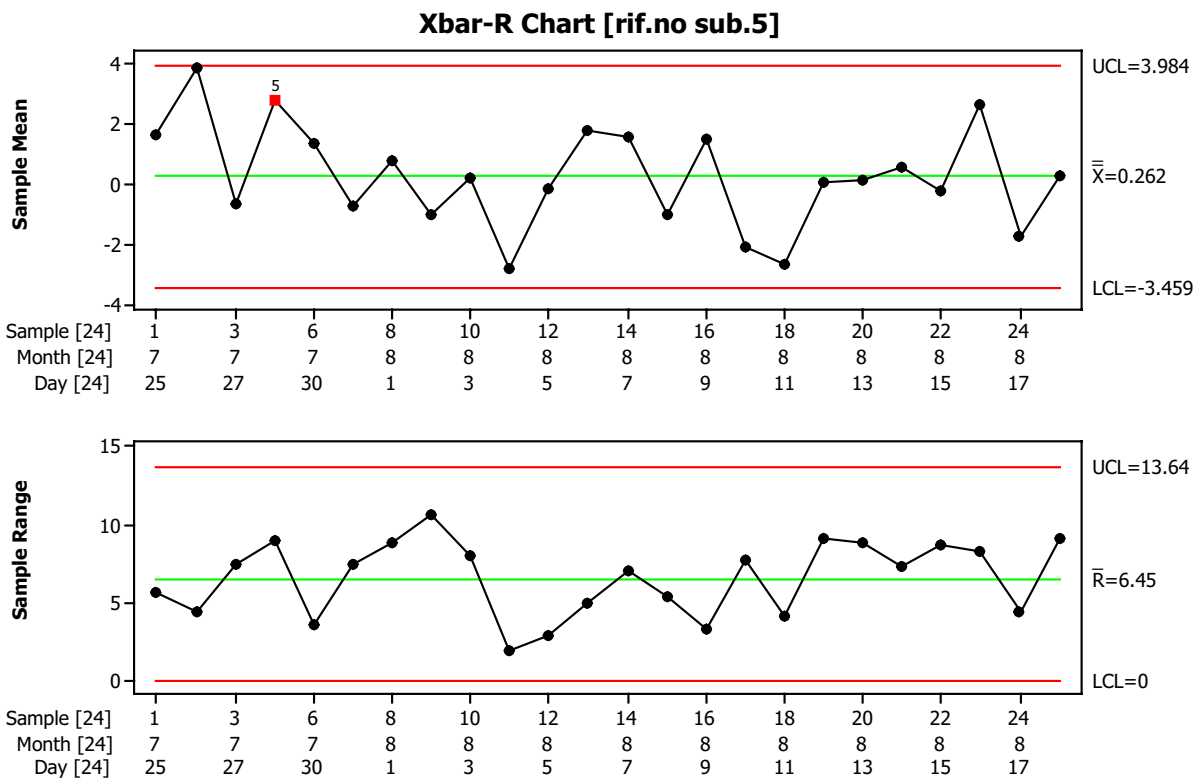


Fig. 2.9 x-chart from R. Numerical example (24 samples). Minitab® Statistical Software

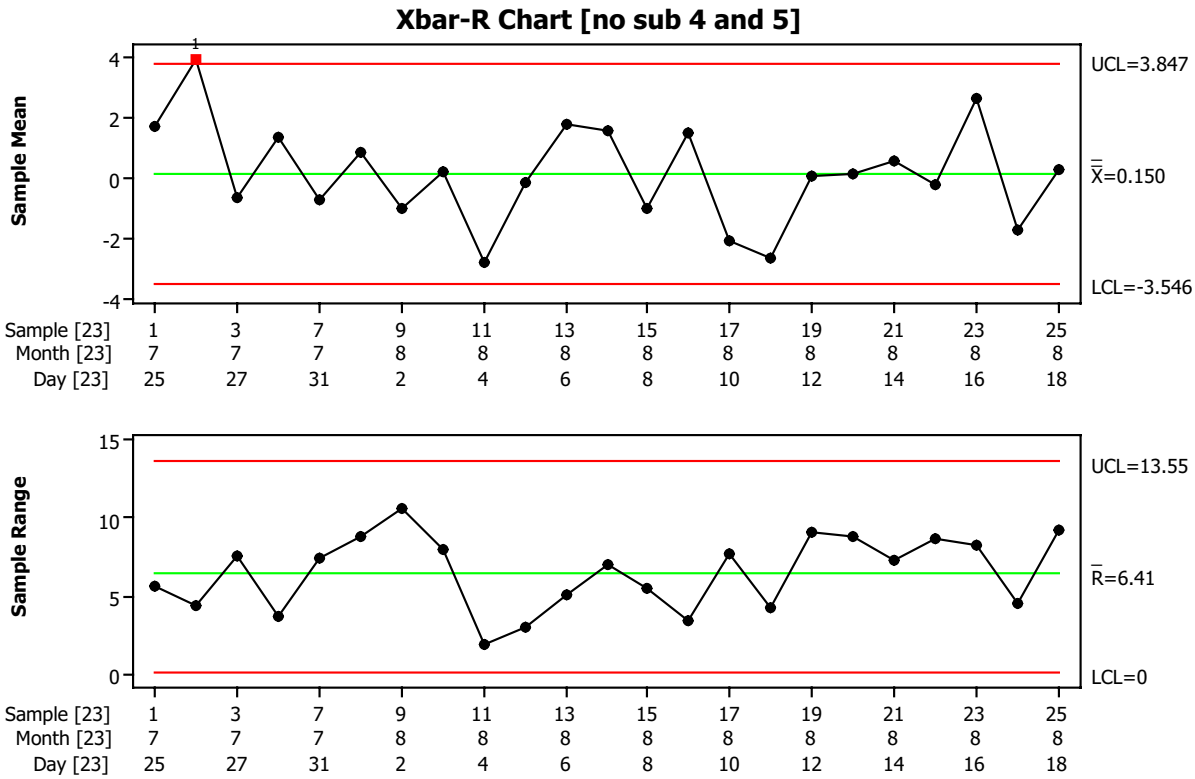


Fig. 2.10 x-chart from R. Numerical example (23 samples). Minitab® Statistical Software

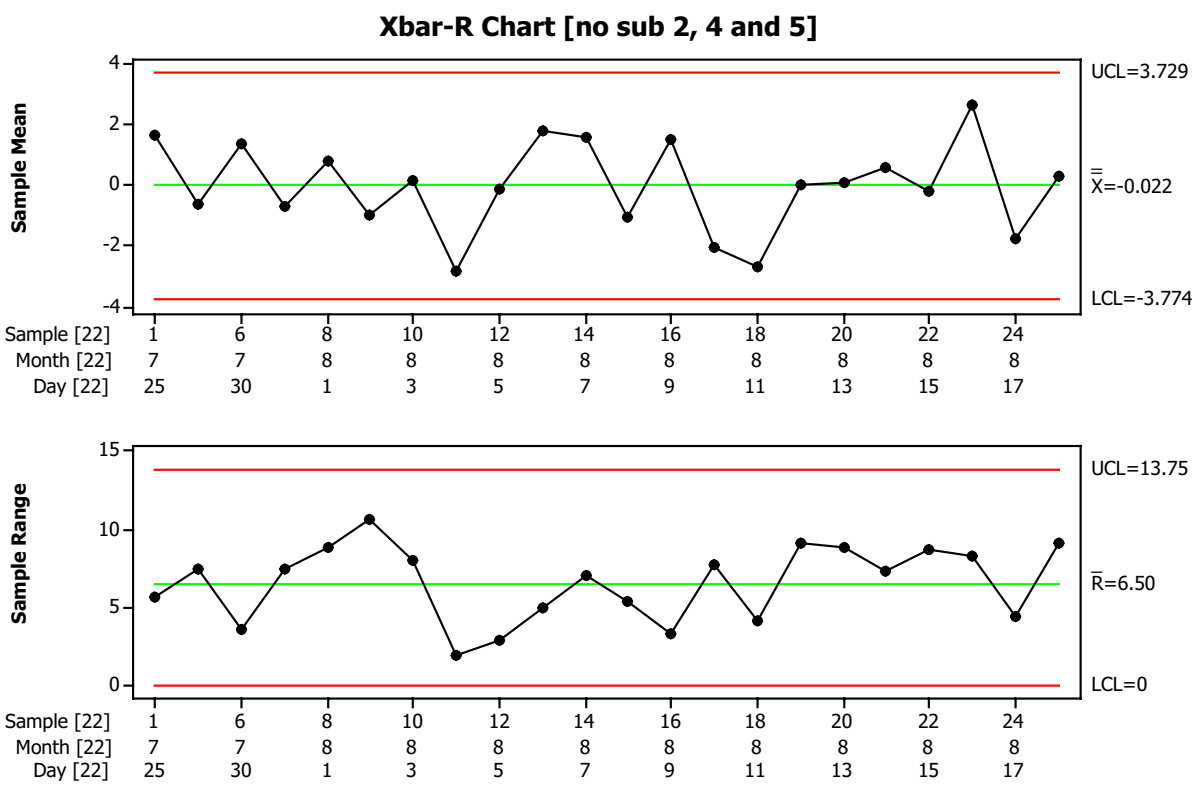


Fig. 2.11 x-chart from R. Numerical example (22 samples). Minitab® Statistical Software

where $\hat{\sigma}(s_i)$ is the estimation of the standard deviation of the variable s_i, the standard deviation of a subgroup, and B_3 and B_4 are constant values reported in Appendix A.2.

The standard deviation of the process measurement is

$$\hat{\sigma} = \hat{\sigma}(\bar{x}_i) = \frac{\bar{s}}{c_4}. \tag{2.12}$$

As a consequence, the control limits of the x-chart are

$$\begin{aligned} \mathrm{UCL}_{\bar{x}} &= \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} + 3\frac{\bar{s}/c_4}{\sqrt{n}} = \bar{\bar{x}} + A_3\bar{s}, \\ \mathrm{LCL}_{\bar{x}} &= \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}} \approx \bar{\bar{x}} - 3\frac{\bar{s}/c_4}{\sqrt{n}} = \bar{\bar{x}} - A_3\bar{s}, \end{aligned} \tag{2.13}$$

where A_3 is a constant value reported in Appendix A.2.

2.7.6 Numerical Example, s-Chart and $\bar{x}$-Chart

Table 2.10 reports a set of measurement data made for 20 samples of size $n = 5$. They are the output of a manufacturing process in the automotive industry. The last three columns report some statistics useful for the construction of the control charts and for verification of the status of the control of the process.

With use of the values of the constant parameters in Appendix A.2, the following control limits and centerlines have been obtained.

Firstly, we propose the results related to the R-chart. By Eq. 2.5 the centerline is

$$\mu_R = \bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i \cong 0.004155.$$

By Eq. 2.6

$$\begin{aligned} \mathrm{UCL}_R &\approx D_4\bar{R} = 2.114 \cdot 0.004155 \cong 0.008784. \\ \mathrm{LCL}_R &\approx D_3\bar{R} = 0 \cdot 0.004155 = 0. \end{aligned}$$

These results are very close to those proposed by the R-chart, as constructed by the tool Minitab® Statistical Software (Fig. 2.12). From the R-chart the process seems to be in the state of statistical control.

The x-chart is now created assuming the centerline of the R-chart and in accordance with Eqs. 2.8 and 2.9:

$$\begin{aligned} \mathrm{UCL}_{x \text{ from } R} &\cong \bar{\bar{x}} + A_2\bar{R} \\ &= 0.009237 + 0.577 \cdot 0.004155 \\ &= 0.0116, \\ \mathrm{LCL}_{x \text{ from } R} &\cong \bar{\bar{x}} - A_2\bar{R} \\ &= 0.009237 - 0.577 \cdot 0.004155 \\ &= 0.0068. \end{aligned}$$

The upper section of Fig. 2.12 presents the x-chart where some subgroups verify a few tests, as illustrated also in Fig. 2.13. Consequently, the process is not in a state of control.

Similarly, by the application of Eqs. 2.10, 2.11, and 2.13,

$$\begin{aligned} \hat{\mu}_S = \bar{s} &= \frac{1}{k}\sum_{i=1}^{k} s_i \cong 0.00170. \\ \mathrm{UCL}_s &\cong B_4\bar{s} = 2.089 \cdot 0.00170 = 0.00355, \\ \mathrm{LCL}_s &\cong B_3\bar{s} = 0 \cdot 0.00170 = 0, \\ \mathrm{UCL}_{x \text{ from } s} &\approx \bar{\bar{x}} + A_3\bar{s} \\ &= 0.009237 + 1.427 \cdot 0.00170 \\ &= 0.01166, \\ \mathrm{LCL}_{x \text{ from } s} &\approx \bar{\bar{x}} - A_3\bar{s} \\ &= 0.009237 - 1.427 \cdot 0.00170 \\ &= 0.00681. \end{aligned}$$

All these values are also reported in Fig. 2.14, showing that the process is not in the state of statistical control. Consequently, a survey for the identification and deletion of special causes of variations, and the subsequent repetition of the control analysis, is required.

2.8 Control Charts for Attribute Data

These charts refer to counted data, also called "attribute data." They support the activities of monitoring and analysis of production processes whose products possess, or do not possess, a specified characteristic or attribute. Attributes measurement is frequently obtained as the result of human judgements.

Table 2.10 Measurement data and subgroup statistics. Numerical example

ID sample – i	Measure – X					$M(x_i)$	R_i	s_i
1	0.0073	0.0101	0.0091	0.0091	0.0053	0.0082	0.0048	0.0019
2	0.0106	0.0083	0.0076	0.0074	0.0059	0.0080	0.0047	0.0017
3	0.0096	0.0080	0.0132	0.0105	0.0098	0.0102	0.0052	0.0019
4	0.0080	0.0076	0.0090	0.0099	0.0123	0.0094	0.0047	0.0019
5	0.0104	0.0084	0.0123	0.0132	0.0120	0.0113	0.0048	0.0019
6	0.0071	0.0052	0.0101	0.0123	0.0073	0.0084	0.0071	0.0028
7	0.0078	0.0089	0.0122	0.0091	0.0095	0.0095	0.0044	0.0016
8	0.0087	0.0094	0.0120	0.0102	0.0099	0.0101	0.0033	0.0012
9	0.0074	0.0081	0.0120	0.0116	0.0122	0.0103	0.0048	0.0023
10	0.0081	0.0065	0.0105	0.0125	0.0136	0.0102	0.0071	0.0029
11	0.0078	0.0098	0.0113	0.0087	0.0118	0.0099	0.0040	0.0017
12	0.0089	0.0090	0.0111	0.0122	0.0126	0.0107	0.0037	0.0017
13	0.0087	0.0075	0.0125	0.0106	0.0113	0.0101	0.0050	0.0020
14	0.0084	0.0083	0.0101	0.0140	0.0097	0.0101	0.0057	0.0023
15	0.0074	0.0091	0.0116	0.0109	0.0108	0.0100	0.0042	0.0017
16	0.0069	0.0093	0.0090	0.0084	0.0090	0.0085	0.0024	0.0010
17	0.0077	0.0089	0.0091	0.0068	0.0094	0.0084	0.0026	0.0011
18	0.0076	0.0069	0.0062	0.0077	0.0067	0.0070	0.0015	0.0006
19	0.0069	0.0077	0.0073	0.0074	0.0074	0.0073	0.0008	0.0003
20	0.0063	0.0071	0.0078	0.0063	0.0088	0.0073	0.0025	0.0011
					Mean	0.009237	0.004155	0.0016832

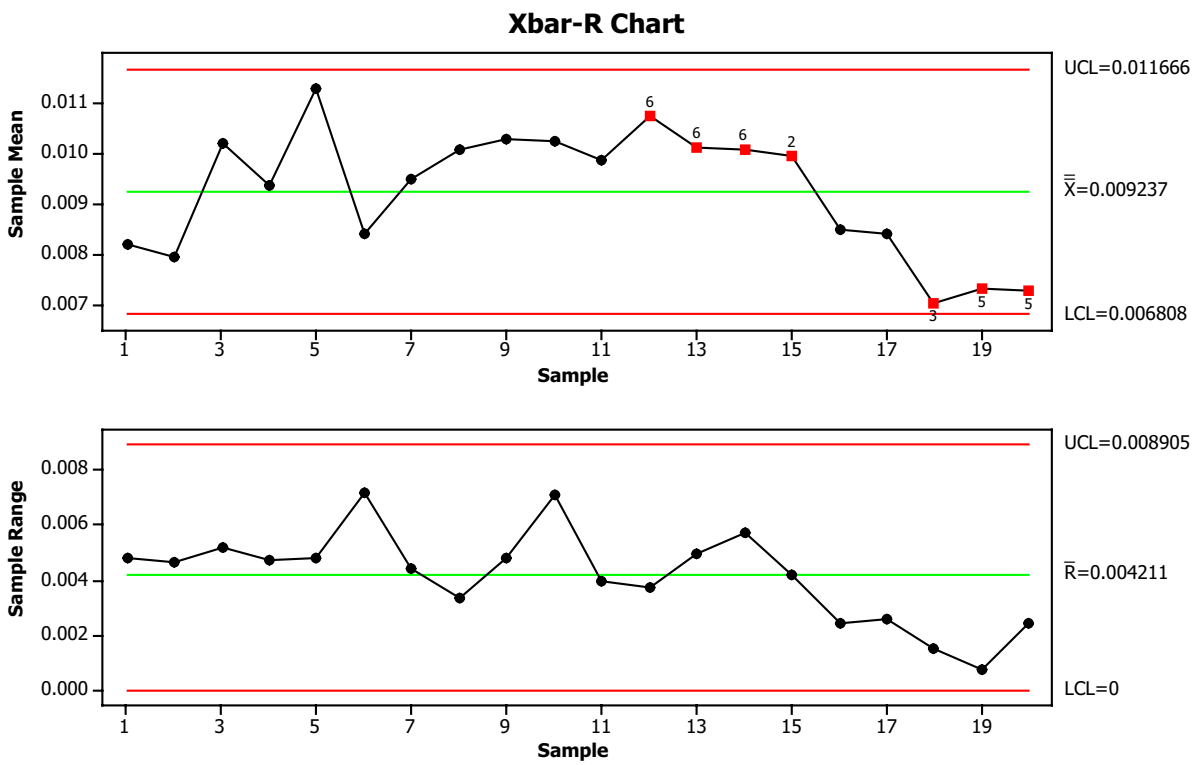


Fig. 2.12 R-chart and x-chart from R. Numerical example. Minitab® Statistical Software

Test Results for Xbar Chart

```
TEST 2. 9 points in a row on same side of center line.
Test Failed at points: 15

TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points: 18

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
     one side of CL).
Test Failed at points:  19; 20

TEST 6. 4 out of 5 points more than 1 standard deviation from center line
(on
     one side of CL).
Test Failed at points:  12; 13; 14; 20
```

Fig. 2.13 x-chart from R, test results. Numerical example. Minitab® Statistical Software

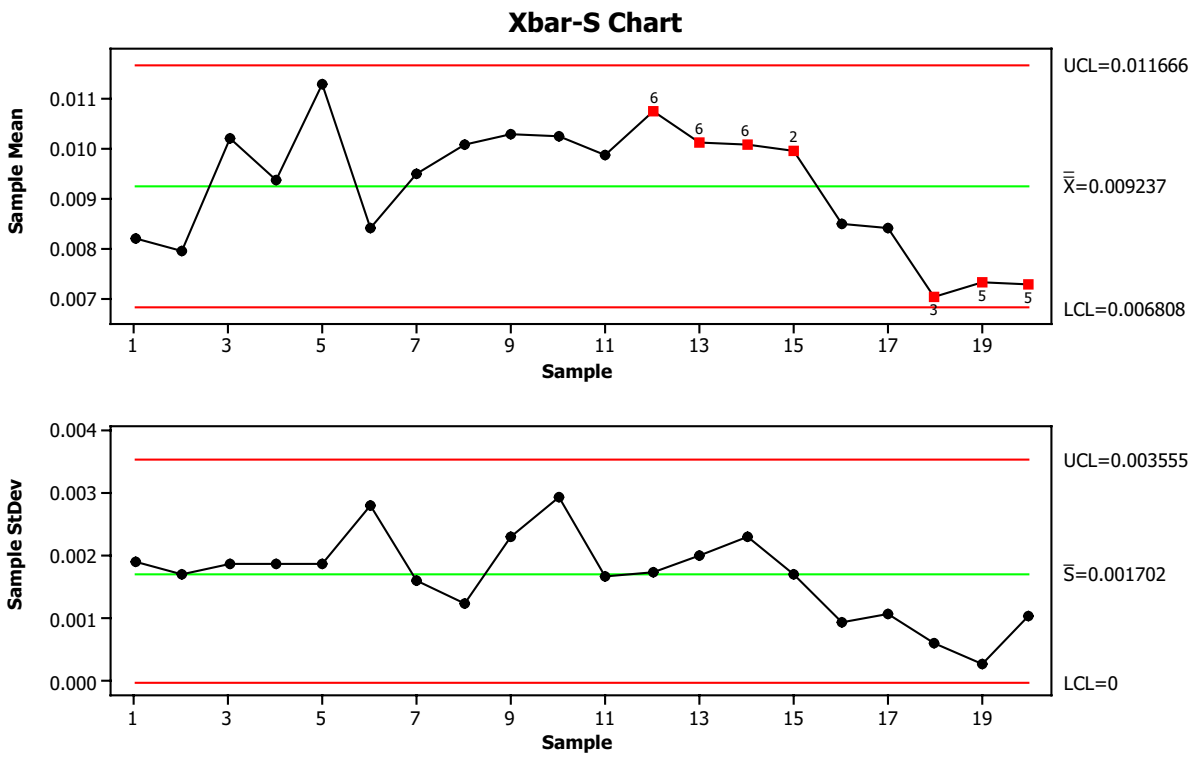


Fig. 2.14 s-chart and x-chart from s. Numerical example. Minitab® Statistical Software

2.8.1 The p-Chart

The p-chart is a control chart for monitoring the proportion of nonconforming items in successive subgroups of size n. An item of a generic subgroup is said to be nonconforming if it possesses a specified characteristic. Given $p_1, p_2, \ldots, p_k$, the subgroups' proportions of nonconforming items, the sampling random variable p_i for the generic sample i has a mean and a standard deviation:

$$\mu_p = \pi,$$
$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}, \tag{2.14}$$

where π is the true proportion of nonconforming items of the process, i. e., the population of items.

The equations in Eq. 2.14 result from the binomial discrete distribution of the variable number of nonconformities x. This distribution function is defined as

$$p(x) = \binom{n}{x} \pi^x (1-\pi)^{n-x}, \tag{2.15}$$

where x is the number of nonconformities and π is the probability the generic item has the attribute.

The mean value of the standard deviation of this discrete random variable is

$$\begin{aligned} \mu &= \sum_x x p(x) = n\pi, \\ \sigma &= \sqrt{\sum_x (x-\mu)^2 p(x)} = n\pi(1-\pi). \end{aligned} \tag{2.16}$$

By the central limit theorem, the centerline, as the estimated value of π, and the control limits of the p-chart are

$$\hat{\mu}_p = \hat{\mu}(p_i) = \bar{p} = \frac{1}{k}\sum_{i=1}^{k} p_i, \tag{2.17}$$

$$\begin{aligned} \mathrm{UCL}_p &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \\ \mathrm{LCL}_p &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}. \end{aligned} \tag{2.18}$$

If the number of items for a subgroup is not constant, the centerline and the control limits are quantified by the following equations:

$$\bar{p} = \frac{x_1 + x_2 + \cdots + x_{k-1} + x_k}{n_1 + n_2 + \cdots + n_{k-1} + n_k}, \tag{2.19}$$

where x_i is the number of nonconforming items in sample i and n_i is the number of items within the subgroup i, and

$$\begin{aligned} \mathrm{UCL}_{p,i} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \\ \mathrm{LCL}_{p,i} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \end{aligned} \tag{2.20}$$

where UCL_i is the UCL for sample i and LCL_i is the LCL for sample i.

Table 2.11 Rejects versus tested items. Numerical example

Day	Rejects	Tested	Day	Rejects	Tested
21/10	32	286	5/11	21	281
22/10	25	304	6/11	14	310
23/10	21	304	7/11	13	313
24/10	23	324	8/11	21	293
25/10	13	289	9/11	23	305
26/10	14	299	10/11	13	317
27/10	15	322	11/11	23	323
28/10	17	316	12/11	15	304
29/10	19	293	13/11	14	304
30/10	21	287	14/11	15	324
31/10	15	307	15/11	19	289
1/11	16	328	16/11	22	299
2/11	21	304	17/11	23	318
3/11	9	296	18/11	24	313
4/11	25	317	19/11	27	302

2.8.2 Numerical Example, p-Chart

Table 2.11 reports the data related to the number of electric parts rejected by a control process considering 30 samples of different size.

By the application of Eqs. 2.19 and 2.20,

$$\begin{aligned} \bar{p} &= \frac{x_1 + x_2 + \cdots + x_{k-1} + x_k}{n_1 + n_2 + \cdots + n_{k-1} + n_k} = \frac{573}{9171} \\ &\cong 0.0625, \end{aligned}$$

$$\begin{aligned} \mathrm{UCL}_{p,i} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \\ &\cong 0.0625 + 3\sqrt{\frac{0.0625(1-0.0625)}{n_i}}, \end{aligned}$$

$$\begin{aligned} \mathrm{LCL}_{p,i} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \\ &\cong 0.0625 - 3\sqrt{\frac{0.0625(1-0.0625)}{n_i}}. \end{aligned}$$

Figure 2.15 presents the p-chart generated by Minitab® Statistical Software and shows that test 1 (one point beyond three standard deviations) occurs for the first sample. This chart also presents the non-continuous trend of the control limits in accordance with the equations in Eq. 2.20.

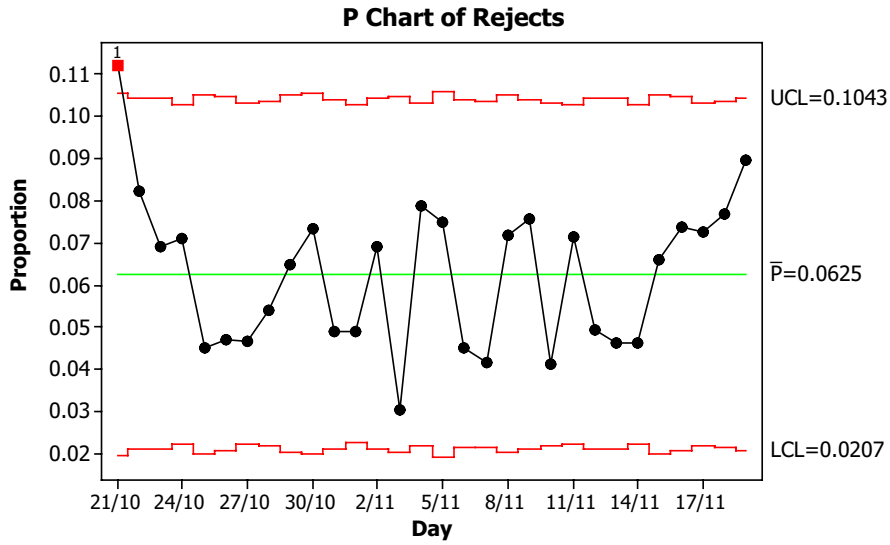


Fig. 2.15 p-chart with unequal sample sizes. Numerical example. Minitab® Statistical Software

2.8.3 The np-Chart

This is a control chart for monitoring the number of nonconforming items in subgroups having the same size. The centerline and control limits are

$$\hat{\mu}_{np} = n\bar{p}, \tag{2.21}$$

$$\begin{aligned} \mathrm{UCL}_{np} &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}, \\ \mathrm{LCL}_{np} &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}. \end{aligned} \tag{2.22}$$

2.8.4 Numerical Example, np-Chart

The data reported in Table 2.12 relate to a production process similar to that illustrated in a previous application, see Sect. 2.8.2. The size of the subgroups is now constant and equal to 280 items. Figure 2.16 presents the np-chart generated by Minitab® Statistical Software: test 1 is verified by two consecutive samples (collected on 12 and 13 November). The analyst has to find the special causes, then he/she must eliminate them and regenerate the chart, as in Fig. 2.17. This second chart presents another anomalous subgroup: 11/11. Similarly, it is necessary to eliminate this sample and regenerate the chart.

Table 2.12 Rejected items. Numerical example

Day	Rejects	Day	Rejects
21/10	19	5/11	21
22/10	24	6/11	14
23/10	21	7/11	13
24/10	23	8/11	21
25/10	13	9/11	23
26/10	32	10/11	13
27/10	15	11/11	34
28/10	17	12/11	35
29/10	19	13/11	36
30/10	21	14/11	15
31/10	15	15/11	19
1/11	16	16/11	22
2/11	21	17/11	23
3/11	12	18/11	24
4/11	25	19/11	27

2.8.5 The c-Chart

The c-chart is a control chart used to track the number of nonconformities in special subgroups, called "inspection units." In general, an item can have any number of nonconformities. This is an inspection unit, as a unit of output sampled and monitored for determination of nonconformities. The classic example is a single printed circuit board. An inspection unit can be a batch, a collection, of items. The monitoring activity of the inspection unit is useful in a continuous pro-

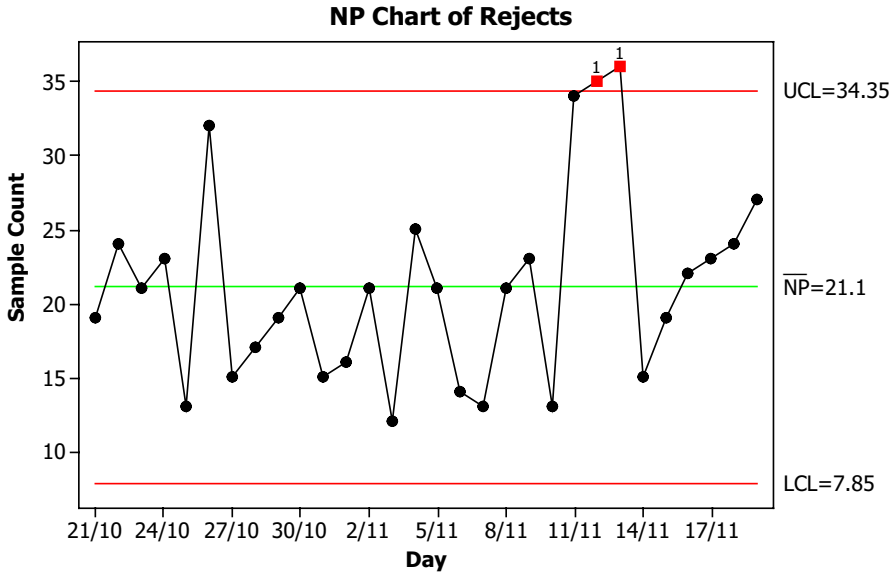


Fig. 2.16 np-chart, equal sample sizes. Numerical example. Minitab® Statistical Software

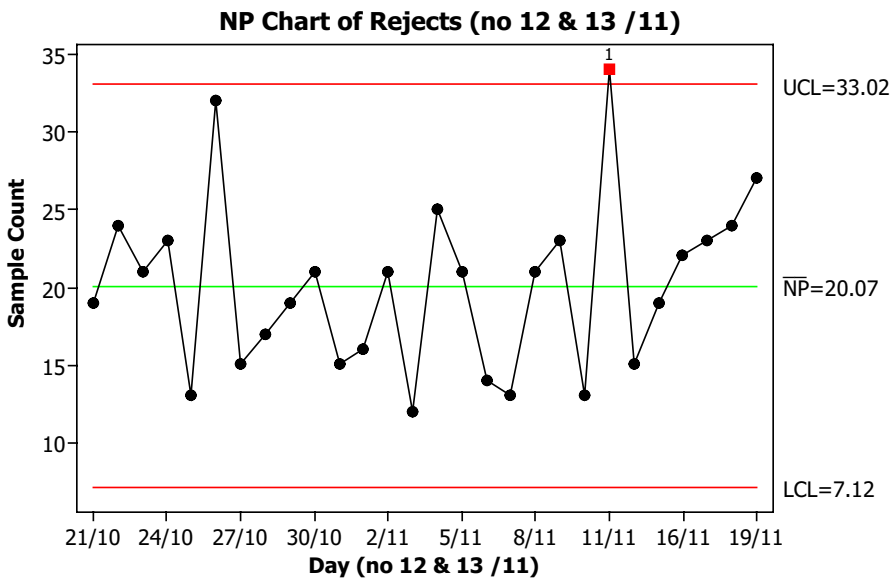


Fig. 2.17 np-chart, equal sample sizes. Numerical example. Minitab® Statistical Software

duction process. The number of nonconformities per inspection unit is called c.

The centerline of the c-chart has the following average value:

$$\hat{\mu}_c = \hat{\mu}(c_i) = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i. \tag{2.23}$$

The control limits are

$$\begin{aligned} \mathrm{UCL}_c &= \bar{c} + 3\sqrt{\bar{c}}, \\ \mathrm{LCL}_c &= \bar{c} - 3\sqrt{\bar{c}}. \end{aligned} \tag{2.24}$$

The mean and the variance of the Poisson distribution, defined for the random variable number of nonconformities units counted in an inspection unit, are

$$\mu\,(c_i) = \sigma(c_i) = \bar{c}. \tag{2.25}$$

The density function of this very important discrete probability distribution is

$$f(x) = \frac{\mathrm{e}^{-\lambda}\lambda^x}{x!}, \tag{2.26}$$

where x is the random variable.

2.8.6 Numerical Example, c-Chart

Table 2.13 reports the number of coding errors made by a typist in a page of 6,000 digits. Figure 2.18 shows the c-chart obtained by the sequence of subgroups and the following reference measures:

$$\bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i = 6.8,$$

$$\mathrm{UCL}_c = \bar{c} + 3\sqrt{\bar{c}} = 6.8 + 3\sqrt{6.8} \cong 14.62,$$

$$\mathrm{LCL}_c = \bar{c} - 3\sqrt{\bar{c}} = \max\{6.8 - 3\sqrt{6.8}, 0\} \cong 0,$$

where c_i is the number of nonconformities in an inspection unit.

From Fig. 2.18 there are no anomalous behaviors suggesting the existence of special causes of variations in the process, thus resulting in a state of statistical control.

A significant remark can be made: why does this numerical example adopt the c-chart and not the p-chart? If a generic digit can be, or cannot be, an object of an error, it is in fact possible to consider a binomial process where the probability of finding a digit with an error is

$$p_i = \frac{c_i}{n} = \frac{c_i}{6{,}000},$$

where n is the number of digits identifying the inspection unit.

The corresponding p-chart, generated by Minitab® Statistical Software and shown in Fig. 2.19, is very similar to the c-chart in Fig. 2.18.

Table 2.13 Errors in inspection unit of 6,000 digits. Numerical example

Day	Errors	Day	Errors
1	10	16	8
2	11	17	7
3	6	18	1
4	9	19	2
5	12	20	3
6	12	21	5
7	14	22	1
8	9	23	11
9	5	24	9
10	0	25	14
11	1	26	1
12	2	27	9
13	1	28	1
14	11	29	8
15	9	30	12

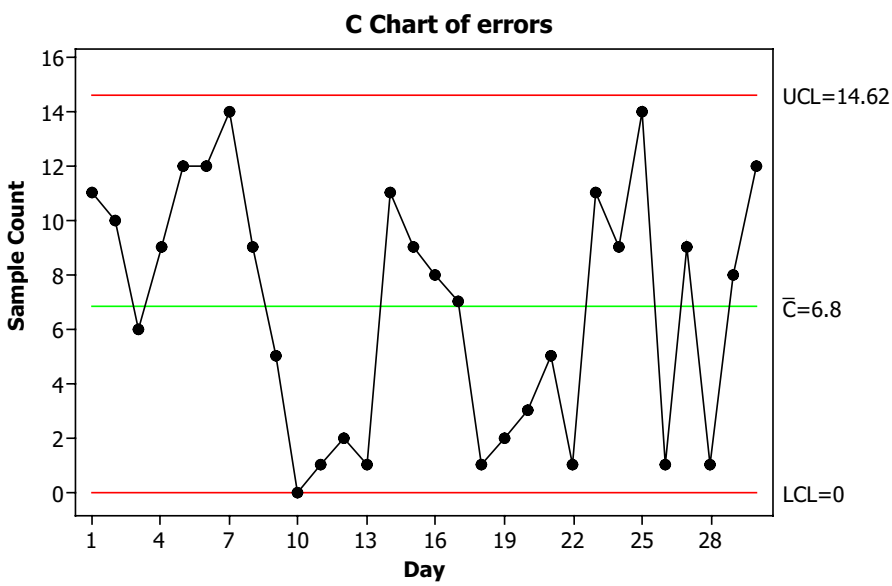


Fig. 2.18 c-chart. Inspection unit equal to 6,000 digits. Numerical example. Minitab® Statistical Software

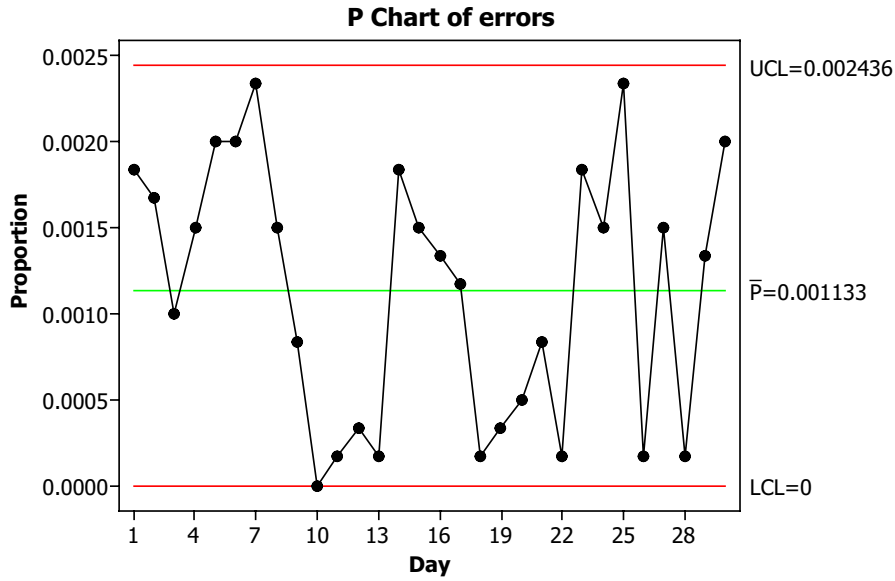


Fig. 2.19 p-chart. Inspection unit equal to 6,000 digits. Numerical example. Minitab® Statistical Software

2.8.7 The u-Chart

If the subgroup does not coincide with the inspection unit and subgroups are made of different numbers of inspection units, the number of nonconformities per unit, u_i, is

$$u_i = \frac{c_i}{n}. \tag{2.27}$$

The centerline and the control limits of the so-called u-chart are

$$\begin{aligned} \hat{\mu}_u = \hat{\mu}(u_i) = \bar{u} &= \frac{1}{k}\sum_{i=1}^{k} u_i, \\ \mathrm{UCL}_{u,i} &= \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}, \\ \mathrm{LCL}_{u,i} &= \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}. \end{aligned} \tag{2.28}$$

2.8.8 Numerical Example, u-Chart

Table 2.14 reports the number of nonconformities as defects on ceramic tiles of different sizes, expressed in feet squared.

Figure 2.20 presents the u-chart obtained; five different subgroups reveal themselves as anomalous. Figure 2.21 shows the chart obtained by the elimination of those samples. A new sample, $i = 30$, is "irregular."

2.9 Capability Analysis

A production process is said to be capable when it is in state of statistical control and products meet the specification limits, i. e., the customers' requirements. In other words, the process is capable when it produces "good" products. This is the first time the lower and upper specifications are explicitly considered in the analysis of the process variations.

Nonconformity rates are the proportions of process measurements above, or below, the USL, or LSL. This proportion can be quantified in parts per million (PPM), as

$$\mathrm{PPM} > \mathrm{USL} = P(x > \mathrm{USL}) \approx P\left(z > \frac{\mathrm{USL} - \hat{\mu}}{\hat{\sigma}}\right), \tag{2.29}$$

$$\mathrm{PPM} < \mathrm{LSL} = P(x < \mathrm{LSL}) \approx P\left(z < \frac{\mathrm{LSL} - \hat{\mu}}{\hat{\sigma}}\right), \tag{2.30}$$

where x is a normal random variable and z is a standard normal variable (see Appendix A.1).

Table 2.14 Errors/defects in ceramic tiles. Numerical example

Sample i	c_i [nonconform. number]	Size [ft^2]	u_i	Sample i	c_i [nonconform. number]	Size [ft^2]	u_i
1	14	7.1	1.972	16	25	9.8	2.551
2	47	3.3	14.242	17	32	8.8	3.636
3	21	5.9	3.559	18	41	7.1	5.775
4	6	5.2	1.154	19	13	3.3	3.939
5	16	5.6	2.857	20	0	6.8	0.000
6	27	8	3.375	21	14	4.4	3.182
7	21	8.9	2.360	22	16	5.6	2.857
8	22	5.6	3.929	23	17	8	2.125
9	43	6.1	7.049	24	18	8.9	2.022
10	17	4.2	4.048	25	26	5.3	4.906
11	32	8.4	3.810	26	14	3.1	4.516
12	14	6.8	2.059	27	23	6.2	3.710
13	9	4.4	2.045	28	35	4.8	7.292
14	16	5.2	3.077	29	42	13.5	3.111
15	19	7.8	2.436	30	31	5.9	5.254

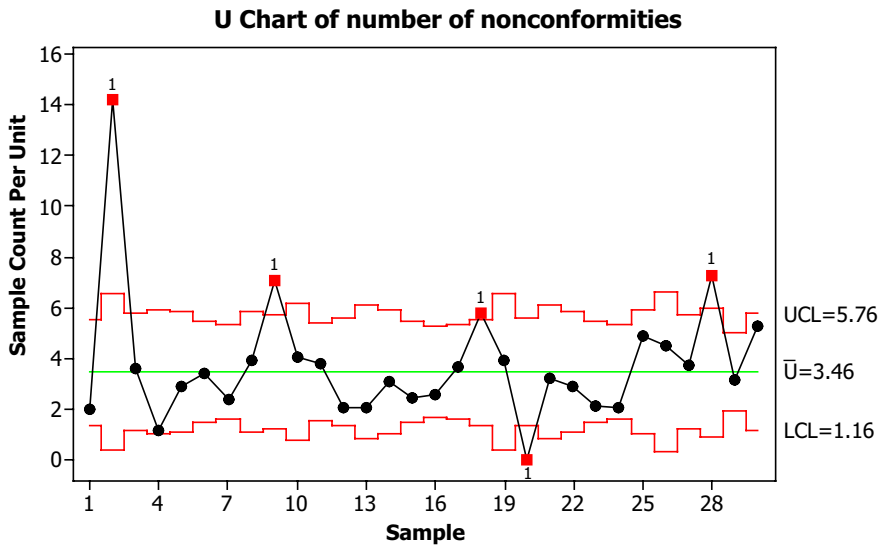


Fig. 2.20 u-chart, tile industry numerical example – chart 1. Minitab® Statistical Software

Consequently, by the application of the central limit theorem, Eqs. 2.29 and 2.30 can be applied to the mean value of the random variable x, $\bar{x}$, assuming the generic statistical probability density function when the size n of the generic sample is over a threshold and critical value.

From Eqs. 2.29 and 2.30 it is necessary to estimate μ and σ, i. e., quantify $\hat{\mu}$ and $\hat{\sigma}$. In particular, in the presence of a normal distribution of values x, in order to quantify $\hat{\sigma}$ it can be useful to use Eq. 2.7 or 2.12.

In general, for a generic statistical distribution of the random variable, i. e., the process characteristic x, there are two different kinds of standard deviations, called "within" and "overall": the first relates to the within-subgroup variation, while the second relates to the between-subgroup variation. In particular, the "overall" standard deviation is a standard deviation of all the measurements and it is an estimate of the overall process variation, while the "within" standard deviation is a measure of the variations of the items within the same group.

In a "in control" process these standard deviation measures are very close to each other. In the following, an in-depth illustration of the statistical models

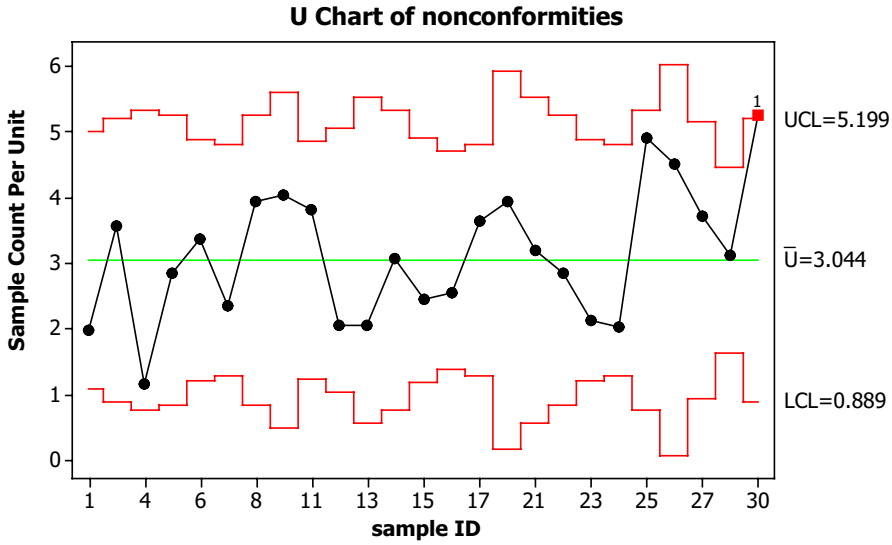


Fig. 2.21 u-chart, tile industry numerical example – chart 2. Minitab® Statistical Software

related to capability analysis is substituted by a few significant numerical examples created with the support of a statistical tool such as Minitab® Statistical Software. For this purpose, it is necessary to introduce the following process capability indexes, specifically designed for normally distributed data, i. e., measurements:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}, \tag{2.31}$$

$$C_{PU} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}}, \tag{2.32}$$

$$C_{PL} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} \tag{2.33}$$

$$C_{pk} = \min\left[\frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}}; \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}}\right]. \tag{2.34}$$

When $C_p < 1$ the process is said to be "noncapable," otherwise it is "capable" because the quality control variability, represented by 6σ, can be included by the specification limits LSL and USL, i. e., the production process can meet the customer requirements. The 6σ variation is also called "process spread," while USL-LSL is called "specification spread." A capable process is able to produce products or services that meet specifications. Nevertheless, this index measures the capability only from a potential point of view, because C_p does not tell us if the range of values $\pm 3\sigma$ above and below the mean value, called "centerline" in the control charts, is really included in the specification range, i. e., in other words it does not tell the analyst if the process is centered on the target value. For this purpose, the index C_{pk} is preferable to C_p because, if we assume values greater than 1, it guarantees the process is centered on the target value, thus telling the analyst what capability the process could achieve if centered, while C_p does not consider the location of the process mean.

Finally, the C_{PU} and C_{PL} indexes relate the process spread, the 3σ variation, to a single-sided specification spread: $\hat{\mu}$-LSL or USL-$\hat{\mu}$, respectively.

A conventionally accepted minimum value for these indexes is 1.33, corresponding to the so-called four sigma production process, as defined in Sect. 2.9.

The performance of an in-control process is predictable. Therefore, the capability analysis following the "in-control analysis" can assess the ability of the production process to produce units that are "in spec" and predict the number of parts "out-of-spec."

2.9.1 Numerical Example, Capability Analysis and Normal Probability

Table 2.15 reports the measurements, in millimeters, obtained on 100 products produced by a manufacturing process of cutting metal bars when the expected

Table 2.15 Measurement data – process 1, numerical example

Sample	Data – process 1					Mean value	Range
1	600.3333	600.8494	600.693	599.2493	600.6724	600.35948	1.6001
2	600.2929	598.789	599.8655	599.3179	599.4127	599.5356	1.5039
3	599.8586	599.706	599.8773	600.8859	600.3385	600.13326	1.1799
4	599.2491	599.537	599.848	600.0593	599.2632	599.59132	0.8102
5	600.4454	599.9179	599.5341	600.3004	598.8681	599.81318	1.5773
6	599.4055	599.5074	599.5099	599.9597	599.2939	599.53528	0.6658
7	600.1634	599.5934	599.9918	600.2792	599.41	599.88756	0.8692
8	600.3021	600.3307	600.6115	599.0412	599.4191	599.94092	1.5703
9	600.1666	599.8434	600.612	600.7174	599.9917	600.26622	0.874
10	600.9336	600.5842	599.7249	599.5842	599.8445	600.13428	1.3494
11	600.3714	601.2756	599.7404	601.0146	600.3568	600.55176	1.5352
12	599.7379	601.112	600.5713	600.287	599.922	600.32604	1.3741
13	599.797	599.9101	599.1727	600.8716	600.1579	599.98186	1.6989
14	600.2411	599.643	599.6155	600.2896	598.6065	599.67914	1.6831
15	599.4932	599.6578	599.9164	600.6215	599.3805	599.81388	1.241
16	600.6162	599.3922	600.6494	599.6583	599.216	599.90642	1.4334
17	599.1419	599.8016	600.4682	599.3786	600.4624	599.85054	1.3263
18	600.5005	599.3184	599.424	600.7875	600.2031	600.0467	1.4691
19	600.7689	599.1993	599.8779	600.7521	599.9077	600.10118	1.5696
20	599.9661	598.7038	600.4608	599.3556	601.4034	599.97794	2.6996
					Average	599.971628	1.40152

values of the target and specification limits are 600, 601, and 599 mm. Consequently, the tolerances are ± 1 mm. First of all, it is useful to conduct the variability analysis by generating the control chart: Figure 2.22 reports the x-chart based on the s-chart. There are no anomalous behaviors of the sequence of subgroups.

It is now possible to quantify the capability indexes and the nonconformity rates by adopting both the overall and the within standard deviations. Figure 2.23 is a report generated by Minitab® Statistical Software for the analysis of the capability of the production process.

The C_p value obtained is 0.55, i. e., the process is not potentially capable, both considering the within capability analysis and the overall capability analysis. Figure 2.23 quantifies also the PPM over and under the specifications by Eqs. 2.29 and 2.30, distinguishing:

- "Observed performance." They are related to the observed frequency distribution of data (see the histogram in Fig. 2.23).
- "Expected within performance."[2] They relate to the parametric distribution, and in particular to the normal distribution, obtained by a best-fitting statistical evaluation conducted with the within standard deviation.
- "Expected overall performance." They relate to the parametric distribution obtained by a best-fitting evaluation conducted with the overall standard deviation.

In particular, the maximum expected value of PPM is about 96,620.

The so-called six-pack capability analysis, illustrated in Fig. 2.24, summarizes the main results presented in Figs. 2.22 and 2.23 and concerning the variability of the process analyzed. The normal probability plot verifies that data are distributed as a normal density function: for this purpose the Anderson–Darling index and the P value are properly quantified. Similarly to the s-chart reported in Fig. 2.22, the R-chart is proposed to support the generation of the x-chart. The standard deviations and capability indexes are hence quantified both in "overall" and "within" hypotheses. Finally, the so-called capability plot illustrates and compares the previously defined process spread and specification spread.

The analyst decides to improve the performance of the production process in order to meet the customer specifications and to minimize the process variations.

[2] Minitab® Statistical Software calls the performance indices P_p and P_{pk} in the "overall capability" analysis to distinguish them from C_p and C_{pk} defined by Eqs. 2.31–2.34 for the "within analysis" (see Fig. 2.23).

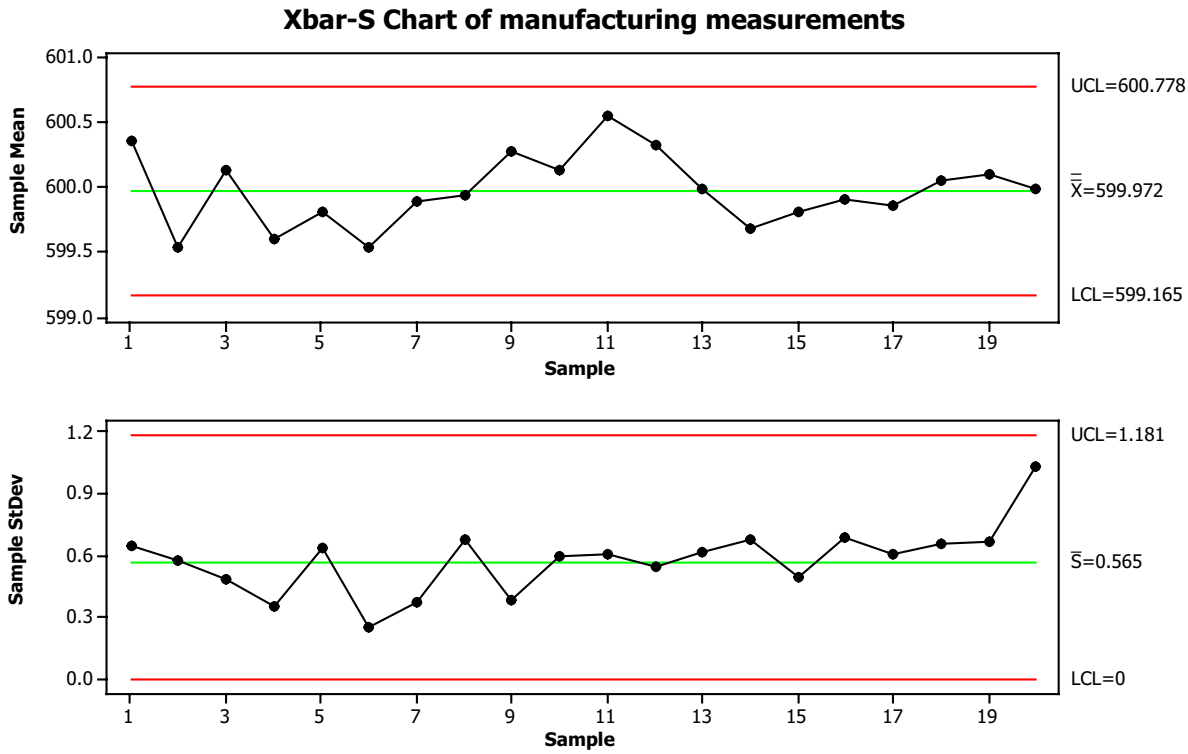


Fig. 2.22 x-chart and s-chart – process 1, numerical example. Minitab® Statistical Software

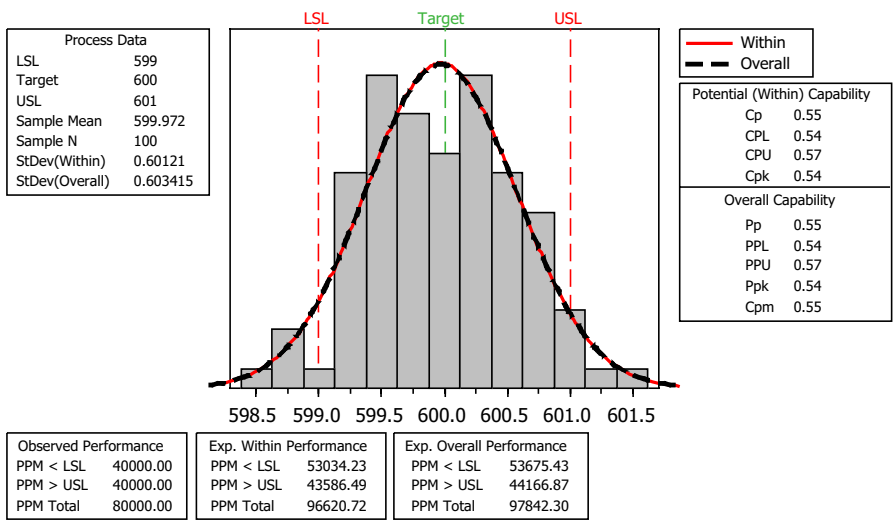


Fig. 2.23 Capability analysis – process 1, numerical example. Minitab® Statistical Software

Table 2.16 reports the process data as a result of the process improvement made for a new set of $k = 20$ samples with $n = 5$ measurements each. Figure 2.25 presents the report generated by the six-pack analysis. It demonstrates that the process is still in statistical control, centered on the target value, 600 mm, and with a C_{pk} value equal to 3.31. Consequently, the negligible expected number of PPM outside the specification

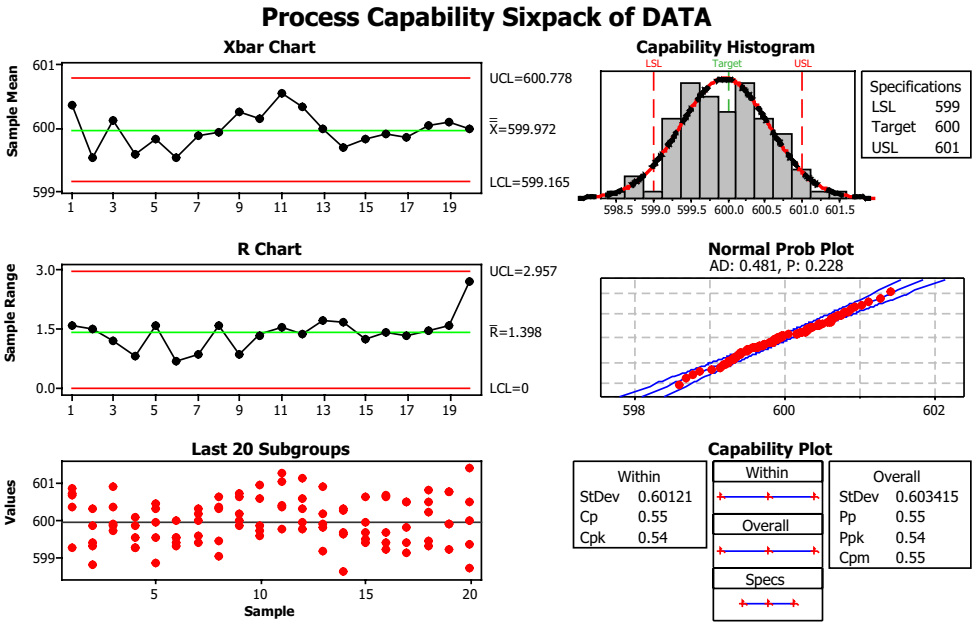


Fig. 2.24 Six-pack analysis – process 1, numerical example. Minitab® Statistical Software

Table 2.16 Measurement data – process 2, numerical example

Sample	Data – process 2					Mean value	Range
2.1	600.041	600.0938	600.1039	600.0911	600.1096	600.08788	0.0686
2.2	599.8219	599.9173	600.0308	600.07	600.0732	599.98264	0.2513
2.3	600.0089	600.075	600.0148	599.9714	600.0271	600.01944	0.1036
2.4	600.1896	600.1723	599.8368	600.0947	599.9781	600.0543	0.3528
2.5	600.1819	600.0538	599.9957	600.0995	599.9639	600.05896	0.218
2.6	599.675	599.9778	599.9633	599.9895	599.8853	599.89818	0.3145
2.7	600.0521	600.1707	599.9446	599.8487	600.012	600.00562	0.322
2.8	600.0002	600.0831	599.9298	599.9329	599.9142	599.97204	0.1689
2.9	600.02	599.9963	599.9278	599.9793	600.0456	599.9938	0.1178
2.10	600.1571	600.0212	599.9061	599.9786	600.0626	600.02512	0.251
2.11	600.0934	599.9554	599.7975	600.0221	599.8821	599.9501	0.2959
2.12	599.8668	599.8757	600.0414	599.7939	600.1153	599.93862	0.3214
2.13	599.9859	599.9269	599.8124	600.0288	600.0261	599.95602	0.2164
2.14	599.9456	600.0405	600.0576	599.7819	600.0603	599.97718	0.2784
2.15	600.0487	600.0569	599.9321	599.9164	599.9984	599.9905	0.1405
2.16	599.8959	599.979	600.1418	600.1157	599.9525	600.01698	0.2459
2.17	600.1891	600.1168	600.1106	599.9148	600.0013	600.06652	0.2743
2.18	600.0002	600.1121	599.93	599.9924	600.0458	600.0161	0.1821
2.19	599.9228	600.092	599.9225	600.1062	600.1794	600.04458	0.2569
2.20	599.7843	599.9597	600.011	600.0409	600.0436	599.9679	0.2593
					Average	600.001124	0.23198

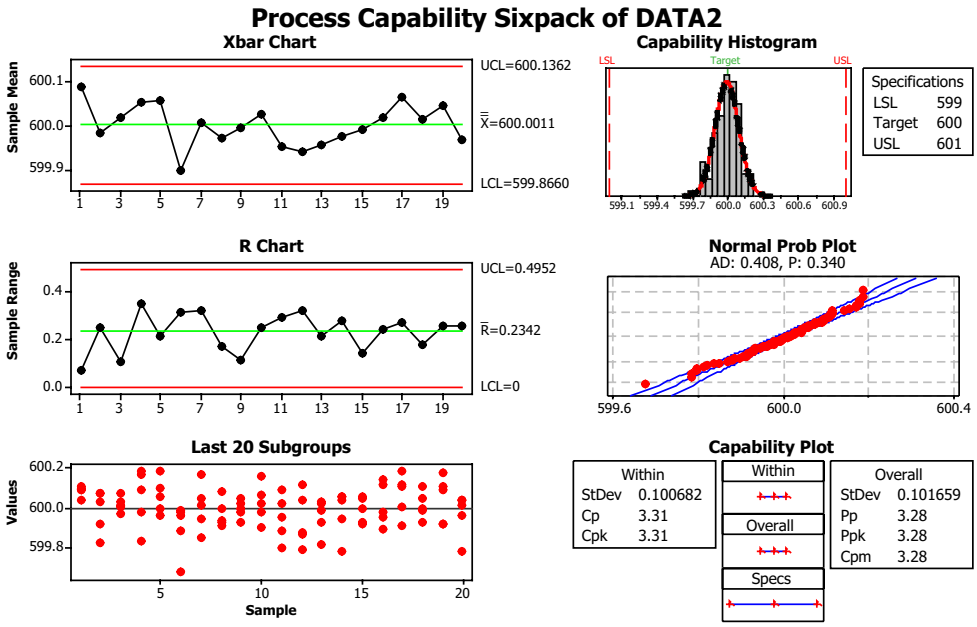


Fig. 2.25 Six-pack analysis – process 2, numerical example. Minitab® Statistical Software

limits is quantified as

$$\text{Total PPM} = \left| P\left(z > \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right) + P\left(z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right) \right|_{\substack{\hat{\sigma}=0.101659 \\ \hat{\mu}=\bar{\bar{x}}=600.0011}} \cong 0.$$

2.9.2 Numerical Examples, Capability Analysis and Nonnormal Probability

These numerical examples refer to data nondistributed in accordance with a normal density function. Consequently, different parametric statistical functions have to be adopted.

Table 2.17 Measurement data (mm/10), nonnormal distribution. Numerical example

Sample	Measurement data									
1	1.246057	0.493869	2.662834	5.917727	3.020594	3.233249	0.890597	1.107955	1.732582	2.963924
2	0.432057	1.573958	2.361707	0.178515	1.945173	3.891315	2.222251	3.295799	2.521666	2.398454
3	3.289106	4.26632	3.597959	1.511217	3.783617	0.323979	5.367135	0.429597	2.179387	1.945532
4	4.740917	1.38156	1.618083	5.597763	3.05798	2.404994	1.409824	1.266203	3.864219	0.735855
5	1.03499	6.639968	6.071461	1.552255	0.151038	1.659891	3.580737	6.482635	2.282011	3.062937
6	4.864409	1.546174	3.875799	1.098431	5.50208	1.281942	0.921708	4.884044	3.054542	3.225921
7	3.045406	3.160609	2.901201	6.760744	6.04942	1.39276	3.495365	2.494509	3.865445	1.390489
8	0.936205	0.940518	3.15243	4.550744	1.732531	5.629206	0.397718	6.539783	4.46137	2.886115
9	4.55721	1.902965	4.462141	3.509317	1.995514	4.803485	1.95335	2.53267	4.884973	0.882012
10	5.635049	1.851431	5.076608	1.630322	2.673297	0.777941	7.998625	0.864797	5.338903	6.03149
11	4.693689	1.903728	6.866619	3.064651	0.565978	2.093118	5.058873	4.96973	4.40998	1.459153
12	1.063906	0.821599	1.658612	5.847757	4.024718	3.41589	2.196106	2.153251	1.59855	3.074742
13	2.902382	2.769513	4.439952	0.912794	3.192323	0.774273	3.936241	2.605119	6.360237	5.220038
14	4.24421	4.099892	0.813895	4.460482	3.007995	3.84575	3.755018	3.018857	2.535924	3.867536
15	1.667182	0.717635	1.420329	2.365193	2.011729	4.629	1.934723	1.844031	6.976545	1.01383

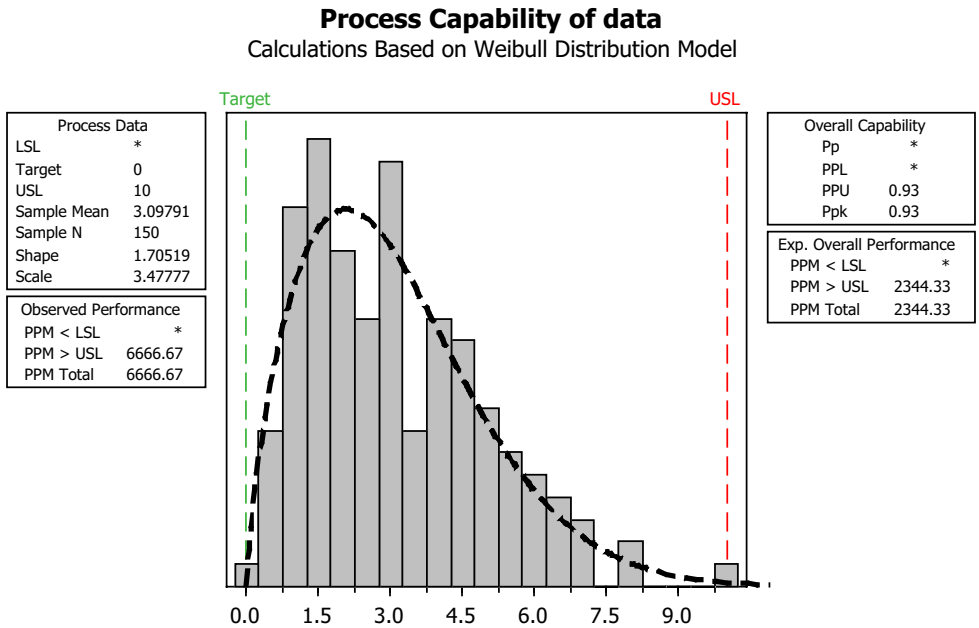


Fig. 2.26 Capability analysis – Weibull distribution, numerical example. Minitab® Statistical Software

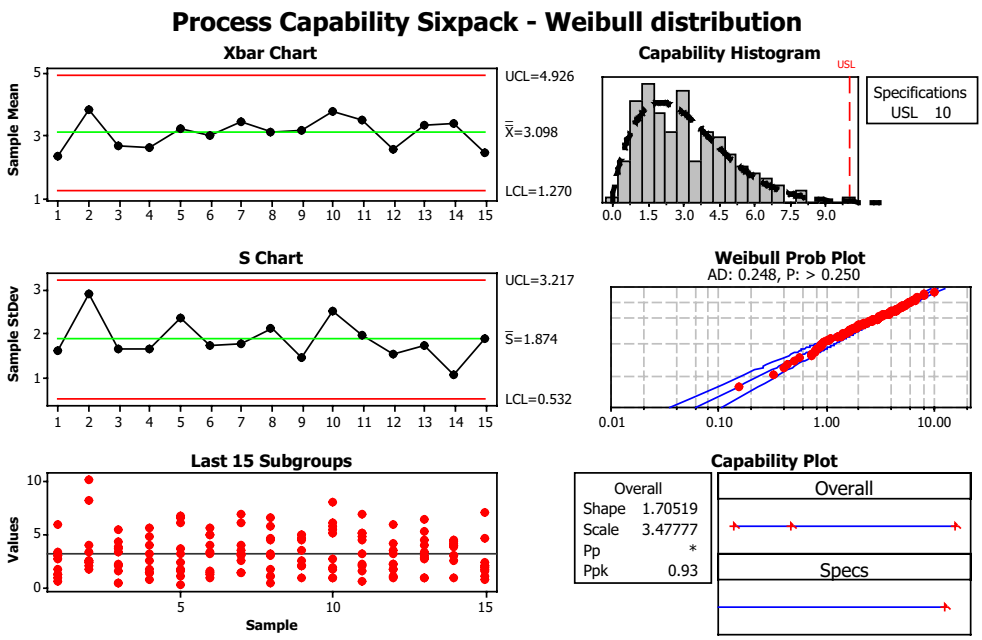


Fig. 2.27 Six-pack analysis – Weibull distribution, numerical example. Minitab® Statistical Software

2.9.2.1 Weibull Distribution

Table 2.17 reports data regarding the output of manufacturing process of tile production in the ceramics industry. This measurement refers to the planarity of the tile surface as the maximum vertical distance of couples of two generic points on the surface, assuming as the USL a maximum admissible value of 1 mm.

Figures 2.26 and 2.27 present the report generated by Minitab® Statistical Software for the capability analysis. The production process generates products, i. e., output, that are "well fitted" by a Weibull statisti-

cal distribution, shape parameter $\beta = 1.71$ and scale parameter $\sigma = 3.48$. The process is therefore "in statistical control" but it does not meet customer requirements in terms of an admissible USL. In other words, the process is "predictable" but "not capable." In particular, the number of expected items over the USL is about 6,667 PPM.

2.9.2.2 Binomial Distribution

This application deals with a call center. Table 2.18 reports the number of calls received in 1 h, between 3 and 4 p.m., and the number of calls that were not answered by the operators. The measurement data can be modeled by assuming a binomial distribution of values. Figure 2.28 presents the results of the capability analysis conducted on this set of values, called "data set 1." The process is not in statistical control because sample 15 is over the UCL. As a consequence, it is not correct to quantify the production process capability. This figure nevertheless shows that the process is difficultly capable, also in the absence of sample 15. In order to meet the demand of customers properly it is useful to increase the number of operators in the call center.

Table 2.18 Number of calls and "no answer", numerical example

Sample	No answer	Calls	Sample	No answer	Calls
day 1	421	1935	day 11	410	1937
day 2	392	1945	day 12	386	1838
day 3	456	1934	day 13	436	2025
day 4	436	1888	day 14	424	1888
day 5	446	1894	day 15	497	1894
day 6	429	1941	day 16	459	1941
day 7	470	1868	day 17	433	1868
day 8	455	1894	day 18	424	1894
day 9	427	1938	day 19	425	1933
day 10	424	1854	day 20	441	1862

2.10 Six Sigma

"Six Sigma" stands for six standard deviations and can be defined as a business management strategy, originally developed by Motorola, that enjoys widespread application in many sectors of industry and services. Six Sigma was originally developed as a set of practices designed to improve manufacturing processes and eliminate defects. This chapter presents a synthetic recall of the basic purpose of Six Sigma, assuming that a large number of the models and methods illustrated here and in the following can properly

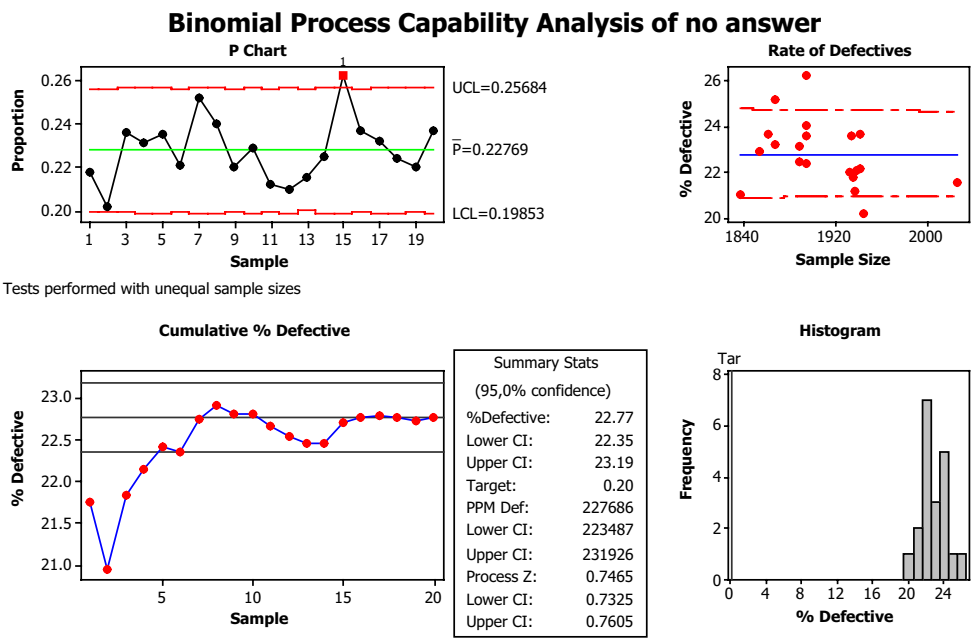


Fig. 2.28 Binomial process capability, numerical example. Minitab® Statistical Software

support it. Nevertheless, there are a lot of ad hoc tools and models specifically designed by the theorists and practitioners of this decisional and systematic approach, as properly illustrated in the survey by Black and Hunter (2003).

Six Sigma is a standard and represents a measure of variability and repeatability in a production process. In particular, the 6σ specifications, also known as Six Sigma capabilities, ask a process variability to be capable of producing a very high proportion of output within specification. The "process spread" has to be included twice in the "specification spread" and centered on the target value.

Figure 2.29 presents the results generated by a process capability conducted on an "in control" process in accordance with the Six Sigma philosophy. Configuration c identifies a capable process, as previously defined, whose variability meets the Six Sigma requirements. In other words, in a Six Sigma process there is a number of defects lower than two parts per billion,

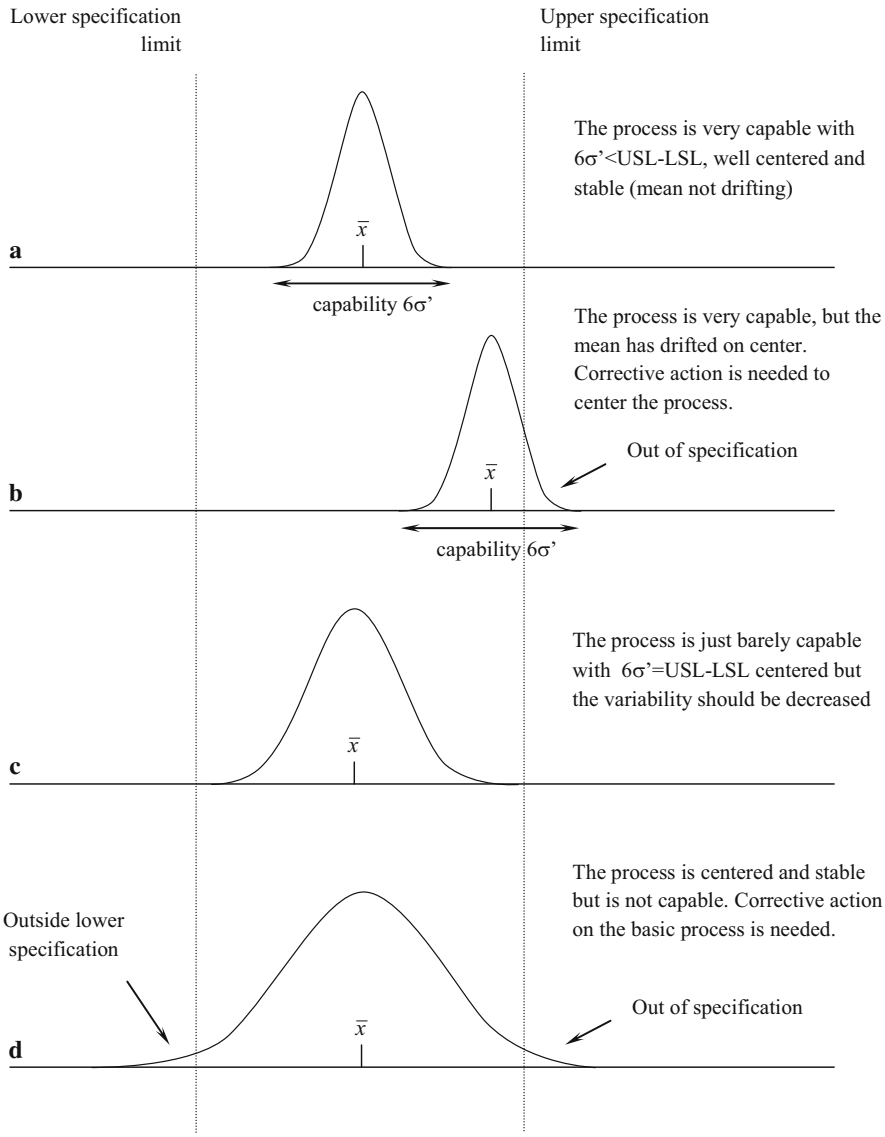


Fig. 2.29 Process capability and Six Sigma

i. e., 0.002 PPM:

$$1 - \int_{-6\sigma}^{+6\sigma} f(x)\,\mathrm{d}x = 2[1 - \Phi(z = 6)] \cong 0.00000000198024 \cong 2 \times 10^{-9}, \quad (2.35)$$

where σ is the standard deviation of the process, $f(x)$ is the density function of the variable x, a measure of the output of the process (process characteristic) – x is assumed to be normally distributed – and Φ is a cumulative function of the standard normal distribution.

Figure 2.30, proposed by Black and Hunter (2003), compares the performance of a capable process with $C_p = C_{pk} = 1.33$, known also as "four sigma capability," and a process with $C_p = C_{pk} = 2$, which guarantees "Six Sigma capability."

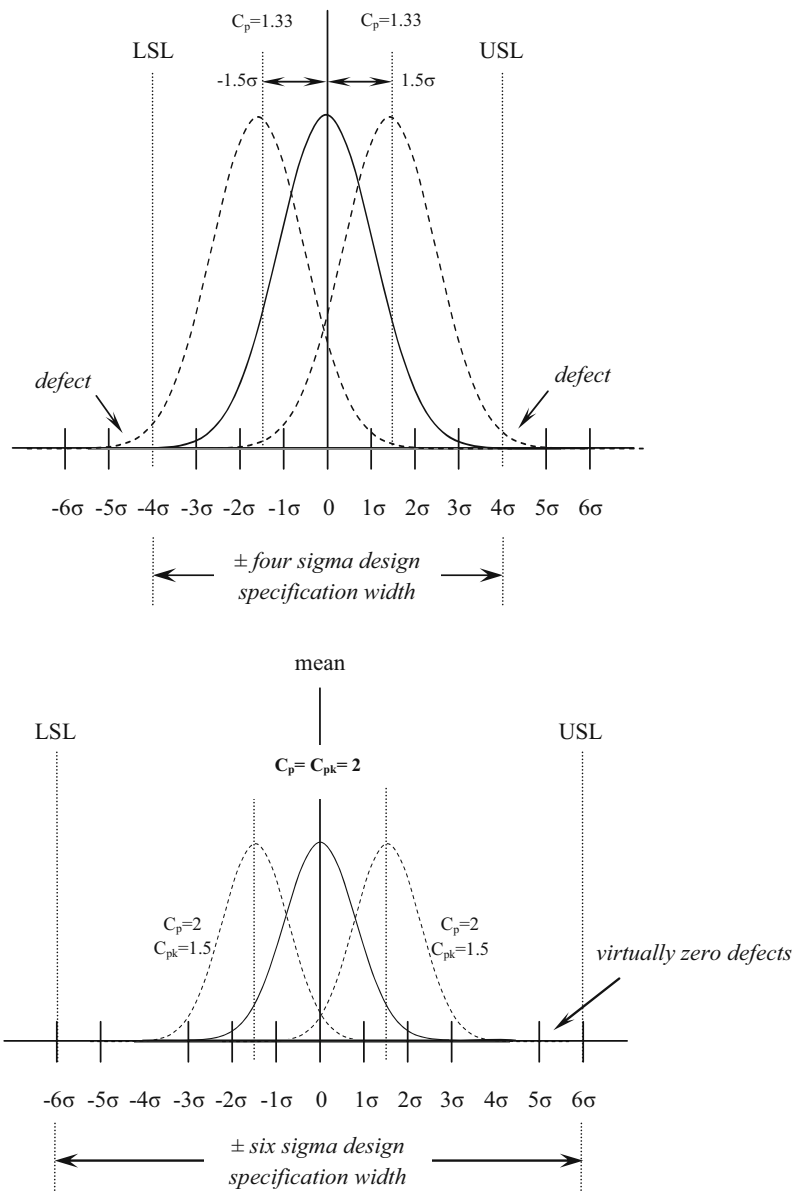


Fig. 2.30 Four sigma ($C_p = C_{pk} = 1.33$) versus Six Sigma ($C_p = C_{pk} = 2$)

2.10.1 Numerical Examples

Among the previously illustrated numerical examples only the one discussed in Sect. 2.9.1 (process 2) verifies the Six Sigma hypotheses, because $C_p = C_{pk} = 3.31$.

2.10.2 Six Sigma in the Service Sector. Thermal Water Treatments for Health and Fitness

In this subsection we present the results obtained by the application of the Six Sigma philosophy to the health service sector of thermal water treatments. This instance demonstrates how this methodological approach is effective also for the optimization of service processes. In particular, in this case study several health and fitness treatments are offered and they are grouped in three divisions, each with a proper booking office and dedicated employees: hotel, wellness, and thermal services.

Employees are nominated to have contact with the costumers, to identify their requirements, to accept the requests, and to finalize the booking process. Customers can have contact via telephone, e-mail, Web site, or by presenting themselves at the reception. Every kind of service has its own booking procedure, depending on the customer request. Before the application of Six Sigma methodologies the process was divided into the following five subroutines, depending on the service:

- single thermal booking;
- group thermal booking;
- single hotel booking;
- tour operator hotel booking;
- wellness booking.

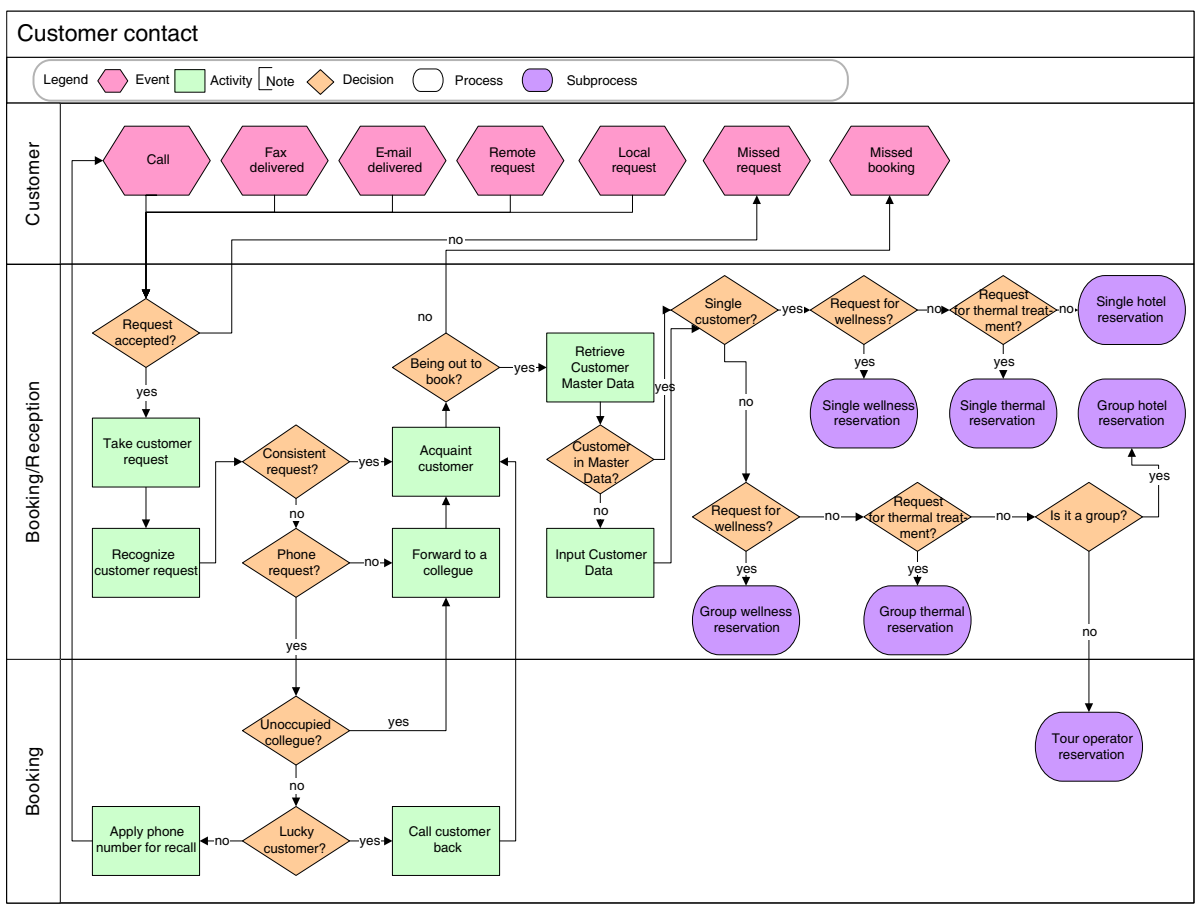


Fig. 2.31 Booking procedure

Once the booking procedure has been completed the staff will wait for the customer. On his/her arrival, the related booking data are recalled from the system and the customer is sent to the so-called "welcome process," which is common to hotel, wellness, and thermal services. By the next check-in stage the customer is accepted and can access the required service. There is a specific check-in stage with its own dedicated rules and procedures for every kind of service. Once the customer has enjoyed the service, he/she will leave the system and go to the checkout stage, with its own procedures too.

The whole process, from the admittance to the exit of the customer, can be displayed as a flowchart; Fig. 2.31 exemplifies the detail of the booking procedure.

The analysis of the whole process has emphasized the existence of significant improvement margins, related to costs and time. For example, a particular service, e. g., thermal mud, may need a medical visit before the customer is allowed to access the treatment. By the Six Sigma analysis it was possible to reduce the lead time of the customer during the visit, through the optimization of the work tasks and processes. Sometimes this can be performed by very simple tricks and expedients.

For example, the aural test can be invalidated by the presence of a plug of ear wax in a patient. Teaching the technician how to recognize and remove this obstruction reduces the probability of null tests, and consequently there is a reduction of costs and lead times.