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Douglas H. Beck, Dieter Haidt, John F. Negele

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Hadronic parity violation and effective field theory

B.R. Holstein[a]

Department of Physics-LGRT, University of Massachusetts, Amherst, MA 01003, USA

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Abstract. The history and phenomenology of hadronic parity violation is reviewed and a new model-independent approach based on effective field theory is developed. Possible future developments are discussed.

PACS. 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 21.30.Fe Forces in hadronic systems and effective interactions – 25.20.-x Photonuclear reactions – 25.40.Lw Radiative capture

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1 Introduction

The strong parity-conserving nucleon-nucleon interaction has, of course, been well studied since the beginning of quantum mechanics. Indeed, the cornerstone of traditional nuclear physics is the study of the nuclear force and, over the years, phenomenological forms of the nuclear potential have become increasingly sophisticated. In the nucleon-nucleon (NN) system, where data abound, the present state of the art is indicated, for example, by phenomenological potentials such as AV18 that are able to fit phase shifts in the energy region from threshold to 350 MeV in terms of ~ 40 parameters. Great progress has also been made in the description of few-nucleon systems [1].

At the same time, in recent years a new technique —effective field theory (EFT)— has been used in order to attack this problem using the symmetries of QCD [2]. In this approach the nuclear interaction is separated into long- and short-distance components. In its original formulation [3], designed for processes with typical momenta comparable to the pion mass —$Q \sim m_\pi$— the long-distance component is described fully quantum mechanically in terms of pion exchange, while the short-distance piece is described in terms of a small number of phenomenologically determined contact couplings. The resulting potential [4,5] is approaching [6,7] the degree of accuracy of purely phenomenological potentials. Even higher precision can be achieved at lower momenta —$Q \ll m_\pi$— where all interactions can be taken as short-ranged, as has been demonstrated not only in the NN system [8,9], but also in the three-nucleon system [10,11]. Precise —$\sim 1\%$— values have been generated for low-energy, astrophysically important cross-sections for reactions such as $n + p \to d + \gamma$ [12] and $p + p \to d + e^+ + \nu_e$ [13]. However, besides providing reliable values for such quantities, the use of EFT techniques allows for a realistic estimation of the size of possible corrections.

Because of the presence of the weak interactions, there exists, of course, in addition to the parity-conserving strong force, a parity-*violating* NN interaction, the study of which began in 1957 with an experiment by Tanner seeking (but not finding) parity violation in the $^{19}\mathrm{F}(p,\alpha)^{16}\mathrm{O}$ reaction [14]. Since that time there have been numerous additional experiments involving both nucleons and nuclei as well as considerable theoretical work. However, despite more than a half century of effort, there remain considerable problems in understanding this weak hadronic PV interaction. The first systematic theoretical basis for understanding this interaction was a pion exchange plus local interaction picture posited by Blin-Stoyle in 1960 [15]. The local interaction piece was developed into a vector-meson-exchange term by Michel in

[a] e-mail: holstein@physics.umass.edu

1964 [16]. Then in 1980 this approach was developed into a comprehensive theoretical framework by Desplanques, Donoghue, and Holstein (DDH) [17]. The latter is the basis of the analysis of nearly all experimental work which has been done during the past quarter century.

As will be discussed in more detail below, the goal of this work has been to measure the phenomenological weak parity-violating meson-nucleon coupling constants defined by DDH. However, in spite of a great deal of effort on this problem there is still no way to describe all the experimental results in terms of the DDH picture. Of particular interest is the size of the weak πNN coupling constant, where there is disagreement as to whether it is of the same general size or is considerably smaller than the value (gu)estimated by DDH. In order to sort out whether the problems in analyzing such experiments are due to the model-dependent meson exchange picture used by DDH or on account of some deeper issue, in recent years Zhu *et al.* have developed a systematic effective field theory approach to study of the PV NN interaction, and that is the subject which is outlined below [18].

Since EFT methods are somewhat unfamiliar to some physicists, in the next section we contrast the conventional and EFT approach to study of the familiar low-energy parity-conserving NN interactions, with and without Coulomb effects, and demonstrate via either approach that near-threshold observables are expressible in terms of just *two* phenomenological parameters —the singlet and triplet S-wave scattering lengths. Then, we show how such EFT methods can be extended to the PV NN interaction at the cost of introducing *five* new parameters —the Danilov coefficients [19]. In sect. 3 we indicate how these parameters are related to the underlying effective PV NN Lagrangian and in sect. 4 we describe how they can be extracted from experiments on light nuclei. (We emphasize the use of *light* nuclei in order to ameliorate nuclear-physics uncertainties.) We conclude our paper with a look into the future, when such a program is reality, and suggest new directions for work at that time. We close with a brief recapitulation.

2 Parity-conserving NN scattering

We begin our discussion with a brief look at conventional scattering theory [20], where in the usual partial-wave expansion, we can write the scattering amplitude as

$$f(\theta) = \sum_\ell (2\ell+1) a_\ell(k) P_\ell(\cos\theta). \qquad (1)$$

Here the partial-wave amplitude $a_\ell(k)$ has the form

$$a_\ell(k) = \frac{1}{k} e^{i\delta(k)} \sin\delta(k) = \frac{1}{k\cot\delta(k) - ik}\,. \qquad (2)$$

Below we contrast the conventional and EFT approaches to the analysis of this scattering process. We begin with the conventional potential model technique.

2.1 Conventional analysis

Working in the potential model picture, one specifies a potential $V(r)$ describing the interaction of two particles, taken for simplicity here to be spinless, yielding a general expression for the scattering phase shift $\delta_\ell(k)$

$$\sin\delta_\ell(k) = -k\int_0^\infty \mathrm{d}r' r' j_\ell(kr') 2m_r V(r') u_{\ell,k}(r'), \qquad (3)$$

where m_r is the reduced mass and

$$\begin{aligned} u_{\ell,k}(r) = {} & r\cos\delta_\ell(k) j_\ell(kr) \\ & + kr\int_0^r \mathrm{d}r' r' j_\ell(kr') n_\ell(kr) u_{\ell,k}(r') 2m_r V(r') \\ & + kr\int_r^\infty \mathrm{d}r' r' j_\ell(kr) n_\ell(kr') u_{\ell,k}(r') 2m_r V(r') \end{aligned} \qquad (4)$$

is the scattering wave function [20]. At very low energies one can characterize the analytic function $k^{2\ell+1}\cot\delta(k)$ via the effective range expansion [21]

$$k^{2\ell+1}\cot\delta_\ell(k) = -\frac{1}{a_\ell} + \frac{1}{2} r_\ell^e k^2 + \dots\,. \qquad (5)$$

Then, from eq. (3) we can calculate *all* of the effective range parameters —*e.g.*, in the case of a weak potential the scattering length a_ℓ is given by

$$a_\ell = \frac{1}{[(2\ell+1)!!]^2}\int_0^\infty \mathrm{d}r' (r')^{2\ell+2} 2m_r V(r') + \mathcal{O}(V^2). \qquad (6)$$

As a specific example of the use of potential methods, suppose we utilize a simple square well potential to describe the interaction

$$V(r) = \begin{cases} -V_0, & r \le R, \\ 0, & r > R. \end{cases} \qquad (7)$$

For the S-wave scattering the wave function in the interior and exterior regions can then be written as

$$\psi^{(+)}(r) = \begin{cases} N j_0(Kr), & r \le R, \\ N' e^{i\delta_0}(j_0(kr)\cos\delta_0 - n_0(kr)\sin\delta_0), & r > R, \end{cases} \qquad (8)$$

where $j_0(kr)$, $n_0(kr)$ are spherical Bessel functions and the interior, exterior wave numbers are given by $k = \sqrt{2m_r E}$, $K = \sqrt{2m_r(E+V_0)}$, respectively. The connection between the two forms can be made by matching logarithmic derivatives at the boundary, yielding

$$k\cot\delta \simeq -\frac{1}{R}\left[1 + \frac{1}{KRF(KR)}\right] \quad \text{with} \quad F(x) = \cot x - \frac{1}{x}\,. \qquad (9)$$

Making the effective range expansion —eq. (5)— we can find expressions for the scattering length, effective range, and higher moments. Thus, defining $K_0 = \sqrt{2m_r V_0}$

$$\begin{aligned} a_0 &= R\left[1 - \frac{\tan K_0 R}{K_0 R}\right], \\ r_0^e &= -\frac{1}{K_0}\left[\frac{K_0 R\sec^2 K_0 R - \tan K_0 R}{(K_0 R - \tan K_0 R)^2}\right], \quad \text{etc.} \end{aligned} \qquad (10)$$

Note that for weak potentials —$K_0R \ll 1$— this expression for the scattering length agrees with the general result, eq. (6)

$$a_0 = \int_0^\infty \mathrm{d}r' r'^2 2m_r V(r') = -\frac{2m_r}{3}R^3V_0 + \mathcal{O}(V_0^2). \quad (11)$$

The important feature here is that because we have chosen a specific form of the potential, *all* terms in the effective range expansion are predicted. Of course, the forms given above in the case of weak potentials are modified in the general case, but the entire system of parameters is determined to all orders and can be determined numerically.

Our application of this formalism will be to the two-nucleon system, so that we must also introduce spin degrees of freedom. We note then that at very low energies, where only the scattering length is relevant, we can write the S-wave scattering matrix in the phenomenological form [22]

$$\mathcal{M}_{PC}(\boldsymbol{k}', \boldsymbol{k}) = m_t(k)P_1 + m_s(k)P_0, \quad (12)$$

where

$$P_1 = \frac{1}{4}(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \qquad P_0 = \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

are spin-triplet, -singlet spin projection operators and

$$m_t(k) \simeq \frac{-a_t}{1 + ika_t}, \qquad m_s(k) \simeq \frac{-a_s}{1 + ika_s} \quad (13)$$

are the S-wave partial-wave amplitudes in the lowest-order effective range approximation, keeping only the scattering lengths a_t, a_s. Here the scattering cross-section is found via

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \mathrm{Tr}\,\mathcal{M}^\dagger\mathcal{M}, \quad (14)$$

so that we reproduce the familiar result

$$\frac{\mathrm{d}\sigma_{s,t}}{\mathrm{d}\Omega} = \frac{|a_{s,t}|^2}{1 + k^2a_{s,t}^2}. \quad (15)$$

The corresponding scattering wave functions are then given by

$$\psi_{\boldsymbol{k}}^{(+)}(\boldsymbol{r}) = \left[e^{i\boldsymbol{k}\cdot\boldsymbol{r}} - \frac{M}{4\pi}\int \mathrm{d}^3r' \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}U(\boldsymbol{r}')\psi_{\boldsymbol{k}}^{(+)}(\boldsymbol{r})\right]\chi$$
$$\xrightarrow{r\to\infty} \left[e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \mathcal{M}(-i\boldsymbol{\nabla}, \boldsymbol{k})\frac{e^{ikr}}{r}\right]\chi, \quad (16)$$

where χ is the spin function. In Born approximation then we can write the scattering wave function in terms of an effective delta function potential

$$U_{t,s}(\boldsymbol{r}) = \frac{4\pi}{M}(a_tP_1 + a_sP_0)\delta^3(\boldsymbol{r}) \quad (17)$$

as can be confirmed by substitution into eq. (16). (Strictly speaking, the use of the Born approximation is not legitimate for the case of singular potentials such as we use, and must be properly defined as in [23].) Of course, before applying this result we need to enforce the stricture of unitarity, which requires that

$$2\,\mathrm{Im}\,T = T^\dagger T. \quad (18)$$

In the case of the S-wave partial-wave amplitude $m_t(k)$ this condition reads

$$\mathrm{Im}\,m_t(k) = k|m_t(k)|^2 \quad (19)$$

and requires the form

$$m_t(k) = \frac{1}{k}e^{i\delta_t(k)}\sin\delta_t(k). \quad (20)$$

Since at zero energy we have

$$\lim_{k\to 0} m_t(k) = -a_t, \quad (21)$$

it is clear that unitarity can be enforced by modifying this lowest-order result via

$$m_t(k) = \frac{-a_t}{1 + ika_t}, \quad (22)$$

which is simply the lowest-order effective range result, so everything seems to hang together.

For the case of nucleon-nucleon interactions, one finds[1]

$$a_0^s = -23.715 \pm 0.015\,\mathrm{fm}, \qquad r_0^s = 2.73 \pm 0.03\,\mathrm{fm},$$
$$a_0^t = 5.423 \pm 0.005\,\mathrm{fm}, \qquad r_0^t = 1.73 \pm 0.02\,\mathrm{fm}, \quad (23)$$

for scattering in the spin-singlet ($S = 0$) and spin-triplet ($S = 1$) channels, respectively. The existence of a bound state with energy $E = -\gamma^2/2m_r$ is indicated by the presence of a pole along the positive-imaginary k-axis —*i.e.* $\gamma > 0$ under the analytic continuation $k \to i\gamma$—

$$\frac{1}{a_0} + \frac{1}{2}r_0\gamma^2 - \gamma = 0. \quad (24)$$

We see from eq. (23) that there exists no bound state in the np spin-singlet channel, but in the spin-triplet system there exists a solution

$$\kappa = \frac{1 - \sqrt{1 - \frac{2r_0^t}{a_0^t}}}{r_0^t} = 45.7\,\mathrm{MeV}, \quad \textit{i.e.} \quad E_B = -2.23\,\mathrm{MeV} \quad (25)$$

corresponding to the deuteron.

2.2 Coulomb effects

When Coulomb interactions are included, the analysis becomes more challenging but remains straightforward. Suppose first that only same charge (*e.g.*, proton-proton) scattering is considered and that we describe the interaction in terms of a potential of the form

$$V(r) = \begin{cases} U(r), & r < R, \\ \frac{\alpha}{r}, & r > R, \end{cases} \quad (26)$$

[1] Note that the large scattering lengths found here show that this is certainly *not* a weak potential situation.

i.e. an attractive component —$U(r)$— at short distances, in order to mimic the strong interaction, and the repulsive Coulomb potential —α/r— at large distance, where $\alpha \simeq 1/137$ is the fine-structure constant. The analysis of scattering then proceeds as above but with the replacement of the exterior spherical Bessel functions j_0, n_0 by the corresponding Coulomb wave functions F_0^+, G_0^+

$$j_0(kr) \to F_0^+(r), \qquad n_0(kr) \to G_0^+(r), \tag{27}$$

whose explicit form can be found in ref. [24]. For our purposes we require only the form of these functions in the limit $kr \ll 1$:

$$\begin{aligned} F_0^+(r) &\xrightarrow{kr\ll 1} C(\eta_+(k))\left(1+\frac{r}{2a_B}+\dots\right) \\ G_0^+(r) &\xrightarrow{kr\ll 1} -\frac{1}{C(\eta_+(k))}\left\{\frac{1}{kr}\right. \\ &\left. + 2\eta_+(k)\left[h(\eta_+(k))+2\gamma_E-1+\ln\frac{r}{a_B}\right]+\dots\right\}. \end{aligned} \tag{28}$$

Here $\gamma_E = 0.577215\dots$ is Euler's constant,

$$C^2(\eta_+(k)) = \frac{2\pi\eta_+(k)}{\exp(2\pi\eta_+(k))-1} \equiv K_s \tag{29}$$

is the usual Coulombic enhancement factor, $a_B = 1/m_r\alpha$ is the Bohr radius, $\eta_+(k) = 1/ka_B$, and

$$\begin{aligned} h(\eta_+(k)) &= \mathrm{Re}\, H(i\eta_+(k)) \\ &= \eta_+^2(k)\sum_{n=1}^{\infty}\frac{1}{n(n^2+\eta_+^2(k))} - \ln\eta_+(k) - \gamma_E, \end{aligned} \tag{30}$$

where $H(x)$ is the analytic function,

$$H(x) = \psi(x) + \frac{1}{2x} - \ln(x). \tag{31}$$

Equating interior and exterior logarithmic derivatives as before, we find now

$$\begin{aligned} KF(KR) &= \frac{\cos\delta_0 F_0^{+\prime}(R) - \sin\delta_0 G_0^{+\prime}(R)}{\cos\delta_0 F_0^+(R) - \sin\delta_0 G_0^+(R)} \\ &= \frac{k\cot\delta_0 K_s \frac{1}{2a_B} - \frac{1}{R^2}}{k\cot\delta_0 K_s + \frac{1}{R} + \frac{1}{a_B}[h(\eta_+(k)) - \ln\frac{a_B}{R} + 2\gamma_E - 1]}. \end{aligned} \tag{32}$$

Since $R \sim 1\,\mathrm{fm} \ll a_B \sim 50\,\mathrm{fm}$, eq. (32) can be written in the form

$$\begin{aligned} k\cot\delta_0 K_s &+ \frac{1}{a_B}\left[h(\eta_+(k)) - \ln\frac{a_B}{R} + 2\gamma_E - 1\right] \\ &\simeq -\frac{1}{a_0}. \end{aligned} \tag{33}$$

The scattering length a_C in the presence of the Coulomb interaction is conventionally defined as [25]

$$k\cot\delta_0 K_s + \frac{1}{a_B}h(\eta_+(k)) = -\frac{1}{a_C} + \dots, \tag{34}$$

so that we have the relation

$$-\frac{1}{a_0} = -\frac{1}{a_C} - \frac{1}{a_B}\left(\ln\frac{a_B}{R} + 1 - 2\gamma_E\right) \tag{35}$$

between the experimental scattering length —a_C— and that which would exist in the absence of the Coulomb interaction —a_0.

As an aside, we note that a_0 is *not* itself an observable since the Coulomb interaction *cannot* be turned off. However, we can imagine a "gedanken scattering" in which there exists no Coulomb repulsion. In this case isotopic spin invariance requires the equality of the S-wave pp and nn scattering lengths —$a_0^{pp} = a_0^{nn}$— yielding the prediction

$$-\frac{1}{a_0^{nn}} = -\frac{1}{a_C^{pp}} - \alpha M_N\left(\ln\frac{1}{\alpha M_N R} + 1 - 2\gamma_E\right). \tag{36}$$

While this is a model-dependent result, Jackson and Blatt have shown, by treating the interior Coulomb interaction perturbatively, that a version of this result with $1 - 2\gamma_E \to 0.824 - 2\gamma_E$ is approximately valid for a wide range of strong-interaction potentials [24] and the correction indicated in eq. (36) is essential in restoring agreement between the widely discrepant —$a_0^{nn} = -18.8\,\mathrm{fm}$ *vs.* $a_C^{pp} = -7.82\,\mathrm{fm}$— values obtained experimentally.

Returning to the problem at hand, the experimental scattering amplitude can then be written as

$$\begin{aligned} f_C^+(k) &= \frac{e^{2i\sigma_0}K_s}{-\frac{1}{a_C} - \frac{1}{a_B}h(\eta_+(k)) - ikK_s} \\ &= \frac{e^{2i\sigma_0}K_s}{-\frac{1}{a_C} - \frac{1}{a_B}H(i\eta_+(k))}, \end{aligned} \tag{37}$$

where $\sigma_0 = \arg\Gamma(1 - i\eta_+(k))$ is the S-wave Coulomb phase.

The above analysis is standard and can be found in many quantum mechanics texts [20]. In the next section we reanalyze the NN system using the ideas of effective field theory.

2.3 Effective field theory analysis

Identical results may be obtained using a parallel effective field theory (EFT) analysis and in many ways the derivation is clearer and more intuitive [26][2]. The basic idea here is that since we are only interested in interactions at very low energy, a scattering length description is quite adequate and it is unnecessary to identify a specific form for the potential —everything can be done in terms of observables. From eq. (11) we see that, at least for weak potentials, the scattering length has a natural representation in terms of the momentum space potential $\tilde{V}(\boldsymbol{p} = 0)$:

$$a_0 = \frac{m_r}{2\pi}\int \mathrm{d}^3 r V(r) = \frac{m_r}{2\pi}\tilde{V}(\boldsymbol{p} = 0) \tag{38}$$

[2] Two interesting didactic introductions to EFT methods can be found in refs. [27] and [28].

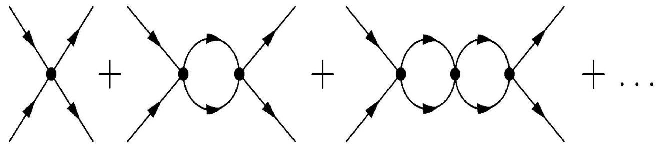

Fig. 1. The multiple scattering series.

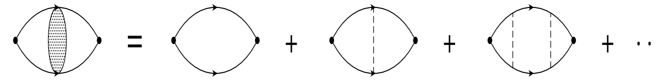

Fig. 2. The Coulomb-corrected bubble.

and it is thus natural to perform our analysis using a simple contact interation. First, consider the situation that we have two particles A, B interacting only via a *local* strong interaction, so that the effective Lagrangian can be written as

$$\mathcal{L} = \sum_{i=A}^{B} \Psi_i^\dagger \left(i\frac{\partial}{\partial t} + \frac{\boldsymbol{\nabla}^2}{2m_i} \right) \Psi_i - C_0 \Psi_A^\dagger \Psi_A \Psi_B^\dagger \Psi_B + \dots \quad (39)$$

The T-matrix is then given in terms of the multiple scattering series shown in fig. 1

$$\begin{aligned} T_{fi}(k) &= -\frac{2\pi}{m_r} f(k) = C_0 + C_0^2 G_0(k) + C_0^3 G_0^2(k) + \dots \\ &= \frac{C_0}{1 - C_0 G_0(k)}\,, \end{aligned} \quad (40)$$

where $G_0(k)$ is the amplitude for particles A, B to travel from zero separation to zero separation —*i.e.*, the propagator $D_F(k; \boldsymbol{r'} = 0, \boldsymbol{r} = 0)$—

$$\begin{aligned} G_0(k) &= \lim_{\boldsymbol{r'},\boldsymbol{r}\to 0} \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \frac{e^{i\boldsymbol{s}\cdot\boldsymbol{r'}} e^{-i\boldsymbol{s}\cdot\boldsymbol{r}}}{\frac{k^2}{2m_r} - \frac{s^2}{2m_r} + i\epsilon} \\ &= \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \frac{2m_r}{k^2 - s^2 + i\epsilon} \end{aligned} \quad (41)$$

(Equivalently $T_{fi}(k)$ satisfies a Lippman-Schwinger equation

$$T_{fi}(k) = C_0 + C_0 G_0(k) T_{fi}(k), \quad (42)$$

whose solution is given in eq. (40).)

The complication here is that the function $G_0(k)$ is divergent and must be defined via some sort of regularization scheme. There are a number of ways by which to accomplish this, but perhaps the simplest is to use a cutoff regularization with $k_{max} = \mu$, which simply eliminates the high-momentum components of the wave function completely. Then

$$G_0(k) = -\frac{m_r}{2\pi} \left(\frac{2\mu}{\pi} + ik \right) \quad (43)$$

(Other regularization schemes are similar. For example, one could subtract at an unphysical momentum point, as proposed by Gegelia [29]

$$G_0(k) = \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \left(\frac{2m_r}{k^2 - s^2 + i\epsilon} + \frac{2m_r}{\Lambda^2 + s^2} \right) = -\frac{m_r}{2\pi}(\Lambda + ik), \quad (44)$$

which has been shown by Mehen and Stewart [30] to be equivalent to the power divergence subtraction scheme proposed by Kaplan, Savage and Wise [26].) In any case, the would-be linear divergence is canceled by the introduction of a counterterm, which accounts for the omitted high-energy component of the theory and modifies C_0 to $C_0(\mu)$. (That $C_0(\mu)$ should be a function of the cutoff is clear because by varying the cutoff energy we are varying the amount of higher-energy physics which we are including in our effective description.) The scattering amplitude then becomes

$$f(k) = -\frac{m_r}{2\pi} \left(\frac{1}{\frac{1}{C_0(\mu)} - G_0(k)} \right) = \frac{1}{-\frac{2\pi}{m_r C_0(\mu)} - \frac{2\mu}{\pi} - ik}\,. \quad (45)$$

Comparing with eq. (2) we identify the scattering length as

$$-\frac{1}{a_0} = -\frac{2\pi}{m_r C_0(\mu)} - \frac{2\mu}{\pi}\,. \quad (46)$$

Of course, since a_0 is a physical observable, it must be *cutoff-independent* —the μ-dependence of $1/C_0(\mu)$ is precisely canceled by the cutoff dependence in the Green's function.

2.4 Coulomb effects in EFT

More interesting (and challenging) is the case where we restore the Coulomb interaction between the particles. The derivatives in eq. (39) then become covariant and the bubble sum is evaluated with static photon exchanges between each of the lines —each bubble is replaced by one involving a sum of zero, one, two, etc. Coulomb interactions, as shown in fig. 2.

The net result in the case of same charge scattering is the replacement of the free propagator by its Coulomb analog

$$\begin{aligned} G_0(k) \to G_C^+(k) &= \lim_{\boldsymbol{r'},\boldsymbol{r}\to 0} \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \frac{\psi_{\boldsymbol{s}}^+(\boldsymbol{r'}) \psi_{\boldsymbol{s}}^{+*}(\boldsymbol{r})}{\frac{k^2}{2m_r} - \frac{s^2}{2m_r} + i\epsilon} \\ &= \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \frac{2m_r K_s}{k^2 - s^2 + i\epsilon}\,, \end{aligned} \quad (47)$$

where

$$\psi_{\boldsymbol{s}}^+(\boldsymbol{r}) = C(\eta_+(s)) e^{i\sigma_0} e^{i\boldsymbol{s}\cdot\boldsymbol{r}} {}_1F_1(-i\eta_+(s), 1, isr - i\boldsymbol{s}\cdot\boldsymbol{r}) \quad (48)$$

is the outgoing Coulomb wave function for repulsive Coulomb scattering [31]. Also in the initial and final states the influence of static photon exchanges must be included to all orders, which produces the factor $K_s \exp(2i\sigma_0)$. Thus, the repulsive Coulomb scattering amplitude becomes

$$f_C^+(k) = -\frac{m_r}{2\pi} \frac{C_0 K_s \exp 2i\sigma_0}{1 - C_0 G_C^+(k)}\,. \quad (49)$$

The momentum integration in eq. (47) can be performed as before using cutoff regularization, yielding [32]

$$G_C^+(k) = -\frac{m_r}{2\pi}\left\{\frac{2\mu}{\pi} + \frac{1}{a_B}\left[H(i\eta_+(k)) - \ln\frac{\mu a_B}{\pi} - \zeta\right]\right\}, \tag{50}$$

where $\zeta = \ln 2\pi - \gamma$. We have then

$$\begin{aligned} f_C^+(k) &= \frac{K_s e^{2i\sigma_0}}{-\frac{2\pi}{m_r C_0(\mu)} - \frac{2\mu}{\pi} - \frac{1}{a_B}[H(i\eta_+(k)) - \ln\frac{\mu a_B}{\pi} - \zeta]} \\ &= \frac{K_s e^{2i\sigma_0}}{-\frac{1}{a_0} - \frac{1}{a_B}\left[h(\eta_+(k)) - \ln\frac{\mu a_B}{\pi} - \zeta\right] - ikK_s}. \end{aligned} \tag{51}$$

Comparing with eq. (37) we identify the Coulomb scattering length as

$$-\frac{1}{a_C} = -\frac{1}{a_0} + \frac{1}{a_B}\left(\ln\frac{\mu a_B}{\pi} + \zeta\right) \tag{52}$$

which matches nicely with eq. (35) if a reasonable cutoff $\mu \sim m_\pi \sim 1/R$ is employed. The scattering amplitude then has the simple form

$$f_C^+(k) = \frac{K_s e^{2i\sigma_0}}{-\frac{1}{a_C} - \frac{1}{a_B}H(i\eta_+(k))} \tag{53}$$

in agreement with eq. (37).

The important lesson here is that at very low energy, where we can completely characterize the amplitude in terms of the scattering length, we see that only *two* parameters are required in order to completely describe NN scattering —the spin-singlet and triplet scattering lengths. The effective range parameters in these channels provide a way to estimate the size of possible corrections to this scattering length approximation. It is *not* necessary to make any assumptions about the detailed shape of the potential —we can write everything in terms of *observables*.

Our next goal then is to emulate this discussion in the case of the parity-*violating* NN potential, a task which we take up in the following section.

2.5 Parity-violating NN interaction: potential model description

Until recently the standard method by which to treat the parity-violating NN interaction was by use of potential theory. The basic idea is that in the same way in which the low-energy parity-conserving NN interaction can be described quite satisfactorily in terms of a simple light meson exchange picture, we can represent the parity-violating NN interaction in a parallel fashion, wherein one of the parity-conserving NNM vertices is replaced by its parity-violating analog —cf. fig. 3. This is the method pioneered by Blin-Stoyle and by Michel and then followed by DDH in their seminal 1980 paper. Of course, this approach is model dependent and generates a specific form for the potential, but in the days before effective field theory this was the standard way to proceed.

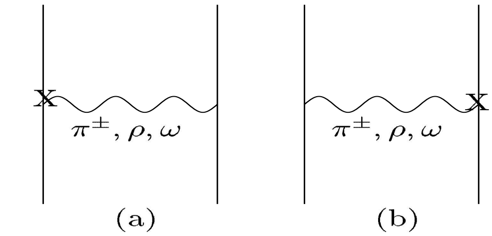

Fig. 3. Parity-violating NN potential generated by meson exchange. Here the symbol "X" indicates a parity-violating vertex.

More specifically, we represent the (parity-conserving) strong coupling of the nucleon to the light vector and pseudoscalar mesons via the effective Lagrangian

$$\begin{aligned} \mathcal{H}_{st} = &\, ig_{\pi NN}\bar{N}\gamma_5\tau\cdot\pi N + g_\rho\bar{N}\left(\gamma_\mu + i\frac{\chi_\rho}{2m_N}\sigma_{\mu\nu}k^\nu\right)\tau\cdot\rho^\mu N \\ &+ g_\omega\bar{N}\left(\gamma_\mu + i\frac{\chi_\omega}{2m_N}\sigma_{\mu\nu}k^\nu\right)\omega^\mu N, \end{aligned} \tag{54}$$

whose values are determined from strong-interaction studies. Typical —though not universally accepted [33]— values are $g_{\pi NN}^2/4\pi \simeq 13.5$ and $g_\rho^2/4\pi = \frac{1}{9}g_\omega^2/4\pi \simeq 0.67$ and, with the use of vector dominance to connect with the electromagnetic interaction, $\chi_\rho = \kappa_p - \kappa_n = 3.7$ and $\chi_\omega = \kappa_p + \kappa_n = -0.12$. For the parity-violating couplings we can write a general phenomenological interaction of the form [17]

$$\begin{aligned} \mathcal{H}_{wk} = &\, i\frac{f_\pi^1}{\sqrt{2}}\bar{N}(\tau\times\pi)_z N \\ &+ \bar{N}\left(h_\rho^0\tau\cdot\rho^\mu + h_\rho^1\rho_z^\mu + \frac{h_\rho^2}{2\sqrt{6}}(3\tau_z\rho_z^\mu - \tau\cdot\rho^\mu)\right)\gamma_\mu\gamma_5 N \\ &+ \bar{N}\left(h_\omega^0\omega^\mu + h_\omega^1\tau_z\omega^\mu\right)\gamma_\mu\gamma_5 N - h_\rho^{'1}\bar{N}(\tau\times\rho^\mu)_z\frac{\sigma_{\mu\nu}k^\nu}{2m_N}\gamma_5 N, \end{aligned} \tag{55}$$

where here we have used the stricture from Barton's theorem that any CP-conserving parity-violating coupling to neutral pseudoscalar mesons such as π^0, η^0 must vanish [34]. We see then that there exist, in this model, *seven* unknown weak couplings f_π^1, $h_\rho^{(0)}$, $h_\rho^{(1)}$, $h_\rho^{(2)}$, $h_\omega^{(0)}$, $h_\omega^{(1)}$, $h_\rho^{(1)'}$. However, quark model calculations suggest that $h_\rho^{(1)'}$ is quite small [35], so this term is generally omitted, leaving parity-violating observables described in terms of just six phenomenological constants —f_π, $h_\rho^{(0)}$, $h_\rho^{(1)}$, $h_\rho^{(2)}$, $h_\omega^{(0)}$, $h_\omega^{(1)}$[3]. In their paper DDH attempted to evaluate these basic PV couplings using basic quark-model and $SU(6)$-symmetry techniques, but they encountered significant theoretical challenges and uncertainties. For this reason their results were presented in terms of an allowable range for each, accompanied by a "best value" representing their reasonable guess for each coupling. These

[3] Another way to view the neglect of the $h_\rho^{'1}$ coupling is that it represents simply a short-range correction to the size of the charged-pion exchange coupling.

Table 1. Weak NNM couplings as calculated in refs. [17,36,37]. All numbers are quoted in units of the "sum rule" value $g_\pi = 3.8 \cdot 10^{-8}$.

Coupling	DDH [17] Reasonable range	DDH [17] "Best" value	DZ [36]	FCDH [37]
f_π	$0 \to 30$	$+12$	$+3$	$+7$
h_ρ^0	$30 \to -81$	-30	-22	-10
h_ρ^1	$-1 \to 0$	-0.5	$+1$	-1
h_ρ^2	$-20 \to -29$	-25	-18	-18
h_ω^0	$15 \to -27$	-5	-10	-13
h_ω^1	$-5 \to -2$	-3	-6	-6

ranges and "best values" are listed in table 1, together with predictions generated by subsequent groups [36,37]. (This list is not comprehensive, merely representative, and many other estimates have been provided. For example, Kaiser and Meissner utilized a chiral soliton approach to calculate these numbers [38], while Hwang and Wen employed the method of QCD sum rules to yield values for the DDH couplings [39].)"

Before making contact with experimental results, however, it is necessary to convert the NNM couplings generated above into an effective parity-violating NN potential. Inserting the strong and weak couplings, defined above into the meson exchange diagrams shown in fig. 1 and transforming to coordinate space, one finds the DDH parity-violating NN potential

$$\begin{aligned}
V_{DDH}^{PV}(\boldsymbol{r}) &= i\frac{f_\pi^1 g_{\pi NN}}{\sqrt{2}}\left(\frac{\tau_1\times\tau_2}{2}\right)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\left[\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\pi(r)\right]\\
&-g_\rho\left(h_\rho^0\tau_1\cdot\tau_2+h_\rho^1\left(\frac{\tau_1+\tau_2}{2}\right)_z+h_\rho^2\frac{(3\tau_1^z\tau_2^z-\tau_1\cdot\tau_2)}{2\sqrt{6}}\right)\\
&\times\left((\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\left\{\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\rho(r)\right\}\right.\\
&\left.+i(1+\chi_V)\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\left[\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\rho(r)\right]\right)\\
&-g_\omega\left(h_\omega^0+h_\omega^1\left(\frac{\tau_1+\tau_2}{2}\right)_z\right)\\
&\times\left((\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\left\{\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\omega(r)\right\}\right.\\
&\left.+i(1+\chi_S)\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\left[\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\omega(r)\right]\right)-\left(g_\omega h_\omega^1-g_\rho h_\rho^1\right)\\
&\times\left(\frac{\tau_1-\tau_2}{2}\right)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\left\{\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\rho(r)\right\}\\
&-g_\rho h_\rho^{1'} i\left(\frac{\tau_1\times\tau_2}{2}\right)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\left[\frac{\boldsymbol{p}_1-\boldsymbol{p}_2}{2m_N}, w_\rho(r)\right], \qquad (56)
\end{aligned}$$

where $w_i(r) = \exp(-m_i r)/4\pi r$ is the usual Yukawa form, $r = |\boldsymbol{x}_1 - \boldsymbol{x}_2|$ is the separation between the two nucleons, and $\boldsymbol{p}_i = -i\boldsymbol{\nabla}_i$. We observe from eq. (56) that the unknown weak couplings f_π^1, h_V^I always occur multiplied by their strong-interaction counterparts $g_{\pi NN}, g_V, g_V\chi_V$ so that the lack of precise knowledge of these strong couplings alluded to above does not really damage the use of the DDH potential for phenomenological purposes.

It is useful to note at this point that the DDH model is the parity-violating analog of the conventional potential approach to scattering and postulates a *complete* form of the effective parity-violating NN potential —both magnitude and shape— so that in principle even high-energy observables are predicted. (In reality, of course, high-energy forms should include meson exchanges from heavier systems such as the axial mesons.) It is also important to point out that each of the vector-meson–mediated pieces of the potential consists of *both* a convective (anticommutator) *and* magnetic (commutator) component, with the relative strength of these two couplings determined from vector dominance in terms of the anomalous magnetic moments of the nucleons, as outlined above.

Essentially *all* experimental results involving hadronic parity violation have been analyzed using $V_{DDH}^{PV}(r)$ for the past quarter century. There have been a number of previous reviews of this field, beginning with the 1985 *Annual Reviews of Nuclear and Particle Science* article by Adelberger and Haxton [40], continuing with the 1995 review by Holstein and Haeberli appearing in the book *Symmetries and Fundamental Interactions in Nuclei* [41], and in 2006 the field was again surveyed by Page and Ramsey-Musolf in *Annual Reviews* [42]. Because each of these papers *comprehensively* examines the experimental situation in a fashion far deeper than possible in the present article, we defer to them for details of the various experiments and merely report here the conclusions, which are that, despite half a century of experimental and theoretical work, at present there appear to exist significant discrepancies between the values extracted for the various DDH couplings from different experiments.

The problem can be seen in a number of ways, but perhaps the most straightforward is to note that analysis of experiments on the asymmetry in longitudinally polarized $\boldsymbol{p}\alpha$ scattering [43] and the photon asymmetry in the decay of the polarized first excited state of ^{19}F [44] are consistent with each other and (within errors) with values about half of the best guess DDH numbers. The analysis depends predominantly on the long-range pion coupling f_π^1 and on the effective isoscalar vector meson coupling $h_\rho^0 + 0.7h_\omega^0$ and is often presented in terms of a two-dimensional plot —cf. fig. 4. However, at least four

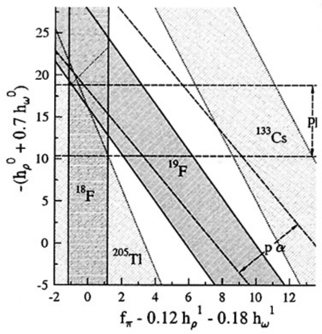

Fig. 4. Experimental limits on weak couplings.

experiments seeking the circular polarization of photons emitted in the decay of the 1.081 MeV excited state of ^{18}F have failed to see any signal [45], which seems to indicate that the pion coupling f_π^1 is considerably *smaller* than its DDH best guess value. One might be tempted to attribute this inconsistency to nuclear uncertainties, but the theoretical analysis of this mode is buttressed by comparison with the two-body contributions to the analog beta decay of ^{18}Ne [46] as well as by a very recent cold neutron experiment which measured the triton asymmetry in the reaction $^6\mathrm{Li}(n,\alpha)^3\mathrm{H}$ [47]. An additional issue is that recent measurements of the anapole moment of ^{133}Cs from atomic parity violation experiments [48] appear to be consistent with a size for f_π^1 in agreement with the DDH "best value". This situation is summarized in fig. 4, which clearly indicates difficulties with the present DDH analysis of the PV NN interaction. These discrepancies possibly suggest a problem with the underlying model-dependent theoretical framework itself, and it is for this reason that a new approach, based on effective field theory, has been developed. This technique is discussed below.

2.6 Parity-violating NN interaction: EFT description

As described above, there presently exist inconsistencies within the DDH analysis of $\Delta S = 0$ hadronic parity-violating experiments. The origin of this problem is unclear, but could certainly be associated with the fact that the extraction of the basic weak couplings requires knowledge of the spatial average of the associated weak parity-violating potential weighted by imperfectly known nuclear wave functions. For this reason and for basic understanding of such processes, the analysis of such experiments has been recently reformulated in terms of an effective field-theoretic parity-violating NN potential, which puts the analysis of these systems into a more rigorous model-independent form. The basic idea here is that there are a number of scales at play in the manifestations of the hadronic weak interaction. There is the momentum transfer Q which is generally much smaller than the chiral scale $\Lambda_\chi \simeq 4\pi F_\pi$ but can be smaller than or comparable to the inverse nucleon size $1/R \sim m_\pi$. First, suppose that $Q \ll m_\pi \ll \Lambda_\chi$. In order to formulate the EFT discussion in parallel to that used in the analysis of the parity-conserving interaction, we begin by writing down the lowest-order (one-derivative) short-range (SR) form of the PV NN potential $V_{1,SR}^{PV}(\boldsymbol{r})$, as given by Zhu *et al.* [18][4],

$$
\begin{aligned}
V_{1,SR}^{PV}(\boldsymbol{r}) = \frac{2}{\Lambda_\chi^3}\Bigg\{ & \left[C_1 + C_2\frac{\tau_1^z+\tau_2^z}{2}\right](\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla}, f_m(r)\} \\
& + \left[\tilde{C}_1 + \tilde{C}_2\frac{\tau_1^z+\tau_2^z}{2}\right] i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[-i\boldsymbol{\nabla}, f_m(r)] \\
& + [C_2 - C_4]\frac{\tau_1^z-\tau_2^z}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla}, f_m(r)\} \\
& + \left[C_3\tau_1\cdot\tau_2 + C_4\frac{\tau_1^z+\tau_2^z}{2} + \mathcal{I}_{ab}C_5\tau_1^a\tau_2^b\right] \\
& \times(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla}, f_m(r)\} \\
& + \left[\tilde{C}_3\tau_1\cdot\tau_2 + \tilde{C}_4\frac{\tau_1^z+\tau_2^z}{2} + \mathcal{I}_{ab}\tilde{C}_5\tau_1^a\tau_2^b\right] \\
& \times i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[-i\boldsymbol{\nabla}, f_m(r)] \\
& + \tilde{C}_6 i(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[-i\boldsymbol{\nabla}, f_m(r)]\Bigg\}, \qquad (57)
\end{aligned}
$$

which is the PV analog of the contact PC interaction eq. (39), the derivative form in eq. (57) being required by the stricture of parity violation. Here

$$
\mathcal{I}^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \qquad (58)
$$

and $f_m(\boldsymbol{r})$ is a function which

i) is strongly peaked, with width $\sim 1/m$ about $r = 0$, and
ii) approaches $\delta^{(3)}(\boldsymbol{r})$ in the zero width —$m \to \infty$— limit.

A convenient (though not unique) form, for example, is the Yukawa-like function

$$
f_m(r) = \frac{m^2}{4\pi r}\exp(-mr), \qquad (59)
$$

where m is a mass chosen to reproduce the appropriate short-range effects. (Actually, for the purpose of carrying out actual calculations, one could just as easily use the momentum space form of V_{SR}^{PV}, thereby avoiding the use of $f_m(\boldsymbol{r})$ altogether.)

The matching of the DDH model to the coefficients C_i, $\tilde{C}_i$ in eq. (57) can be done by writing the Yukawa functions $w_i(r)$ in terms of their Fourier transforms

$$
w_i(r) = \int\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}}{m_i^2+\boldsymbol{Q}^2}\,. \qquad (60)
$$

[4] Note that below, as suggested by Liu [49], we have used the symbol $\tilde{C}_6$ rather than C_6 as used by Zhu *et al.* [18] since it multiplies a commutator rather than an anticommutator.

Working in the limit in which $\boldsymbol{Q}^2 \ll mi^2$ with $i = \pi, \rho, \omega$ we can make the replacement $m_i^2 + \boldsymbol{Q}^2 \longrightarrow m_i^2$, whereby the Yukawa function is replaced by a delta function,

$$w_i(r) \longrightarrow \frac{1}{m_i^2}\delta^3(r).$$

In the Zhu *et al.* formalism this delta function is represented by $f_m(r)$. We observe then that the same set of spin-space and isospin structures appear in both V_{eff}^{PV} and the vector-meson exchange terms in V_{DDH}^{PV}, though the relationship between the various coefficients in V_{eff}^{PV} is more general. In particular, the DDH model is tantamount to assuming

$$\frac{\tilde{C}_1}{C_1} = \frac{\tilde{C}_2}{C_2} = 1 + \chi_\omega \simeq 0.88, \tag{61}$$

$$\frac{\tilde{C}_3}{C_3} = \frac{\tilde{C}_4}{C_4} = \frac{\tilde{C}_5}{C_5} = 1 + \chi_\rho \simeq 4.7, \tag{62}$$

and taking $m \sim m_\rho$, m_ω for $C_1, \ldots, C_5$ but $m \sim m_\pi$ for $\tilde{C}_6$, assumptions which may not be physically realistic. Nevertheless, if this ansatz is posited, the EFT and DDH results coincide provided the identifications

$$\begin{aligned}
C_1^{DDH} &= -\frac{\Lambda_\chi^3}{2m_N m_\omega^2} g_\omega h_\omega^0 \xrightarrow{\text{best guess}} 2.3 \times 10^{-6},\\
C_2^{DDH} &= -\frac{\Lambda_\chi^3}{2m_N m_\omega^2} g_\omega h_\omega^1 \xrightarrow{\text{best guess}} 1.4 \times 10^{-6},\\
C_3^{DDH} &= -\frac{\Lambda_\chi^3}{2m_N m_\rho^2} g_\rho h_\rho^0 \xrightarrow{\text{best guess}} 4.6 \times 10^{-6},\\
C_4^{DDH} &= -\frac{\Lambda_\chi^3}{2m_N m_\rho^2} g_\rho h_\rho^1 \xrightarrow{\text{best guess}} 0.1 \times 10^{-6},\\
C_5^{DDH} &= \frac{\Lambda_\chi^3}{4\sqrt{6} m_N m_\rho^2} g_\rho h_\rho^2 \xrightarrow{\text{best guess}} -0.8 \times 10^{-6},\\
\tilde{C}_6^{DDH} &\simeq \tilde{C}_6^\pi = \frac{\Lambda_\chi^3}{2\sqrt{2} m_N m_\pi^2} g_{\pi NN} f_\pi^1 \xrightarrow{\text{best guess}} 180 \times 10^{-6},
\end{aligned} \tag{63}$$

are made [18] and only the S-P mixing terms in the DDH form are retained. (Note that for use below we have quoted the "best value" numbers for these parameters.)

This form of the effective theory is generally termed the "pionless" picture because, since $Q \ll m_\pi$, the pion does not appear as an explicit degree of freedom.

Of course, the "pionless" approximation breaks down at energies of order $m_\pi^2/M_N \sim 20\,\text{MeV}$ and must be replaced by a somewhat more complex theory which *does* contain an explicit pion. In this "pionful" theory, which should work until energies of order the pion mass, we have $\boldsymbol{Q}^2 \simeq m_\pi^2$, but we still have $Q^2 \ll m_\rho^2 m_\omega^2$ so that the matching of the DDH picture to the effective theory given above is unchanged for the coefficients C_i, $\tilde{C}_i$ $i = 1, 2, \ldots, 5$. However, in the case of $\tilde{C}_6$, which is associated with pion exchange, the replacement of this piece by an effective short-range interaction is no longer justified. Instead the last line of eq. (57) —*i.e.* the term involving $\tilde{C}_6$ must be removed and replaced by four additional types of terms:

i) a long-range one-pion exchange potential, which is two orders *lower* in chiral counting than the corresponding short-range (vector-meson exchange) terms:

$$V_{-1,LR}(r) = \frac{1}{\Lambda_\chi^3}\tilde{C}_6^\pi i(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot[-i\boldsymbol{\nabla}, f_\pi(r)], \tag{64}$$

where $\tilde{C}_6^\pi$ is defined in eq. (63).

ii) a medium-range interaction which arises from the effects of two-pion exchange

$$\begin{aligned}
V_{1,MR}(r) = \frac{\tilde{C}_2^{2\pi}}{\Lambda_\chi^3}\Big\{&(\tau_1 + \tau_2)_z i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot \boldsymbol{y}_{2\pi}^L(r)\\
&-\frac{3}{4k}(\tau_1 \times \tau_2)_z(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\\
&\times\left[\left(1 - \frac{1}{3g_A^2}\right)\boldsymbol{y}_{2\pi}^L(r) - \frac{1}{3}\boldsymbol{y}_{2\pi}^H(r)\right]\Big\},
\end{aligned} \tag{65}$$

where

$$\tilde{C}_2^{2\pi} = -4\sqrt{2}\pi g_A^3 f_\pi^1 \tag{66}$$

and the functions $\boldsymbol{y}_{2\pi}^{H,R}(r)$ are defined via

$$\boldsymbol{y}_{2\pi}^{H,L}(r) = [-i\boldsymbol{\nabla}, H, L(r)] \tag{67}$$

with H, $L(r)$ being the Fourier transform of the functions

$$\begin{aligned}
L(\boldsymbol{q}) &= \frac{\sqrt{4m_\pi^2 + \boldsymbol{q}^2}}{|\boldsymbol{q}|}\log\left(\frac{\sqrt{4m_\pi^2 + \boldsymbol{q}|^2} + |\boldsymbol{q}}{2m_\pi}\right),\\
H(\boldsymbol{q}) &= \frac{4m_\pi^2}{4m_\pi^2 + \boldsymbol{q}^2}L(\boldsymbol{q}),
\end{aligned} \tag{68}$$

respectively. (Note that these terms include only the *nonanalytic* pieces of the full two-pion exchange amplitude, since it is only these pieces which yield medium-range effects. Any analytic component of the two-pion exchange amplitude generates a short-distance contribution, which is subsumed into the phenomenological coefficients of the contact terms already written down.)

iii) a long-range component generated from one-loop corrections to the leading vertices, which is two orders higher in the counting than the leading pion-exchange potential:

$$V_{1,LR}(\boldsymbol{p}_1,\boldsymbol{p}_1',\boldsymbol{p}_2,\boldsymbol{p}_2') = \frac{g_A h_\pi^1}{\Lambda_\chi F_\pi^2}\frac{1}{2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z \times\left[\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{p}_1'\times\boldsymbol{p}_1\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{q^2+m_\pi^2}+(1\leftrightarrow 2)\right] +i\frac{g_A f_\pi^1}{\sqrt{2}m_N^2F_\pi}\frac{1}{2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\frac{1}{q^2+m_\pi^2} \times\left\{\frac{1}{4}[(\boldsymbol{p}_1^2-\boldsymbol{p}_1^{'2})\boldsymbol{\sigma}_1\cdot(\boldsymbol{p}_1'+\boldsymbol{p}_1)-(1\leftrightarrow 2)] -\frac{1}{8}[(\boldsymbol{p}_1^2+\boldsymbol{p}_1^{'2})\boldsymbol{\sigma}_1\cdot\boldsymbol{q}+(1\leftrightarrow 2)] +\frac{1}{4}[\boldsymbol{\sigma}\cdot\boldsymbol{p}_1^1\boldsymbol{q}\cdot\boldsymbol{p}_1+\boldsymbol{\sigma}\cdot\boldsymbol{p}_1\boldsymbol{q}\cdot\boldsymbol{p}_1'+(1\leftrightarrow 2)]\right\}, \quad (69)$$

where $\boldsymbol{q}_i=\boldsymbol{p}_i'-\boldsymbol{p}_i$.

iv) a PV "Kroll-Ruderman"-like $NN\pi\gamma$ coupling $\tilde{C}_\pi$ that leads to a new independent current operator:

$$\boldsymbol{J}(\boldsymbol{x}_1,\boldsymbol{x}_2)=\frac{\sqrt{2}g_A\tilde{C}_\pi m_\pi^2}{\Lambda_\chi^2F_\pi}e^{-i\boldsymbol{q}\cdot\boldsymbol{x}_1}\tau_1^+\tau_2^-\boldsymbol{\sigma}_1\times\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\hat{r}H_\pi(r) +(1\leftrightarrow 2), \quad (70)$$

where

$$H_\pi(r)=\frac{e^{-m_\pi r}}{m_\pi r}\left(1+\frac{1}{m_\pi r}\right). \quad (71)$$

In this pionful theory the parameter m which characterizes nonpionic pieces of the potential (which is of order m_π in the pionless theory) should presumably assume a value of order $m\sim m_\rho\sim m_\omega$, since the pionic degrees of freedom are included explicitly in the potentials i), ii), iii), iv) described above.

It is interesting to note here that the "best guess" parameter $\tilde{C}_6^\pi$ is at least an order of magnitude larger than any of its short-distance (vector-meson–dominated) counterparts or than the medium-range "best guess" values, as might be suspected from its lower order in the chiral counting scheme.

3 Danilov parameters

The discussion in the previous section might make it appear that the low-energy analysis of the PV NN interaction must involve the determination of *ten* parameters[5] —a daunting task indeed. However, this assumption is misleading. *In fact, it is easy to see that at the very lowest energies there can be only five phenomenological constants involved.* This is because at threshold energies, we can neglect all but S-P-wave mixing, in which case there exist only *five* independent phenomenological amplitudes:

i) $d_t(k)$ representing $^3S_1$-$^1P_1$ mixing with $\Delta I=0$;

ii) $d_s^{0,1,2}(k)$ representing $^1S_0$-$^3P_0$ mixing with $\Delta I=0,1,2$ respectively;

iii) $c_t(k)$ representing $^3S_1$-$^3P_1$ mixing with $\Delta I=1$.

These *five* independent transition amplitudes are the PV analogs of the *two* (singlet and triplet) S-wave amplitudes $m_s(k)$, $m_t(k)$ involved in the PC case.

Following Danilov [19], the low-energy parity-violating scattering matrix in the presence of parity violation can be written then as[6]

$$\mathcal{M}_{PV}(\boldsymbol{k}',\boldsymbol{k})=c_t(k)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}'+\boldsymbol{k})\frac{1}{2}(\tau_1-\tau_2)_z +(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}'+\boldsymbol{k}) \times\left(P_0d_s^0(k)+\frac{1}{2}(\tau_1+\tau_2)_zd_s^1(k)+\frac{3\tau_{1z}\tau_{2z}-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{2\sqrt{6}}d_s^2(k)\right) +d_t(k)(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}'+\boldsymbol{k})P_1. \quad (72)$$

Note that under spatial inversion —$\boldsymbol{\sigma}\to\boldsymbol{\sigma}$, $\boldsymbol{k},\boldsymbol{k}'\to-\boldsymbol{k},-\boldsymbol{k}'$— each of these pieces is P-odd, while under time reversal —$\boldsymbol{\sigma}\to-\boldsymbol{\sigma}$, $\boldsymbol{k},\boldsymbol{k}'\to-\boldsymbol{k}',-\boldsymbol{k}$— each term is T-even. At very low energies the coefficients in the T-matrix become real and we can define [22]

$$\lim_{k\to 0}c_t(k),d_s(k),d_t(k)\equiv\rho_ta_t,\lambda_s^ia_s,\lambda_ta_t. \quad (73)$$

The motivation for inclusion of the S-wave scattering lengths a_t, a_s will be described presently. The five real numbers (Danilov parameters) ρ_t, λ_s^i, λ_t then completely characterize the lowest-energy parity-violating interaction and can in principle be determined experimentally, as we shall discuss below[7]. Alternatively, instead of a total isotopic spin representation, we can write things in terms of the equivalent notation

$$\lambda_s^{pp}=\lambda_s^0+\lambda_s^1+\frac{1}{\sqrt{6}}\lambda_s^2,\quad \lambda_s^{np}=\lambda_s^0-\frac{2}{\sqrt{6}}\lambda_s^2,\quad \lambda_s^{nn}=\lambda_s^0-\lambda_s^1+\frac{1}{\sqrt{6}}\lambda_s^2. \quad (74)$$

In Born approximation we can represent this interaction in terms of a simple effective NN potential. Integrating by parts, we have

$$\int \mathrm{d}^3r'\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}\{-i\boldsymbol{\nabla},\delta^3(\boldsymbol{r}')\}e^{i\boldsymbol{k}\cdot\boldsymbol{r}'}=(-i\boldsymbol{\nabla}+\boldsymbol{k})\frac{e^{ikr}}{r} \quad (75)$$

which represents the parity-violating contribution to the scattering wave function in terms of an S-wave admixture to the scattering P-wave state —$\sim\boldsymbol{\sigma}\cdot\boldsymbol{k}e^{ikr}/r$—

[5] There appear to exist *eleven* terms —C_i, $\tilde{C}_i$, $i=1,\dots,5$ plus $\tilde{C}_6$. However, $\tilde{C}_2$ and $\tilde{C}_4$ appear only in the combination $\tilde{C}_2+\tilde{C}_4$.

[6] An alternative low-energy form based on the Bethe-Goldstone equation has been given by Desplanques and Missimer [50].

[7] Note that there exists no singlet analog to the spin-triplet constant c_t since the combination $\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2$ is proportional to the total spin operator and vanishes when operating on a spin-singlet state.

plus a P-wave admixture to the scattering S-state —$\sim -i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}e^{ikr}/r$. We observe then that the parity-violating component of the scattering wave function can be described via the effective potential

$$U(\boldsymbol{r}) = \frac{4\pi}{M}\Bigg[\lambda_t a_t(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla},\delta^3(\boldsymbol{r})\}P_1 \\ +\rho_t a_t(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla},\delta^3(\boldsymbol{r})\}\frac{1}{2}(\tau_1-\tau_2)_z \\ +(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{-i\boldsymbol{\nabla},\delta^3(\boldsymbol{r})\}a_s \\ \times\left(P_0\lambda_s^0+\frac{1}{2}(\tau_1+\tau_2)_z\lambda_s^1+\frac{3\tau_{1z}\tau_{2z}-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{2\sqrt{6}}\lambda_s^2\right)\Bigg], \tag{76}$$

which is the PV analog of eq. (17).

Before application of this effective potential we must worry about the stricture of unitarity, which we have seen can be enforced in effective field theory language by using a Lippman-Schwinger solution. However, things become more interesting in the case of the parity-violating transitions, for which the requirement of unitarity reads, *e.g.*, for the case of scattering in the $^3S_1$-$^1P_1$ channel

$$\mathrm{Im}\, d_t(k) = k[m_t^*(k)d_t(k)+d_t^*(k)m_p(k)], \tag{77}$$

where $m_p(k)$ is the $^1P_1$ analog of $m_t(k)$. Equation (77) is satisfied by the solution

$$d_t(k) = |d_t(k)|e^{i(\delta_{^3S_1}(k)+\delta_{^1P_1}(k))} \tag{78}$$

i.e., the phase of the parity-violating transition amplitude should be the sum of the strong-interaction phases in the incoming and outgoing channels [51]. At very low energies we can neglect the P-wave phase and can write, following Danilov, the (approximately) unitarized forms

$$c_t(k)\simeq\rho_t m_t(k),\quad d_s^i(k)\simeq\lambda_s^i m_s(k),\quad d_t(k)\simeq\lambda_t m_t(k). \tag{79}$$

Since at threshold $m_t(k), m_s(k)\to a_t, a_s$, the threshold values of the parity-violating amplitudes become

$$c_t(0)=\rho_t a_t,\qquad d_s^i(0)\simeq\lambda_s^i a_s,\qquad d_t(0)\simeq\lambda_t a_t \tag{80}$$

and it is for this reason that the empirical S-P mixing parameters are defined by multiplying Danilov parameters by the relevant S-wave scattering lengths.

This result is also easily seen in the language of EFT, wherein the full transition matrix must include the weak amplitude to lowest order accompanied by rescattering in both incoming and outgoing channels to all orders in the strong interaction. If we represent the lowest-order weak contact interaction as

$$T_{0tp}(k) = D_{0tp}(\mu)(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}+\boldsymbol{k}'), \tag{81}$$

then the full amplitude is given by

$$T_{tp}(k) = \frac{D_{0tp}(\mu)}{(1-C_{0t}(\mu)G_0(k))(1-C_{0p}(\mu)G_1(k))} \\ \times(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}+\boldsymbol{k}'), \tag{82}$$

where we have introduced a lowest-order contact term C_{0p} which describes the $^1P_1$-wave nn interaction. Since the phase of the combination $1-C_0(\mu)G_0(k)$ is simply the negative of the strong-interaction phase the unitarity stricture is clear, and we can define the physical transition amplitude A_{tp} via

$$A_{tp}\equiv(1+ika_t)(1+ik^3a_p) \\ \times\frac{D_{0tp}(\mu)}{(1-C_{0t}(\mu)G_0(k))(1-C_{0p}(\mu)G_1(k))}. \tag{83}$$

Making the identification $\lambda_t = -\frac{m_N}{4\pi}A_{tp}$ and noting that

$$\frac{1}{1+ika_t} = \cos\delta_t(k)e^{i\delta_t(k)}$$

the Danilov parameter λ_t is seen to be identical to the R-matrix element defined by Miller and Driscoll [51].

The "mystery" of how *ten* contact terms —C_i, $\tilde{C}_i$— can be related to only *five* Danilov parameters can be solved by noting that the matrix elements of the commutator and anticommutator are identical in the zero-range (contact interaction) approximation —ZRA— in which $m\to\infty$[8]

$$\lim_{m\to\infty}\langle P|[-i\boldsymbol{\nabla}, f_m(r)]|S\rangle = \lim_{m\to\infty}\langle P|\{-i\boldsymbol{\nabla}, f_m(r)\}|S\rangle. \tag{84}$$

In this limit the contribution of the various operators characterized by $C_i\,\tilde{C}_i$ to observables can only occur in five different combinations, which may be found by use of the identity

$$\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2 = -\frac{i}{2}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2). \tag{85}$$

Thus for the $^1S_0$-$^3P_0$ parameters $d_s^{0,1,2}(k)$, we have $\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2 = i\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2$ and we find that C_i and $\tilde{C}_i$ appear in the combination $C_i+\tilde{C}_i$ —*i.e.*, in the ZRA, the dependence in the different channels upon the EFT parameters C_i, $\tilde{C}_i$ must be

i) $^1S_0\to{}^3P_0$ pp: $C_1+C_2+C_3+C_4-2C_5+(C_i\to\tilde{C}_i)$,
ii) $^1S_0\to{}^3P_0$ nn: $C_1-C_2+C_3-C_4-2C_5+(C_i\to\tilde{C}_i)$,
iii) $^1S_0\to{}^3P_0$ pn: $C_1+C_3+4C_5+(C_i\to\tilde{C}_i)$.

On the other hand, in the case of the $^3S_1$ parameter d_t we have $\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2=-i\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2$ and so that C_i and $\tilde{C}_i$ appear in the combination $C_i-\tilde{C}_i$ —*i.e.*, in the ZRA,

iv) $^3S_1\to{}^3P_1$ pn: $C_1-3C_3-(C_i\to\tilde{C}_i)$.

Finally, in the case of the $^3S_1$ parameter $c_t(k)$, we exploit the isotopic spin analog of eq. (85):

$$\boldsymbol{\tau}_1-\boldsymbol{\tau}_2 = -\frac{i}{2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2), \tag{86}$$

so that, in the ZRA, the dependence must be

v) $^3S_1\to{}^1P_1$ pn: $\tilde{C}_6+\frac{1}{2}(C_2-C_4)$.

[8] This is clear since a gradient operator acting on an S-state wave function yields a term linear in r, which vanishes at the origin.

An alternate way to understand the feature that there can be only five independent low-energy observables has recently been presented by Girlanda [52]. Using the feature that in the nonrelativisitic limit the twelve forms given in the definition of the EFT potential eq. (57) can be replaced by the twelve relativistic operators

$$\begin{aligned}
\mathcal{O}_1 &= \bar\psi\gamma^\mu\psi\bar\psi\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_1 &= \bar\psi\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi),\\
\mathcal{O}_2 &= \bar\psi\gamma^\mu\psi\bar\psi\tau_3\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_2 &= \bar\psi\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\tau_3\sigma_{\mu\nu}\psi),\\
\mathcal{O}_3 &= \bar\psi\tau_a\gamma^\mu\psi\bar\psi\tau^a\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_3 &= \bar\psi\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\tau^a\sigma_{\mu\nu}\psi),\\
\mathcal{O}_4 &= \bar\psi\tau_3\gamma^\mu\psi\bar\psi\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_4 &= \bar\psi\tau_3\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\sigma_{\mu\nu}\psi),\\
\mathcal{O}_5 &= \mathcal{I}_{ab}\bar\psi\tau_a\gamma^\mu\psi\bar\psi\tau_b\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_5 &= \mathcal{I}_{ab}\bar\psi\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\tau_b\sigma_{\mu\nu}\psi),\\
\mathcal{O}_6 &= i\epsilon_{ab3}\bar\psi\tau_a\gamma^\mu\psi\bar\psi\tau_b\gamma_\mu\gamma_5\psi,\\
\tilde{\mathcal{O}}_6 &= i\epsilon_{ab3}\bar\psi\tau_a\gamma^\mu\gamma_5\psi\partial^\nu(\bar\psi\tau_b\sigma_{\mu\nu}\psi),
\end{aligned}\tag{87}$$

then, with the use of Fierz transformations and the free particle equation of motion, one finds the six conditions

$$\begin{aligned}
\mathcal{O}_3 &= \mathcal{O}_1,\\
\mathcal{O}_2-\mathcal{O}_4 &= 2\mathcal{O}_6,\\
\tilde{\mathcal{O}}_3+3\tilde{\mathcal{O}}_1 &= 2m_N(\mathcal{O}_1+\mathcal{O}_3),\\
\tilde{\mathcal{O}}_2+\tilde{\mathcal{O}}_4 &= m_N(\mathcal{O}_2+\mathcal{O}_4),\\
\tilde{\mathcal{O}}_2-\tilde{\mathcal{O}}_4 &= -2m_N\mathcal{O}_6-\tilde{\mathcal{O}}_6,\\
\tilde{\mathcal{O}}_5 &= \mathcal{O}_5.
\end{aligned}\tag{88}$$

Finally, using the feature that the operators $\mathcal{O}_6$ and $\tilde{\mathcal{O}}_6$ have the same form in the lowest-order nonrelativisitic expansion, we find an effective (pionless) potential

$$\begin{aligned}
V_{EFT} = \frac{2\mu^2}{\Lambda_\chi^3}\Bigg[&\left(C_1+(C_2+C_4)\left(\frac{\tau_1+\tau_2}{2}\right)_z+C_5\mathcal{I}_{ab}\tau_{1a}\tau_{2b}\right)\\
&\times(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{\boldsymbol{p}_1-\boldsymbol{p}_2,f_\mu(r)\}\\
&+i\tilde C_1(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[\boldsymbol{p}_1-\boldsymbol{p}_2,f_\mu(r)]\\
&+iC_6\epsilon_{ab3}\tau_{1a}\tau_{2b}[\boldsymbol{p}_1-\boldsymbol{p}_2,f_\mu(r)]\Bigg]
\end{aligned}\tag{89}$$

expressed in terms of just five independent constants, as required. Of course, the use of the free particle equation of motion means that binding effects are omitted in this reduction. Binding effects arise in terms higher order in the chiral expansion, so this omission is consistent with the use of the LO form of the effective potential, as done in our analysis. The Girlanda and Zhu *et al.* forms are then completely equivalent and one can choose to use either form as the low-energy parity-violating potential. In this article, we shall continue to use the conventional analysis of Zhu *et al.*, since it can be connected straightforwardly to the DDH picture, within which nearly all experimental results have been presented.

The effect of higher-order terms, which are omitted in our lowest-order expansion, can be gauged by the use of finite-range and realistic nucleon wave functions, whereby the simple dependences of Danilov parameters on Zhu coefficients expounded above are modified. For example, Desplanques and Benayoun quote the approximate results [53]

$$\begin{aligned}
\lambda_s^{pp} &= -K_p\Big[B_6(C_1+C_2+C_3+C_4-2C_5+(C_i\to\tilde C_i))\\
&\quad+B_7(C_1+C_2+C_3+C_4-2C_5-(C_i\to\tilde C_i))\Big],\\
\lambda_s^{nn} &= -K_p\Big[B_6(C_1-C_2+C_3-C_4-2C_5+(C_i\to\tilde C_i))\\
&\quad+B_7(C_1-C_2+C_3-C_4-2C_5-(C_i\to\tilde C_i))\Big],\\
\lambda_s^{np} &= -K_p\Big[B_6(C_1+C_3+4C_5+(C_i\to\tilde C_i))\\
&\quad+B_7(C_1+C_3+4C_5-(C_i\to\tilde C_i))\Big],\\
\lambda_t &= -K_p\Big[B_4(C_1-3C_3+(C_i\to\tilde C_i))\\
&\quad+B_5(C_1-3C_3-(C_i\to\tilde C_i))\Big],\\
\rho_t &= -K_p\left[B_2\left(\frac{1}{2}(C_2-C_4)+\tilde C_6\right)\right.\\
&\quad\left.+B_3\left(\frac{1}{2}(C_2-C_4)-\tilde C_6\right)\right],
\end{aligned}\tag{90}$$

where $K_p = 2\Lambda_\chi^9/m_N^4m_\rho^6$ and the Reid soft core potential values for the B_i are found to be

$$B_i = [-0.0043, 0.0005, -0.0009, -0.0022, -0.0067, 0.0003]\quad \text{for } i=2,3,\ldots,7.$$

We see then that the size of the finite-range corrections to the lowest-order results are given by

$$|B_3/B_2| = 0.12, \qquad |B_7/B_6| = 0.04$$

and

$$|B_4/B_5| = 0.41.$$

The first two ratios are rather small and suggest that zero range is a reasonable first approximation. In the case of the ratio $|B_4/B_5|$ there are sizable corrections to the zero-range result as a consequence of important tensor force contributions.

Alternatively, for example, in a "hybrid" pionless theory which uses the pionless potential but with AV18 wave functions, choosing $m\sim m_\pi$ Liu has evaluated these cobinations and finds [49]

$$B_i = [0.0014, 0.0008, 0.0005, 0.0008, 0.0023, 0.0003]\quad \text{for } i=2,3,\ldots,7,$$

which are somewhat different from the estimates of Desplanques and Beneyoun and indicate some of the uncertainties associated with such analyses.

In any case we see that NLO and higher-order corrections can be of order 25% or so, which is suggestive of omitted terms in the LO chiral expansion of order nuclear binding energy or Fermi momentum —$\sim 250\,\mathrm{MeV}$— over the usual chiral expansion parameter $\Lambda_\chi \sim 4\pi F_\pi \sim 1\,\mathrm{GeV}$. An important goal of future analyses should be to include such effects by proceeding beyond the simple LO analysis given herein.

We now address the form in which to present predictions of the theory. As emphasized above, in the past most experimental numbers are interpreted in terms of the DDH parameters f_π^1, $h_{\rho,\omega}^i$. However, in an effective field-theoretic framework one wants to express predictions in terms of the parameters of the theory —in our case $C_i\,\tilde{C}_i$. However, because these ten constants must appear only in the combinations given above in analysis of threshold processes, it is more convenient to represent all predictions in terms of the five Danilov parameters, which have a rather direct connection to observables. Before presenting these predictions, however, we first show how these five parameters can be (approximately) analytically connected to the underlying low-energy constants. As an example, consider the parameter λ_t. Since the associated interaction is short-ranged, we can use this feature in order to simplify the analysis. For example, we can determine the shift in the deuteron wave function associated with parity violation by demanding orthogonality with the ${}^3S_1$ scattering state, which yields, using the simple asymptotic form of the bound-state wave function [54,55]

$$\psi_d(r) = [1 + \rho_t(\boldsymbol{\sigma}_p + \boldsymbol{\sigma}_n)\cdot -i\boldsymbol{\nabla} + \lambda_t(\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_n)\cdot -i\boldsymbol{\nabla}] \times \sqrt{\frac{\gamma}{2\pi}}\frac{1}{r}e^{-\gamma r}, \tag{91}$$

where $E = -\gamma^2/M = -2.23\,\mathrm{MeV}$ is the deuteron binding energy. Now the shift generated by $V^{PV}(\boldsymbol{r})$ is found to be [54,55]

$$\begin{aligned}\delta\psi_d(\boldsymbol{r}) &\simeq \int \mathrm{d}^3r' G(\boldsymbol{r},\boldsymbol{r}')V^{PV}(\boldsymbol{r}')\psi_d(r')\\ &= -\frac{m_N}{4\pi}\int \mathrm{d}^3r' \frac{e^{-\gamma|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}V^{PV}(\boldsymbol{r}')\psi_d(r')\\ &\simeq \frac{m_N}{4\pi}\boldsymbol{\nabla}\left(\frac{e^{-\gamma r}}{r}\right)\cdot\int \mathrm{d}^3r'\boldsymbol{r}'V^{PV}(\boldsymbol{r}')\psi_d(r'),\end{aligned} \tag{92}$$

where the last step is permitted by the short range of $V^{PV}(\boldsymbol{r}')$. Comparing eqs. (92) and (91) yields then the identification

$$\sqrt{\frac{\gamma}{2\pi}}\lambda_t\chi_t \equiv i\frac{m_N}{16\pi}\xi_0^\dagger\int \mathrm{d}^3r'(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{r}'V^{PV}(\boldsymbol{r}')\psi_d(r')\chi_t\xi_0, \tag{93}$$

where we have included the normalized isoscalar wave function ξ_0 since the potential involves $\boldsymbol{\tau}_1, \boldsymbol{\tau}_2$. When operating on such an isosinglet np state the PV potential can be written as

$$\begin{aligned}V^{PV}(\boldsymbol{r}') = \frac{2}{\Lambda_\chi^3}&[(C_1 - 3C_3)(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\\ &\cdot(-i\boldsymbol{\nabla}f_m(r) + 2f_m(r)\cdot -i\boldsymbol{\nabla})\\ &+(\tilde{C}_1 - 3\tilde{C}_3)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{\nabla}f_m(r)],\end{aligned} \tag{94}$$

whereby eq. (93) becomes

$$\begin{aligned}\sqrt{\frac{\gamma}{2\pi}}\lambda_t\chi_t \simeq{}& \frac{2m_N}{16\pi\Lambda_\chi^3}\frac{4\pi}{3}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)^2\chi_t\int_0^\infty \mathrm{d}r r^3\\ &\times\left[-2(3C_3 - C_1)f_m(r)\frac{\mathrm{d}\psi_d(r)}{\mathrm{d}r}\right.\\ &\left.+(3\tilde{C}_3 - 3C_3 - \tilde{C}_1 + C_1)\frac{\mathrm{d}f_m(r)}{\mathrm{d}r}\psi_d(r)\right] =\\ &\sqrt{\frac{\gamma}{2\pi}}\cdot 4\chi_t\frac{1}{12}\frac{4m_N m^3}{4\pi\Lambda_\chi^3}\frac{K_c}{(\gamma+m)^2},\end{aligned} \tag{95}$$

where

$$\begin{aligned}K_c = 2m&(6C_3 - 3\tilde{C}_3 - 2C_1 + \tilde{C}_1)\\ &+\gamma(15C_3 - 3\tilde{C}_3 - 5C_1 + \tilde{C}_1),\end{aligned} \tag{96}$$

or

$$\lambda_t \simeq -\frac{m_N m^3}{3\pi\Lambda_\chi^3}\frac{K_c}{(\gamma+m)^2}. \tag{97}$$

Performing the indicated integration and using $m \sim m_\rho$ we find the result

$$\lambda_t \simeq -0.020(-2C_1 + \tilde{C}_1) + 0.060(-2C_3 + \tilde{C}_3). \tag{98}$$

However, this is clearly an overestimate because it was obtained i) using the asymptotic form of the wave function and ii) omits short-range correlation effects. In order to deal approximately with the short-distance properties of the deuteron wave function, we modify the exponential form to become constant inside the deuteron radius R [54,55]

$$\sqrt{\frac{\gamma}{2\pi}}\frac{1}{r}e^{-\gamma r} \to N\begin{cases}\frac{1}{R}e^{-\gamma R}, & r \le R,\\ \frac{1}{r}e^{-\gamma r}, & r > R,\end{cases} \tag{99}$$

where

$$N = \sqrt{\frac{\gamma}{2\pi}}\frac{\exp\gamma R}{\sqrt{1+\frac{2}{3}\gamma R}}$$

is the modified normalization factor and we use $R = 1.6\,\mathrm{fm}$. As to the short-range (Jastrow) correlation, we multiply the wave function by the simple phenomenological form [56]

$$\phi(r) = 1 - ce^{-dr^2}, \quad \text{with} \quad c = 0.6,\ d = 3\,\mathrm{fm}^{-2}. \tag{100}$$

With these modifications we determine the much more reasonable values for the Danilov parameter λ_t

$$\lambda_t = [0.003(-2C_3 + \tilde{C}_3) - 0.002(-2C_1 + \tilde{C}_1)]m_N^{-1}. \tag{101}$$

In this way approximate analytic forms can also be found for the remaining Danilov parameters [57].

However, it is obviously preferable to use estimates obtained using the best available wave functions. In this way Liu determines that [49]

$$\lambda_t = [0.0045(-2.23C_3 + \tilde{C}_3) - 0.0015(-2.23C_1 + \tilde{C}_1)]m_N^{-1}. \tag{102}$$

The similarity with the approximate analytic expression is obvious —the discrepancy with the coefficients involving $C_3, \tilde{C}_3$ is again due to effects from the tensor interaction— and the other coefficients are found in this way to be

$$\begin{aligned}
\lambda_s^{pp} &= 0.0043[(C_1 + C_2 + C_3 + C_4 - 2C_5) \\
&\quad +1.27(C_i \to \tilde{C}_i)]m_N^{-1}, \\
\lambda_s^{nn} &= 0.0046[(C_1 - C_2 + C_3 - C_4 - 2C_5) \\
&\quad +1.22(C_i \to \tilde{C}_i)]m_N^{-1}, \\
\lambda_s^{np} &= 0.0047[(C_1 + C_3 + 4C_5) + 1.24(C_i \to \tilde{C}_i)]m_N^{-1} \\
\rho_t &= 0.0031[\tilde{C}_6 + 0.60(C_2 - C_4)]m_N^{-1}.
\end{aligned} \tag{103}$$

A connection between the underlying EFT Lagrangian and the empirical Danilov parameters has thus been established.

In the next section we shall describe how the results of various low-energy experiments can be expressed in terms of the Danilov parameters. In the design of such experiments, it is obviously useful to have at hand numerical values for the size of these quantities and the use of the DDH estimates for C_i, $\tilde{C}_i$ provides a reasonable way to provide such numbers. Of course, the completely consistent and correct way to accomplish this is to use a pionful theory, with one- and two-pion exchange pieces described in terms of f_π^1 and the remaining terms written in terms of short-distance quantities C_i, $\tilde{C}_i$. However, for the reasonable and simple estimates needed below we shall instead employ approximate values for the Danilov parameters which include the effects of heavy meson exchange for λ_t, $\lambda_s^{0,1,2}$ and pion exchange for the parameter ρ_t, yielding

$$\begin{aligned}
{}^{DDH}\lambda_s^{pp} &= 2.3 \times 10^{-7} m_N^{-1}, \\
{}^{DDH}\lambda_s^{nn} &= 2.1 \times 10^{-7} m_N^{-1}, \\
{}^{DDH}\lambda_s^{np} &= 0.8 \times 10^{-7} m_N^{-1}, \\
{}^{DDH}\lambda_t &= 0.6 \times 10^{-7} m_N^{-1}, \\
{}^{DDH}\rho_t &= 5.6 \times 10^{-7} m_N^{-1}.
\end{aligned} \tag{104}$$

These numbers, of course, should *not* be treated as being in any sense precise. However, they are useful in estimating the possible size of experimental effects, as will be seen below.

4 Experimental program

Having developed a connection of the five Danilov parameters with the underlying effective Lagrangian, our next task is to develop a program whereby experimental values of these quantities can be reliably determined. Since we desire to generate *definitive* values for these constants, we certainly do not wish to introduce nuclear-physics uncertainties into the analysis. Thus we shall require *only* experiments involving systems with $A \leq 4$, for which nuclear wave functions are well determined. We recognize that, by imposing this requirement we eliminate the opportunity to enhance experimental signals via the careful choice of near-degenerate opposite parity levels that has permitted experiments in ^{18}F [45], ^{19}F [44], ^{21}Ne [58] with precision of parts in 100000 or even in 10000 to provide useful input into the parity-violating interaction puzzle. As a consequence, the experimental signals we need to analyze will be a part in 10000000 or even smaller! Nevertheless, we consider this a price worth paying in order to have confidence in the interpretation of the experimental signals.

Since there is a need to determine *five* parameters, we clearly require a minimum of *five* independent measurements. To the extent that all experiments are performed at threshold, the analysis can only involve five independent combinations of EFT parameters. We elect to present our predictions in terms of the five Danilov coefficients, because of their phenomenological signifcance, but this choice is somewhat arbitrary and there is no implication that this representation is superior to other possibilities. (Of course, if we stray above threshold, additional terms can come in.) As discussed above, we do not possess at this time a modern first principles theoretical analysis of each of the experimental possibilities. Hence, in the discussion below we present approximate existing estimates within the simple pionless theory, with an effective contact potential in terms of the Danilov coefficients. A fully consistent pionless calculation would then evaluate corresponding diagrams generated within this framework. However, this does not exist at this time, so that in order to match onto experiment we shall have to use results from a variety of different calculational schemes. *Providing a rigorous theoretical analysis of each experiment within the same rigorous calculational framework should be a priority for future work.* With these caveats, we shall consider five such possible reactions in turn:

i) ***pp*** scattering asymmetry: the first and simplest such reaction has already been performed and involves the asymmetry in the scattering cross-section for longitudinally polarized protons on an unpolarized proton target. The experimental signal is the difference in the right- and left-handed total scattering cross-sections divided by their sum

$$A_h = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}\,.$$

Results are available from a Bonn experiment at lab energy 13.6 GeV and from a PSI experiment at 45 MeV[9]:

$$\begin{aligned}
A_h(13.6\,\text{MeV}) &= -(0.93 \pm 0.20 \pm 0.05) \times 10^{-7}\ [60], \\
A_h(45\,\text{MeV}) &= -(1.57 \pm 0.23) \times 10^{-7}\ [61].
\end{aligned} \tag{106}$$

[9] Note that there also exists a Los Alamos measurement at 15 MeV

$$A_h(15\,\text{MeV}) = -(1.7 \pm 0.8) \times 10^{-7}\ [59] \tag{105}$$

which is quite consistent with the asymmetry measured at 13.6 MeV. However, because of its superior precision, we shall use only the Bonn result.

The feature that the PSI number is about 60% greater than its Bonn analog is consistent with the feature that the asymmetry should depend roughly linearly on the proton momentum which depends on energy as

$$\frac{k_{PSI}}{k_{Bonn}} \sim \sqrt{\frac{45}{13.6}} = 1.8. \tag{107}$$

Thus these two experiments should *not* be considered as yielding independent numbers.

The connection with the Danilov parameters can be found by calculating the helicity-correlated cross-sections which, since the initial state must be in a spin-singlet, must have the form [57]

$$\begin{aligned}\sigma_\pm = &\int \mathrm{d}\Omega \frac{1}{2}\,\mathrm{Tr}\,\mathcal{M}(\boldsymbol{k}', \boldsymbol{k})\frac{1}{2}(1+\boldsymbol{\sigma}_2\cdot\hat{k})\\ &\cdot\frac{1}{4}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathcal{M}^\dagger(\boldsymbol{k}', \boldsymbol{k}) =\\ &|m_s(k)|^2 \pm 4k\,\mathrm{Re}\, m_s^*(k)d_s^{nn}(k) + \mathcal{O}(d_s^2).\end{aligned} \tag{108}$$

Defining the asymmetry via the sum and difference of such helicity cross sections and neglecting the tiny P-wave scattering, we have then

$$\begin{aligned}A_h(E_{\text{threshold}}) &= \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}\\ &= -\frac{8k\,\mathrm{Re}\,m_s^*(k)d_s^{nn}(k)}{2|m_s(k)|^2} = -4k\lambda_s^{pp}.\end{aligned} \tag{109}$$

Thus the threshold helicity-correlated pp scattering asymmetry provides a direct measure of the parity-violating parameter λ_s^{pp}.

Of course, the actual experiments are performed not at threshold but rather at the finite laboratory energies quoted above. Converting to momentum we find that the corresponding numbers are 160 MeV/c and 290 MeV/c respectively, so that the simple threshold relation above must be modified and the relation of the asymmetry to the Danilov parameter λ_s^{pp} becomes somewhat more complex [49]:

$$\begin{aligned}A_h(13.6\,\mathrm{MeV}) &= -0.449 m_N\lambda_s^{pp},\\ A_h(45\,\mathrm{MeV}) &= -0.795 m_N\lambda_s^{pp}.\end{aligned} \tag{110}$$

We see that the dominant dependence is on the Danilov parameter λ_s^{pp}, with corrections relatively small at 45 MeV and tiny at 13.6 MeV. However, the momentum involved in the 45 MeV experiment certainly gives one pause and a recent careful EFT analysis by Phillips *et al.* suggests that the agreement between the simple momentum-scaled experimental numbers may be fortuitous, as NLO corrections are expected to be significant [62].

It should be noted for completeness that there exist additional measurements of the $\boldsymbol{pp}$ asymmetry at 221.3 MeV and at 800 MeV performed at TRIUMF and LANL, respectively. In the case of the former, the energy was carefully selected in order that S-P parity mixing effects vanish, leaving sensitivity to P-D mixing and allowing separation of the DDH parameters involving isoscalar rho exchange and omega exchange. A precise number [63]

$$A_h(221.3\,\mathrm{MeV}) = (0.83 \pm 0.29 \pm 0.17)\times 10^{-7} \tag{111}$$

was obtained. However, P-D mixing is beyond the scope of our parity-violating EFT and would have to be accounted for by inclusion of a an entirely new set of phenomenological parameters. Nevertheless Carlson *et al.* have reported being able to fit both the low- and higher-energy data within the DDH scheme [64]. The 800 MeV result [65]

$$A_h(800\,\mathrm{MeV}) = (2.4 \pm 1.1)\times 10^{-7} \tag{112}$$

is positive, as expected from the feature that the S-P and P-D interference terms both contribute positively above 220 MeV, but again a detailed analysis requires input which is well beyond the scope of our low-energy EFT methods.

Considering only the low-energy results then, we find from the above measurements the result

$$\lambda_s^{pp} \simeq (2.0 \pm 0.3)\times 10^{-7} m_N^{-1} \tag{113}$$

which is the only really solid experimental measurement of a Danilov parameter which exists at present. Note that this limit is quite consistent with the DDH "best value" estimate

$$^{DDH}\lambda_s^{pp} = 2.3\times 10^{-7} m_N^{-1}. \tag{114}$$

ii) $\boldsymbol{p\alpha}$ scattering asymmetry: a second experiment which is relevant to this program is a 45 MeV proton helicity asymmetry experiment on a ^{4}He target, which was performed at PSI, yielding [43]

$$A_h(45\,\mathrm{MeV}) = -(3.3 \pm 0.9)\times 10^{-7}. \tag{115}$$

The problem here is that we do not yet have a precise theoretical prediction in terms of Danilov parameters. There does exist, however, a Desplanques and Missimer calculation [50]

$$\begin{aligned}A_h(45\,\mathrm{MeV}) = -&\left[0.48\left(\lambda_s^{pp} + \frac{1}{2}\lambda_s^{pn}\right)\right.\\ &\left.+1.07\left(\rho_t + \frac{1}{2}\lambda_t\right)\right] m_N\end{aligned} \tag{116}$$

which provides a constraint

$$0.48\lambda_s^{pn} + 2.14\left(\rho_t + \frac{1}{2}\lambda_t\right) = (4.6 \pm 2.0)\times 10^{-7} m_N^{-1} \tag{117}$$

quite consistent with the DDH "best value" estimate

$$0.48\,{}^{DDH}\lambda_s^{pn} + 2.14\left({}^{DDH}\rho_t + \frac{1}{2}\,{}^{DDH}\lambda_t\right) = 4.8\times 10^{-7} m_N^{-1}. \tag{118}$$

However, what is needed here is a *definitive* theoretical analysis. (Of course, the rather deep 28 MeV binding energy of ^{4}He might give one pause as to whether a calculation in terms of simply the threshold (Danilov) parameters is adequate. On the other hand, a recent paper is successful in explaining a correlation between the triton and alpha binding energies in various pionless theories [66], so this issue invites further study.)

An additional source of information is provided by experiments involving the radiative capture of neutrons on a proton target

$$n + p \to d + \gamma$$

for which a solid theoretical analysis *is* available. There exist two independent parity-violating observables in this reaction:

iii) Asymmetry in $\boldsymbol{np}$ capture: one possible measurement involves the photon asymmetry in the case of polarized neutron capture, for which one finds [49]

$$A_\gamma = -0.093 m_N \rho_t. \tag{119}$$

On the experimental side, there exist already two results, one from an old Grenoble measurement and one from a new LANSCE experiment:

$$\begin{aligned} A_\gamma &= (0.6 \pm 2.1) \times 10^{-7}\ [67], \\ A_\gamma &= (-1.1 \pm 2.0 \pm 0.2) \times 10^{-7}\ [68]. \end{aligned} \tag{120}$$

While these two numbers are in good agreement and are of impressive precision, considerable improvement is still called for. That is because the dominant piece of the Danilov parameter ρ_t comes from one-pion exchange and therefore depends upon the PV pion emission amplitude f_π^1. Using the DDH "best value" for this number, the corresponding experimental prediction for A_γ is

$${}^{DDH}A_\gamma^{th} = -5 \times 10^{-8} \tag{121}$$

which is a full order of magnitude *smaller* than the levels probed by the existing experiments! For this reason, the LANSCE experiment has been disassembled and moved to the new fundamental neutron physics beamline at SNS, where the associated increased intensity should allow a measurement at the level of a few parts per billion. In fact this experiment is the commissioning experiment for this beamline and should commence later this year.

The theoretical prediction is in good agreement with previous calculations —cf. [69]— and depends predominantly on the parity-violating pion coupling f_π^1. Thus measurement of the $\boldsymbol{np} \to d\gamma$ asymmetry with the hoped for precision should finally resolve the burning question of whether this long-range coupling is of the order or considerably smaller than its DDH prediction.

iv) Circular polarization in np capture: an independent probe of parity violation in radiative neutron capture is provided by the possibility of measuring the circular polarization of the outgoing photon resulting from the capture of *unpolarized* neutrons, for which the prediction in terms of Danilov parameters is [49]

$$P_\gamma = -0.161 m_N \lambda_s^{np} + 0.670 m_N \lambda_t. \tag{122}$$

This is an old idea and the first attempt to measure this parameter was done in 1972 by Lobashov *et al.* who reported a value

$$P_\gamma = -(1.3 \pm 0.45) \times 10^{-6}\ [70]. \tag{123}$$

It was later realized that this experiment was contaminated by polarized bremsstrahlung photons from fission products in the reactor and the number was subsequently revised downward to

$$P_\gamma = (1.8 \pm 1.8) \times 10^{-7}\ [71]. \tag{124}$$

Again, however, despite its impressive precision, a considerably improved measurement is needed, since use of eq. (122) with DDH "best values" for the Danilov parameters yields a prediction

$${}^{DDH}P_\gamma^{th} = 2.7 \times 10^{-8} \tag{125}$$

considerably *below* the current experimental precision. Improvement of the existing limit will be challenging, however, because of the relatively low efficiencies of circular polarization detectors, and it may be advantageous to use the time-reversed reaction

$$\boldsymbol{\gamma} + d \to n + p$$

for which the asymmetry using circularly polarized photons is equal, using detailed balance, to the circular polarization in the radiative capture reaction. Nevertheless, either experiment will be extraordinarily difficult since the theoretical expectation is so small.

Note that the predicted value depends only on the short-distance–dominated Danilov parameters λ_s^{np} and λ_t and is independent of the PV pion coupling. Nevertheless the predicted DDH value is in reasonable agreement with previous estimates —cf. [69].

v) As a fifth experiment in this program, one can utilize neutron spin rotation when passing through a parahydrogen target, for which the rotation rate is predicted to be [49]

$$\frac{\mathrm{d}\phi^{np}}{\mathrm{d}z} = [2.500\lambda_s^{np} - 0.571\lambda_t + 1.412\rho_t] m_N \,\mathrm{rad/m}. \tag{126}$$

The use of the DDH "best value" numbers then predicts the small number

$${}^{DDH}\left(\frac{\mathrm{d}\phi^{np}}{\mathrm{d}z}\right)^{th} = 9.6 \times 10^{-7}\,\mathrm{rad/m}. \tag{127}$$

but a planned experiment at SNS anticipates a precision at the level of 2.7×10^{-7} rad/m and will provide an important data point. However, such experiments are very challenging, since one must shield the system from external magnetic fields, for which Faraday rotation in the Earth's magnetic field yields a rotation considerably larger than those being sought due to the weak interactions.

The theoretical prediction here is of the same rough size and sign as that given in [72] and [49] but differs in sign from an earlier prediction —[73].

We see then that in principle there do indeed exist a complete set of independent low-energy measurements which could be utilized in order to determine the five Danilov parameters. However, since each of the experiments is so challenging it is certainly advisable to *overdetermine* these quantities by performing additional parity-violating experiments in $A < 4$ systems. There are a number of possibilities here.

a) Neutron spin rotation on ^{4}He: this is an experiment which is already underway at NIST. As in the case of $p\alpha$ scattering the use of ^{4}He and its $\sim 28\,\mathrm{MeV}$ binding energy means that the use of EFT methods may be a bit of a stretch. Also, a definitive calculation of the rotation angle has not been performed. Nevertheless an estimate

$$\frac{\mathrm{d}\phi^{n\alpha}}{\mathrm{d}z} = [0.60\lambda_s^{np} + 1.34\lambda_t - 2.68\rho_t + 1.2\lambda_s^{nn}]\, m_N \,\mathrm{rad/m} \tag{128}$$

is available [50]. The use of the DDH estimates for the Danilov parameters yields then

$${}^{DDH}\left(\frac{\mathrm{d}\phi^{n\alpha}}{\mathrm{d}z}\right)^{th} = -11.7 \times 10^{-7}\,\mathrm{rad/m},$$

which is larger than and of opposite sign compared to the corresponding np number quoted above. There exists an experimental number for this quantity from a University of Washington Thesis [74]

$$\left(\frac{\mathrm{d}\phi^{n\alpha}}{\mathrm{d}z}\right)^{exp} = (8 \pm 14) \times 10^{-7}. \tag{129}$$

However, it is clear that the precision of this measurement is not high enough to place significant limits on the Danilov parameters.

b) Radiative $\boldsymbol{nd}$ capture —$\boldsymbol{n} + d \to t + \gamma$— is being considered at SNS as a possible followup experiment to the radiative $\boldsymbol{np}$ capture. Again a definitive calculation of the photon asymmetry has not yet been performed. However, an estimate

$$A_\gamma^{\boldsymbol{n}} = [1.35\rho_t + 0.58\lambda_s^{nn} + 1.15\lambda_t + 0.50\lambda_s^{pn}]m_N \tag{130}$$

has been given [53]. Using the DDH "best value" estimates, this yields an effect

$${}^{DDH}A_\gamma^{\boldsymbol{n}th} = 9.9 \times 10^{-7} \tag{131}$$

much larger than the corresponding $\boldsymbol{np}$ value. However, the existing experimental number [75]

$$A_\gamma^{\boldsymbol{n}\,exp} = (4.2 \pm 3.8) \times 10^{-6} \tag{132}$$

will have to be improved by nearly an order of magnitude in order to say something meaningful.

Another possibility is to measure the photon asymmetry following the capture of an unpolarized neutron by a polarized deuteron, for which one finds [53]

$$A_\gamma^{\boldsymbol{d}} = -[3.56\rho_t + 0.24\lambda_s^{nn} + 1.39\lambda_t + 0.71\lambda_s^{pn}]m_N \tag{133}$$

yielding an even larger signal using the DDH "best value" estimates

$${}^{DDH}A_\gamma^{\boldsymbol{d}\,th} = 2.2 \times 10^{-6}. \tag{134}$$

However, a high-polarization deuterium target would be required.

Finally, one can imagine measuring the circular polarization of the photon following the capture of an unpolarized neutron, for which an estimate

$$P_\gamma = -[2.73\rho_t + 0.57\lambda_s^{nn} + 1.56\lambda_t + 0.73\lambda_s^{pn}]m_N \tag{135}$$

has been given [53]. Using the DDH "best value" numbers we find an estimate

$${}^{DDH}P_\gamma^{th} = -1.8 \times 10^{-6} \tag{136}$$

again much larger than its corresponding $np \to d\gamma$ value. However, the efficient detection of circular polarization represents a challenge, and the reverse reaction $\boldsymbol{\gamma} + t \to n + d$ is associated with significant safety issues because of the need for a tritium target and is probably not a serious consideration.

c) $\boldsymbol{p}d$ scattering, for which at $15\,\mathrm{MeV}$ has the longitudinal asymmetry [50]

$$A_L^{pd} = -[0.21\rho_t + 0.07\lambda_s^{pp} - 0.13\lambda_t - 0.04\lambda_s^{pn}]m_N. \tag{137}$$

This calculated value is based on the Desplanques-Missimer/Bethe-Goldstone estimate, and should be updated with a modern three-body calculation. However, the use of the DDH "best values" indicates an effect of the size

$${}^{DDH}A_L^{th} = -1.3 \times 10^{-7}. \tag{138}$$

This experiment has been performed both at LANL at $15\,\mathrm{MeV}$ [76] and at PSI at $45\,\mathrm{MeV}$ [77]. However, the measured asymmetry is available only over a limited range of angles. Also the experiments do not distinguish elastic and breakup events. Thus, a detailed theoretical analysis would be required in order to extract information from the existing numbers.

d) Another possibility being considered is neutron spin rotation on deuterium, although experimentally this presents a number of challenges. However, a new precision theoretical estimate is available [78]:

$$\frac{1}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}z} = 2\frac{m_\pi^3}{\Lambda_\chi^3}[270\tilde{C}_6 + 3.6C_1 - 0.1\tilde{C}_1 - 0.5(C_2 + C_4)] \tag{139}$$

in terms of the effective potential developed by Girlanda —eq. (89). Using a liquid-deuterium density of 0.4×10^{23} atoms/cm^3 one finds a "best value" predicted size of about 5×10^{-6} rad/m which is about an order of magnitude larger than the corresponding np number and thus should be seriously considered as a possible source of information provided the experimental challenges can be overcome.

e) An additional followup experiment at SNS is a measurement of the proton asymmetry in the capture of polarized neutrons by ^{3}He —$\boldsymbol{n}^3\text{He} \to pt$. An estimate by M. Viviani has been provided within the DDH model [79]:

$$A_p = -0.18 f_\pi^1 - 0.14 h_\rho^0 + 0.27 h_\rho^1 + 0.0012 h_\rho^2 \\ -0.13 h_\omega^0 + 0.05 h_\omega^1$$

and use of best values yields the estimate $A_p^{\text{best value}} = 1 \times 10^{-7}$, which involves a considerable cancellation between f_π^1 and h_ρ^0 couplings. R&D is now taking place at LANSCE for such an experiment, to begin in 2011.

The completion of the five core experiments supplemented by one or more of the additional possibilities outlined above would (at last!) provide a solid base of empirically determined PV parameters.

5 Future initiatives

At the present time, we have results for only two of the five necessary experimental results. Therefore it is unknown whether implementation of EFT methods will be able to resolve the inconsistencies which exist in the current DDH analysis of hadronic PV experiments. However, through successful completion and analysis of a set of experiments such as those described above, we can anticipate obtaining a consistent set of Danilov parameters at some point is the (near?) future. An obvious question is: what happens next? To some extent the answer to this question depends on whether the results of the experimental program are in some sense surprising in that they are strongly discrepant with the DDH analysis. Let us suppose that this is *not* the case. Then a number of obvious steps are suggested:

i) Firstly, it will be interesting to determine if the values of the parameters $\tilde{C}_i/C_i$ differ from their vector dominance values suggested via single-meson exchange. This will not be easy to do, however, in that in order to make this determination one will have to go above the threshold region in order to separate matrix elements involving the commutator —$[-i\boldsymbol{\nabla}, f_m(r)]$— from those involving the anticommutator —$\{-i\boldsymbol{\nabla}, f_m(r)\}$. This analysis must be done carefully so that P-D mixing effects are appropriately included.

ii) Another interesting topic is the size of the PV coupling f_π^1, for which the present DDH-based analysis indicates a value considerably smaller than the DDH best estimate from analysis of experiments involving ^{18}F but a value considerably larger than the DDH best estimate from analysis of experiments involving ^{133}Cs. It is always possible that the value determined from the Danilov analysis will agree with neither, but if the new number is consistent with either of the present values, something important will be learned about nuclear effects from the analysis of the "losing" experiment.

iii) Once a fully consistent set of values is obtained for the low-energy constants C_i, $\tilde{C}_i$ it will be important to see if the numbers obtained experimentally can be predicted from purely theoretical considerations. At the simplest level one can compare with the DDH expectations. However, because of the uncertainties inherent in the DDH numbers, this may be a challenge. More fundamental should be an attempt to calculate such couplings via lattice methods. Because these are two-nucleon matrix elements involving both the strong *and* weak interactions this will not be a simple calculation but is a necessary ingredient to any real understanding of hadronic parity violation. Some of the challenges associated with any such calculation have recently been discussed by Beane and Savage in the context of a lattice calculation of the pion-nucleon coupling constant [80].

6 Conclusions

The field of hadronic parity violation began in 1957 with the experiment by Tanner looking for the PV $^{19}\text{F}(p,\alpha)^{16}\text{O}$ reaction. More than fifty years (and many experiments) later we still do not have a comprehensive understanding of the PV NN interaction. Since 1980 nearly all such experiments have been analyzed within the DDH (single-meson exchange) picture, but it has been difficult to resolve the issue of a small PV pionic coupling f_π^1 indicated by measurements of the circular polarization of the photon emitted in the decay of the 1.089 MeV level of ^{18}F with that indicated by the ^{133}Cs anapole moment measurement, as well as others. In order to resolve these issues and to remove nuclear-physics uncertainties from the analysis, an effective field theory approach to the subject together with an experimental program utilizing only $A \le 4$ systems have been developed. Both the theoretical and experimental programs were described above.

However, significant challenges remain. On the theoretical side it is important to develop state-of-the-art calculations which relate empirical results to the underlying theoretical basis. This is especially important for those experiments involving ^{4}He targets. Experimentally, the price that is paid for use of light nuclear systems is the loss of the possibility of nuclear enhancement, meaning that experiments must be done to a precision of a part in 10^8 or better. Nevertheless, this can and *must* be achieved in order to bring understanding to this field. The use of effective field-theoretic methods means that at some point in the near future we will be able to converge on a consistent set of empirical parameters —the Danilov coefficients—

which can characterize the low-energy PV NN interaction. Once this is accomplished the focus can shift to the use of these numbers to understand the previous nuclear experiments and to the theoretical prediction of such numbers from fundamental theory —QCD. Only then can we say that, after more than a half century of effort, the problem of hadronic parity violation is finally solved.

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