Biological and Medical Physics, Biomedical Engineering

Physics of the Human Body

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Statics of the Body

The study of the force balance of an object at rest is called “statics.” Moreover, the study of very slow motion can usually be treated as a series of static conditions – as if there were no motion; this is called “quasistatics.” After reviewing the conditions for static equilibrium in three dimensions, we will examine the useful simplification to two dimensions, examples of which can often be characterized as one of the three types of levers. We will then apply these equilibrium conditions to the lower arm, hip, and the spine (lower back). Statics is one important area in biomechanics [75, 82, 86, 94].

2.1 Review of Forces, Torques, and Equilibrium

Each force $\mathbf{F}$ can be resolved into components in the x, y, and z directions (F_x, F_y, F_z). In a static condition the sum of the forces $\mathbf{F}$ in each the x, y, and z directions is zero:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0. \tag{2.1}$$

The speed of the center of mass of the object in each direction is then constant, and will usually be assumed to be zero here. These forces can be in balance either for the entire body or for any part of the body.

Similarly, each torque τ can be resolved into components in the x, y, and z directions (τ_x, τ_y, τ_z). In a static condition, the torques τ about the x-, y-, and z-axis also each sum to zero for the entire body and for any body part:

$$\sum \tau_x = 0, \qquad \sum \tau_y = 0, \qquad \sum \tau_z = 0. \tag{2.2}$$

The speed of angular rotation of the object about each axis is then constant, and will usually be assumed to be zero here.

What actually is a torque? Forces describe changes in linear motion – which means changes in velocities, while torques describe how these same forces

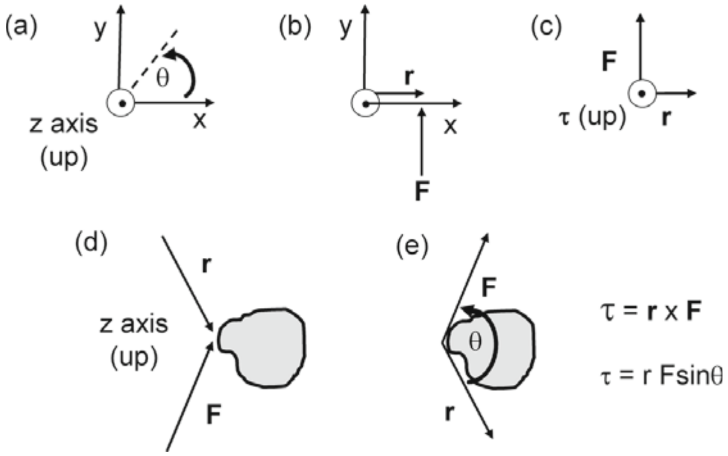

Fig. 2.1. Torques and relevant axes

can change angular motion – which means changes in angular velocities. The diagram in Fig. 2.1b shows that a force **F** applied in the positive y direction (with component F) a distance $+r$ from the z-axis, leads to a torque about the z-axis τ_y of magnitude rF. This leads to motion in the counterclockwise direction, caused by an angular acceleration that increases the angle θ. This is defined as a positive torque about this axis. A negative torque would occur, for example, if the force were applied in the negative y direction. This would lead to motion in the clockwise direction, caused by an angular acceleration that decreases the (signed) angle θ.

In general, the torque (vector τ) about any axis is defined as the vector cross product between the distance vector from that axis to the point where the force is applied **r** and the force vector **F** (Fig. 2.1e)

$$\tau = \mathbf{r} \times \mathbf{F}. \tag{2.3}$$

(You do not need to understand or use this vector cross product, just the results that are given below.)

Because vectors can be translated anywhere, things may be clearer if we move both **r** and **F**, as in Fig. 2.1b, d, so they originate from where the force is applied, as in Fig. 2.1c, e, respectively. We will call the angle from the **r** vector to the **F** vector θ. The torque τ_z about the upward axis is

$$\tau_z = rF \sin\theta, \tag{2.4}$$

where r is the magnitude of vector **r** (the distance from the axis to the point where the force is applied) and F is the magnitude of vector **F**. For $0° < \theta < 180°$ (or $0 < \theta < \pi$ in radians), $\sin\theta$ is positive and the torque is positive (Fig. 2.2a), while for $180° < \theta < 360°$ ($\pi < \theta < 2\pi$), $\sin\theta$ is negative and the torque is negative (Fig. 2.2b). When $\theta = 90° (= \pi/2)$, $\sin\theta = 1$ and the torque

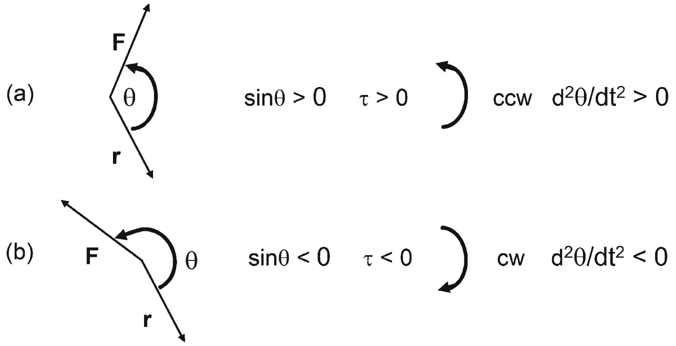

Fig. 2.2. Direction of torques, showing (**a**) positive and (**b**) negative torques

is rF, as above. When **r** and **F** are either parallel ($\theta = 0^\circ (= 0)$) or antiparallel ($\theta = 180^\circ (= \pi)$), the torque is zero.

Clearly, only the component of the **r** normal to the **F**, which we will call r', contributes to the torque action. In fact, as Fig. 2.3 proves, $\tau_z = r'F$. Equivalently, only the component of **F** normal to **r**, i.e., F', contributes to the torque action, and $\tau_z = rF'$. As we will see, sometimes information is provided where these normal components of displacement or force are provided, and the torques can be calculated without explicitly determining the angle between the displacement and force vectors. Consequently,

$$\tau_z = \mathbf{rF}\sin\theta = r'F = rF'. \tag{2.5}$$

This is true for any axis. The axis can be chosen cleverly for a particular problem to simplify analysis.

In linear motion, a force leads to an acceleration $\mathbf{a} = \mathrm{d}^2\mathbf{r}/\mathrm{d}t^2$, which is equivalent to a change in velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ (the magnitude of which is the speed v) or momentum $\mathbf{p} = m\,\mathrm{d}\mathbf{v}/\mathrm{d}t$, by

$$\mathbf{F} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}, \tag{2.6}$$

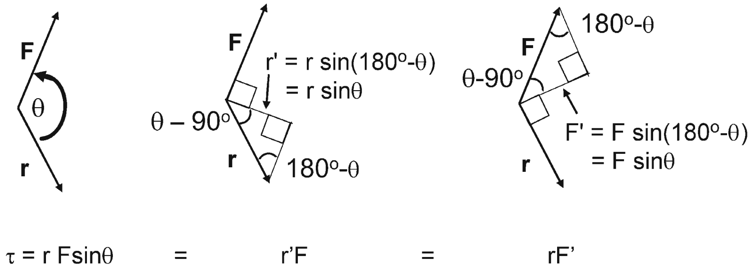

Fig. 2.3. Determining torques from using components of the displacement and force vectors that are normal to the force and displacement vectors, respectively

where m is the mass of the object. Similarly, a torque leads to an analogous change in the angle θ and angular frequency $\Omega = \mathrm{d}\theta/\mathrm{d}t$, and orbital angular momentum L

$$\tau = I\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{\mathrm{d}L}{\mathrm{d}t}, \tag{2.7}$$

where I is the moment of inertia and $L = I\Omega$. In static situations the sum of the forces and torques on the object is zero, so the right-hand side of (2.7) equals zero.

By the way, what we are defining as torques are indeed commonly called "torques" in connection to the rotational and twisting motions of objects, as in this chapter and Chap. 3, but are instead called "moments" in connection to the bending of objects, as in Chap. 4.

2.2 Statics: Motion in One Plane and Levers

Many problems involve motion in one plane, say the xy plane – for which z is a constant. For example, the motion of knees and elbows is in one plane. Some problems involving motion of the leg about the hip can be treated in these two dimensions. The six equations in (2.1) and (2.2) then reduce to three equations:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum \tau_z = 0. \tag{2.8}$$

(We will adopt this xyz coordinate system because it is conventionally used in two-dimensional problems, even though it differs from the coordinate system convention we adopted for the body in Fig. 1.1.)

These types of problems can be classified as one of the three types of levers (Fig. 2.4). There are examples in the body of each. They can be described by how a weight W and a force M, provided by a muscle, act on a solid object, say a bone resting on a fulcrum; this represents an articular joint. The weight can include that of parts of the body as well as external weights. The weight and muscle act at distances d_W and d_M from the joint. For each type of lever the total torque is zero when

$$Md_\mathrm{M} = Wd_\mathrm{W}, \tag{2.9}$$

so

$$M = \frac{d_\mathrm{W}}{d_\mathrm{M}}\, W. \tag{2.10}$$

The relative directions of the forces and the relative distances of the weight and muscle forces from the joint are different for each type of lever.

In a first class lever, the weight and muscle act on opposite sides of the fulcrum and are in the same direction (Fig. 2.4a). This is the least common type of lever in the body. Using the x, y coordinate system shown, there are

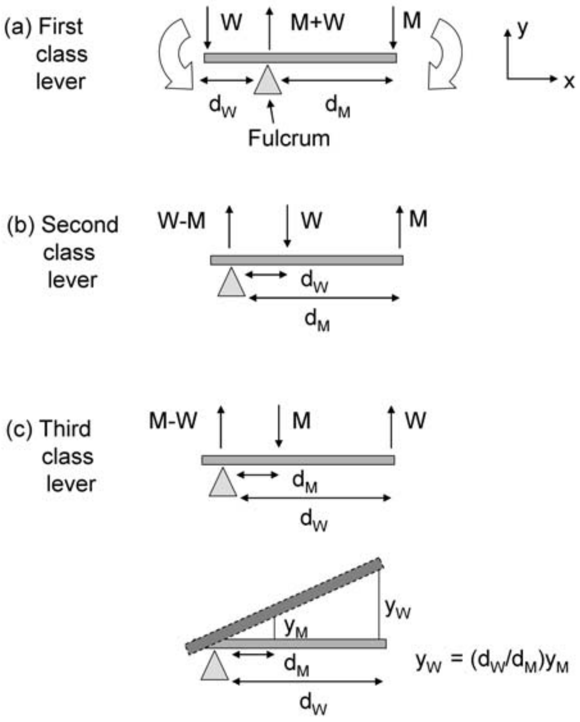

Fig. 2.4. Three types of levers, (**a**) first, (**b**) second, and (**c**) third class levers. The large increase in the distance the weight moves over the change in muscle length in the third class lever is also seen in part (**c**)

clearly no forces in the x direction so the first equation in (2.8) is automatically satisfied. Since the weight and muscle both act in the same direction – downward – force balance in the y direction requires that the fulcrum provides an upward force of $W + M$. Balancing torques in the z direction requires a choice of a z-axis. Any axis normal to the xy plane can be chosen. The simplest one is an axis at the fulcrum. The weight provides a torque of Wd_{W}, while the muscle provides a torque of $-Md_{\mathrm{M}}$. The signs are consistent with the above discussion. The fulcrum provides no torque about this axis because the distance from the fulcrum to the axis is zero. So

$$\sum \tau_z = Wd_{\mathrm{W}} - Md_{\mathrm{M}} = 0. \tag{2.11}$$

This leads to (2.9), which tells how large the muscle force must be to maintain equilibrium. If the muscle cannot provide this large of a force, there can be no static condition. (Example: The lead ball is too heavy to hold up.) If the muscle provides more than this force, there is motion. (Example: The baseball is being thrown, as we will see later.) In these two cases, $\sum \tau_z$ in (2.11) is not zero.

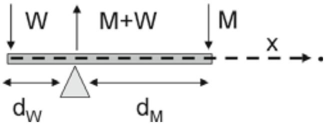

Fig. 2.5. Displacing the axis for calculating torques to the right of the weight by a distance x, as shown for a first class lever. For the axis chosen at the fulcrum $x = d_{\mathrm{W}}$. The axis can be laterally displaced anywhere, to the left or right (as shown) of the lever, above or below it, or in it

It may seem that we cheated by choosing the axis at the fulcrum. Actually, we could have chosen the axis anyway in the xy plane. To prove this let us choose the axis anywhere along the bone, say a distance x to the right of the weight (Fig. 2.5). The torques provided by the weight, fulcrum, and muscle are now Wx, $(W+M)(d_{\mathrm{W}}-x)$, and $-M(d_{\mathrm{W}}+d_{\mathrm{M}}-x)$, respectively. Balance requires

$$\sum \tau_z = Wx + (W+M)(d_{\mathrm{W}}-x) - M(d_{\mathrm{W}}+d_{\mathrm{M}}-x) = 0, \qquad (2.12)$$

which reduces to (2.11) again.

One type of the first class lever is a seesaw or teeter totter. A second type is the head atop the spinal cord, where the weight of the head is balanced by the downward effective force of the muscles (Fig. 2.6a). In a third example, the

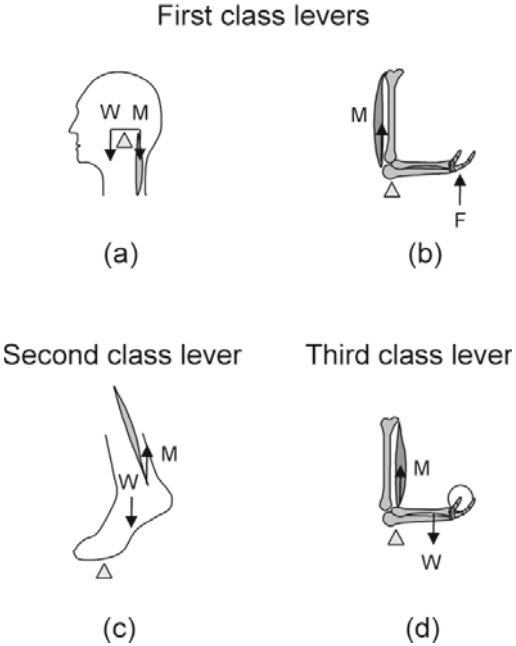

Fig. 2.6. Examples of first (**a**, **b**), second (**c**), and third (**d**) class levers in the body

triceps brachii pull on the ulna about the elbow pivot balanced by the forces on the forearm. With the upper arm down, the triceps brachii can balance an upward force pushing the hand up (Fig. 2.6b). (With the upper arm pointed up, the triceps brachii can also balance the hand holding a weight.) Because $d_\mathrm{M} \ll d_\mathrm{W}$ here, the force that the triceps brachii needs to exert is much greater than the forces exerted at the hand.

In a second class lever, the muscle and weight act on the same side of the fulcrum, and the weight is nearer to the fulcrum, so $M < W$ (Fig. 2.4b). This type of lever is the second most common in the body. One example is standing on tiptoes: the rotation of the foot about the toes (the fulcrum), which would be caused by the weight of the foot, is balanced by the muscle force transmitted by the Achilles tendon (Fig. 2.6c). Another example is pushing down with the triceps brachii.

In third class levers, the muscle and weight are again on the same side of the fulcrum, but now the muscle is nearer to the fulcrum than the weight (Fig. 2.4c). This is the most common example in the body. Because often $d_\mathrm{M} \ll d_\mathrm{W}$, we see that $M = (d_\mathrm{W}/d_\mathrm{M})W \gg W$. This arrangement means that very large forces must be exerted by the muscles because of this $d_\mathrm{W}/d_\mathrm{M}$ amplification, which seems to be a big disadvantage (and is literally a mechanical disadvantage). However, something else is gained in this tradeoff in design. As seen in the Fig. 2.4c, when the bone rotates a given angle, causing a vertical displacement y_M at the muscle, there is an amplification of the distance traveled at the position of the weight by $d_\mathrm{W}/d_\mathrm{M}$. As we will see in Chap. 5, muscles are able to contract only a small fraction of their length – which amounts to at most several cm in many muscles. The length of the biceps is about 25 cm, and the maximum contraction is by ~7–8 cm. With this amplification, the weight can now move much more than this. One example of a third class lever is the balancing of the lower arm by the biceps brachii inserted on the radius (Fig. 2.6d). Another is holding a weight with an outstretched arm.

2.3 Statics in the Body

We will examine the planar forces in the static equilibrium of the lower arm, at the hip, and in the back. In analyzing the lower arm, we will choose successively more complex and realistic models. We will see that the forces in the hip and back are quite large, much more than one would expect, and explains why people often have problems in these parts of the body, problems that can lead to hip replacements and life-long lower back pain. The approach for these problems is the same. We consider all elements in one plane and examine the forces in the (as defined) x and y directions and the torque in the z direction. Some of the approaches of [65] and [86] are followed.

2.3.1 The Lower Arm

We will examine the equilibrium of the forearm balanced by the contraction of the biceps brachii inserted on the forearm long bone called the radius; this is a continuation of the discussion of third class levers. The relevant bones are shown in Figs. 2.7 and 2.8. In equilibrium, the biceps brachii force counters the potential rotation about the elbow joint by the weight held in the hand (Fig. 2.9a). We will examine this example for different models, using Fig. 1.15 and Tables 1.6 and 1.7 to provide anthropometric information. The forearm is $0.146H$ long and the hand length is $0.108H$, where H is the body height, so the weight held in the hand is about $(0.146 + 0.108/2)H = 0.2H$ from the pivot. (The ball is in the middle of the hand.)

Case 1

The biceps brachii insert about 4 cm from the pivot axis. Say there is a weight W_{W} held in the hand, which is $d_{\mathrm{W}} = 36$ cm from the pivot. (With $H = 180\,\mathrm{cm}$, $0.2H = 36\,\mathrm{cm}$.) Therefore $M = (d_{\mathrm{W}}/d_{\mathrm{M}})W_{\mathrm{W}} = (36\,\mathrm{cm}/4\,\mathrm{cm})W_{\mathrm{W}} = 9W_{\mathrm{W}}$. So for a weight of 100 N the muscle must provide a force of 900 N for balance. Here N stands for the MKS/SI unit of newtons. Since $1\,\mathrm{N} \simeq 0.225\,\mathrm{lb}$, equivalently, a 22 lb weight is balanced by 200 lb of force exerted by the biceps brachii (Fig. 2.9b).

We have made several assumptions and approximations in this example without explicitly stating them. It is always good to start with simple models. It is equally important to understand exactly what assumptions and approximations are being made. Then, the model can be made more realistic. Here, we have assumed that the forearm and upper arm make a 90° angle. We have also neglected the mass of the forearm.

Case 2

Now let us improve the model by including the weight of the forearm W_{F} (Fig. 2.9c). This is about $0.022W_{\mathrm{b}}$, (where W_{b} is the body weight) (Table 1.7). For a 70 kg (700 N, 160 lb) person, this is ≈15 N (3.4 lb). We can treat the effect of the weight of the forearm as if it were acting at its center of mass, which is approximately in the middle of the forearm, $d_{\mathrm{F}} = 0.146H/2 = 13\,\mathrm{cm}$ from the pivot:

$$\sum \tau_z = Md_{\mathrm{M}} - W_{\mathrm{W}}d_{\mathrm{W}} - W_{\mathrm{F}}d_{\mathrm{F}} = 0 \tag{2.13}$$

$$Md_{\mathrm{M}} = W_{\mathrm{W}}d_{\mathrm{W}} + W_{\mathrm{F}}d_{\mathrm{F}} \tag{2.14}$$

$$M = \frac{d_{\mathrm{W}}}{d_{\mathrm{M}}}W_{\mathrm{W}} + \frac{d_{\mathrm{F}}}{d_{\mathrm{M}}}W_{\mathrm{F}}. \tag{2.15}$$

The ratio $d_{\mathrm{F}}/d_{\mathrm{M}}$ $(= 13\,\mathrm{cm}/4\,\mathrm{cm})$, so now $M = 9W_{\mathrm{W}} + 3.25W_{\mathrm{F}}$ and the muscle force required to maintain equilibrium has increased to $900\,\mathrm{N} + 3.25\ (15\,\mathrm{N}) = 950\,\mathrm{N}$ (210 lb).

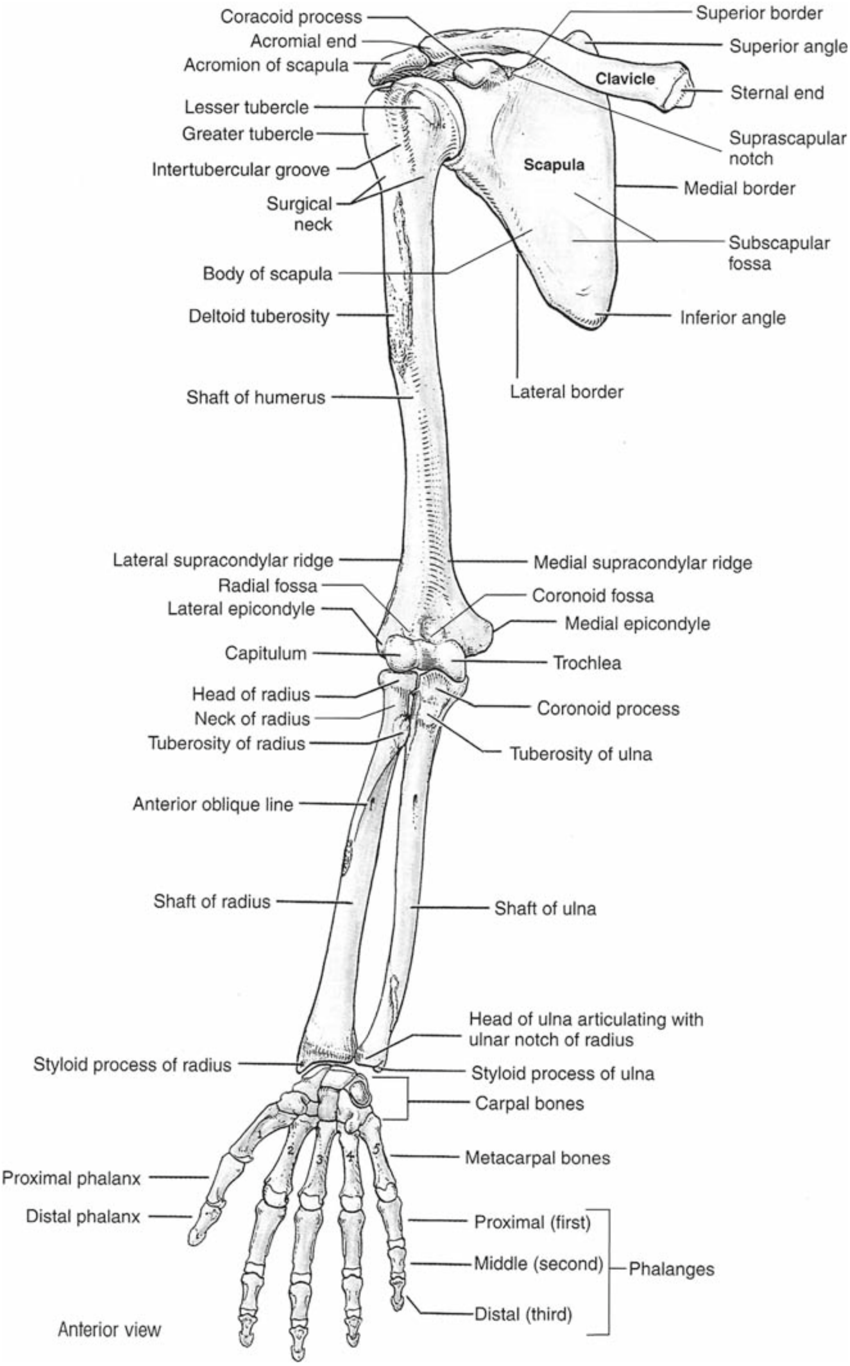

Fig. 2.7. Bones of the arm, anterior view. (From [78]. Used with permission)

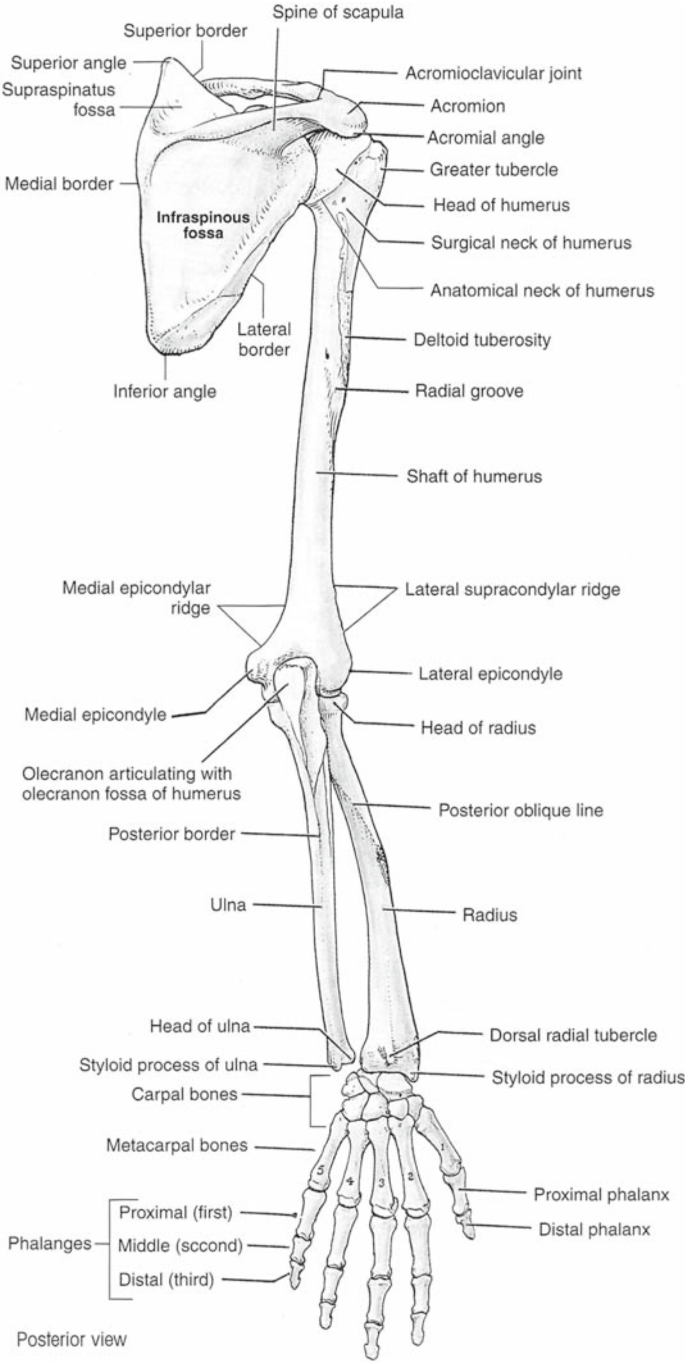

Fig. 2.8. Bones of the arm, posterior view. (From [78]. Used with permission)

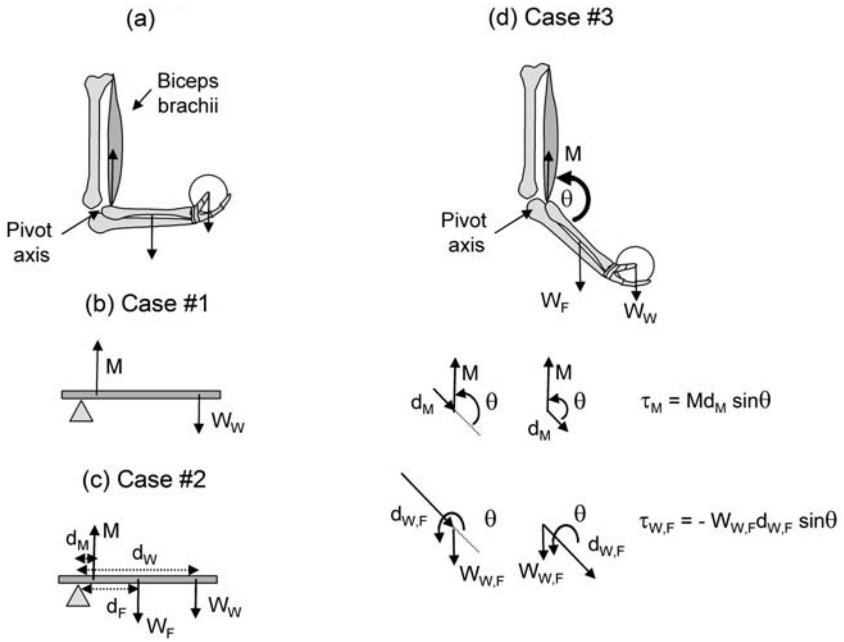

Fig. 2.9. (**a**) Weight held in the hand, showing the biceps brachii muscles. (**b**–**d**) Forces for the equilibrium of a weight held in the hand for Cases 1–3

Case 3

What happens if we no longer assume that the forearm and upper arm make a 90° angle? Let us keep the upper arm vertical and let the forearm make an angle θ, which can range over 142° (Table 1.10). The force due to the muscle is then still vertical, and those due to the weights of the forearm and ball are, of course, downward. From Fig. 2.9d we see that the torque caused by each of these three forces is multiplied by $\sin\theta$. Now

$$\sum \tau_z = M d_{\mathrm{M}} \sin\theta - W_{\mathrm{W}} d_{\mathrm{W}} \sin\theta - W_{\mathrm{F}} d_{\mathrm{F}} \sin\theta = 0 \tag{2.16}$$

and we arrive at the same result that

$$M d_{\mathrm{M}} = W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}}. \tag{2.17}$$

Actually, we made additional assumptions in this example that we will re-examine later. The distance from the pivot where the biceps brachii insert on the radius really changes with θ (Fig. 3.42). Also, while this analysis suggests that the muscle force M required for equilibrium is the same for all angles, there is a subtlety in this result. Equation (2.16) gives the muscle force needed to maintain equilibrium. As we will see in Chap. 5, muscles can exert forces up to a maximum value. If the M from (2.17) can be achieved, then there can be equilibrium; if it cannot, then the static condition cannot be achieved. The maximum force that a muscle can exert depends on its length, which, from Fig. 2.9, is clearly a function of θ. So the M in (2.17) may be achievable at some angles (nearer 90°, where the maximum force turns out to be greatest) and not at others.

Case 4

The biceps brachii are not the only muscles used to flex the elbow. What happens if we also include the contributions of these other muscles? Figure 2.10 shows that the biceps brachii, the brachialis, and the brachioradialis all contribute to this flexing. Assuming that $\theta = 90°$ (which may not be a good assumption for each muscle), (2.13) is modified to

$$\sum \tau_z = M_1 d_{\mathrm{M}_1} + M_2 d_{\mathrm{M}_2} + M_3 d_{\mathrm{M}_3} - W_{\mathrm{W}} d_{\mathrm{W}} - W_{\mathrm{F}} d_{\mathrm{F}} = 0 \tag{2.18}$$

$$M_1 d_{\mathrm{M}_1} + M_2 d_{\mathrm{M}_2} + M_3 d_{\mathrm{M}_3} = W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}} \tag{2.19}$$

where M_1, M_2, and M_3 represent the forces exerted by the three muscles M_i, respectively. If the physiological cross-sectional areas of the three muscles are A_1, A_2, and A_3, respectively (which we usually call PCA), and the muscle force for each can be assumed to be proportional to this area (which is a pretty good assumption), then $M_i = kA_i$, for $i = 1, 2, 3$. (We will see

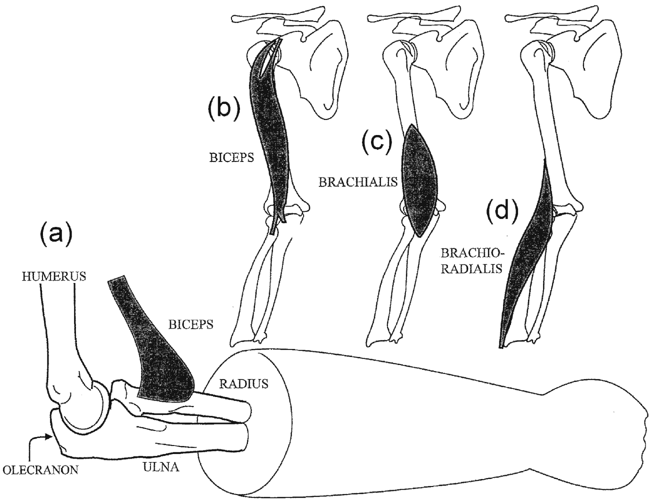

Fig. 2.10. Sketch of the elbow joint for analyzing the statics of the lower arm for Case 4, with the three muscles, the biceps (brachii), brachialis, and brachioradialis, shown in (**b**–**d**). (From [76])

Table 2.1. Data for the three elbow muscles used for flexion. (Using data from [76, 95])

muscle	moment arm d_i (cm)	physiological cross-section (PCA) (cm^2)
biceps (muscle 1)	4.6	4.6
brachialis (muscle 2)	3.4	7.0
brachioradialis (muscle 3)	7.5	1.5

that this is a good assumption with k reaching a maximum of $\sim$40 N/cm^2 or so.) So,

$$kA_1 d_{\mathrm{M}_1} + kA_2 d_{\mathrm{M}_2} + kA_3 d_{\mathrm{M}_3} = W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}} \tag{2.20}$$

$$k = \frac{W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}}}{A_1 d_{\mathrm{M}_1} + A_2 d_{\mathrm{M}_2} + A_3 d_{\mathrm{M}_3}} \tag{2.21}$$

$$M_1 = kA_1 = A_1 \frac{W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}}}{A_1 d_{\mathrm{M}_1} + A_2 d_{\mathrm{M}_2} + A_3 d_{\mathrm{M}_3}} \tag{2.22}$$

$$M_2 = kA_2 = A_2 \frac{W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}}}{A_1 d_{\mathrm{M}_1} + A_2 d_{\mathrm{M}_2} + A_3 d_{\mathrm{M}_3}} \tag{2.23}$$

$$M_3 = kA_3 = A_3 \frac{W_{\mathrm{W}} d_{\mathrm{W}} + W_{\mathrm{F}} d_{\mathrm{F}}}{A_1 d_{\mathrm{M}_1} + A_2 d_{\mathrm{M}_2} + A_3 d_{\mathrm{M}_3}}. \tag{2.24}$$

Using the parameters from Table 2.1, we get $M_1 = 262$ N (biceps), $M_2 = 399$ N (brachialis), and $M_3 = 85$ N (brachioradialis) when we generalize Case 2. This compares to the $M_1 = 696$ N that we would obtain for Case 2 with the biceps alone, using $d_1 = 4.6$ cm (instead of the 4 cm used before, which led to 800 N). The total muscle force is 746 N, which is greater than 696 N because the brachialis has a relatively small moment arm.

Life is a bit more complex than this result suggests because we assumed that k has the same value for each muscle. Really $M_i = k_i A_i$, and all the k_i's need not be the same, as long as k_i is less than the maximum that can be exerted by the muscles. Unfortunately, if all the k_is are not assumed to be equal, we do not have enough information to solve this problem uniquely as posed. The body may solve the indeterminate nature of this problem (with more variables than conditions) by minimizing energy or optimizing the force distribution (shifting the load from one muscle to another) to rest specific muscles or to keep the weight balanced better (so it will not tip in the hand).

2.3.2 Hip Problems

The hip (pelvis) is not a single bone, but several bones that are fused together (Figs. 2.11 and 2.12). The pelvis is composed of the pelvic girdle and two

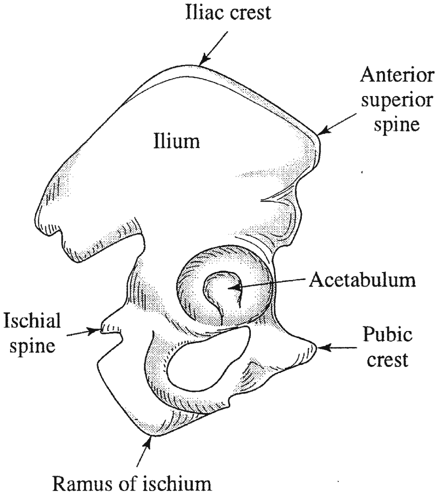

Fig. 2.11. Right hip bone in adult. (From [65])

parts of the spinal cord, the sacrum and coccyx. The pelvic girdle itself is composed of the right and left coxal (or hip) bones. Each coxal (hip) bone is composed of three bones: the ilium (at the top = superior), pubis (bottom front = inferior, anterior), and ischium (iss-kee'-um) (bottom back = inferior, posterior). The acetabulum (a-si-tab-yoo'-lum) is the socket area where the femur of the leg (Fig. 2.13) is attached ("hip joint"). Actually, the head of the femur is in this socket, and is maintained there by the muscles attached at the greater trochanter. These muscles are collectively called the hip abductor muscles. (The hip abductor muscles are not the only one attached at the greater trochanter (see below), but they are the ones that contribute to the force needed for the equilibrium condition in this problem.)

First we will determine the force on the head of the femur and in the hip abductor muscles while the subject is standing on one leg, say the right

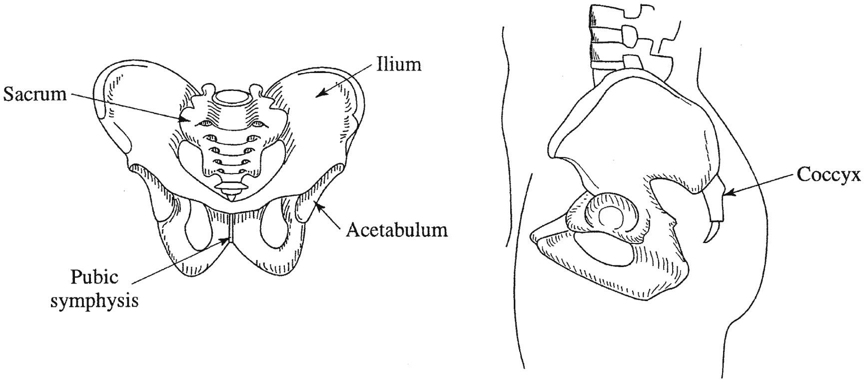

Fig. 2.12. Front and side views of the hip. (From [65])

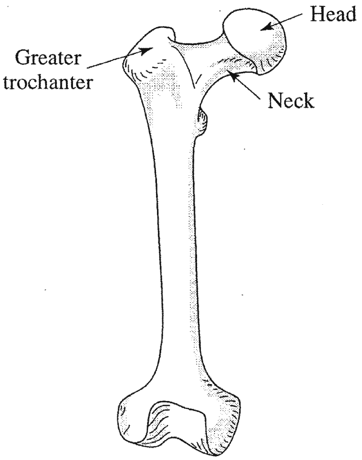

Fig. 2.13. Anterior view of right femur. (From [65])

leg. This is actually a good model for determining these forces during slow walking. The upper and lower leg and the foot are treated as a rigid body. This problem is solved in two steps. First, the forces in the whole body are analyzed and then the rigid leg is treated as a free rigid body, whose only interaction with the rest of the body will be the normal force from the hip [65, 85]. The bones in the leg are shown in Figs. 2.14 and 2.15.

Total Body Equilibrium

There are only two forces on the body. The body weight W_{b} acts downward, and as if it all originated at the center of mass of the body, which is in the midline in the hip (Fig. 2.16). The foot feels an upward normal force from the floor of magnitude N. There are no forces in the x direction, and these two forces in the y direction must balance in equilibrium, so $N = W$. In equilibrium the body cannot start to rotate, so the torques are zero. It is clear from the Fig. 2.16 that this occurs when the foot is directly below the hip, in the midline. If we choose the pivot axis at the center of mass, the torque from the center of mass is zero because the distance term (from the axis to the center of mass) is zero and the torque from the normal force is zero because the normal force is antiparallel to the distance vector ($\theta = 180°$). (You can prove for yourself that the total toque is zero for any other axis normal to the xy plane.)

Equilibrium of the Individual Body Component

There are four external forces on the leg (Figs. 2.16 and 2.17):

(a) **N** is the normal force on the leg from the floor, and we know that $N = W_{\mathrm{b}}$.

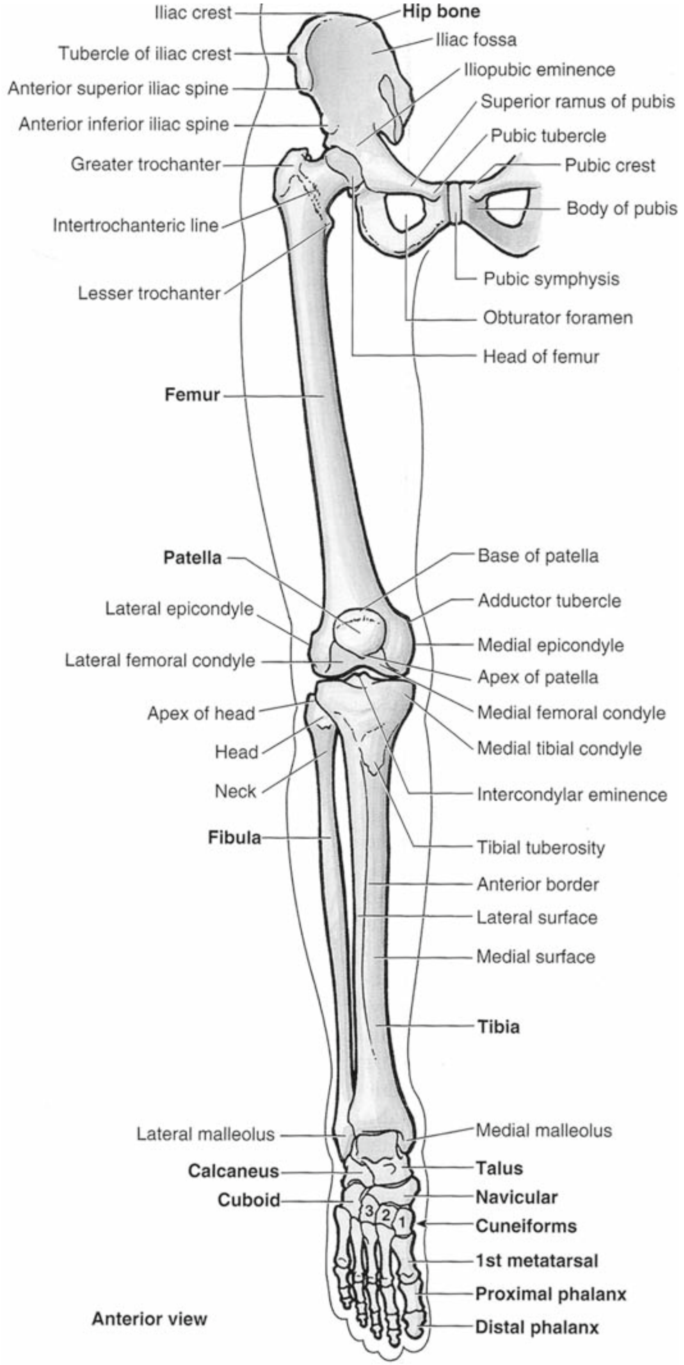

Fig. 2.14. Bones of the leg and hip, anterior view, with names of bones in bold. (From [79]. Used with permission)

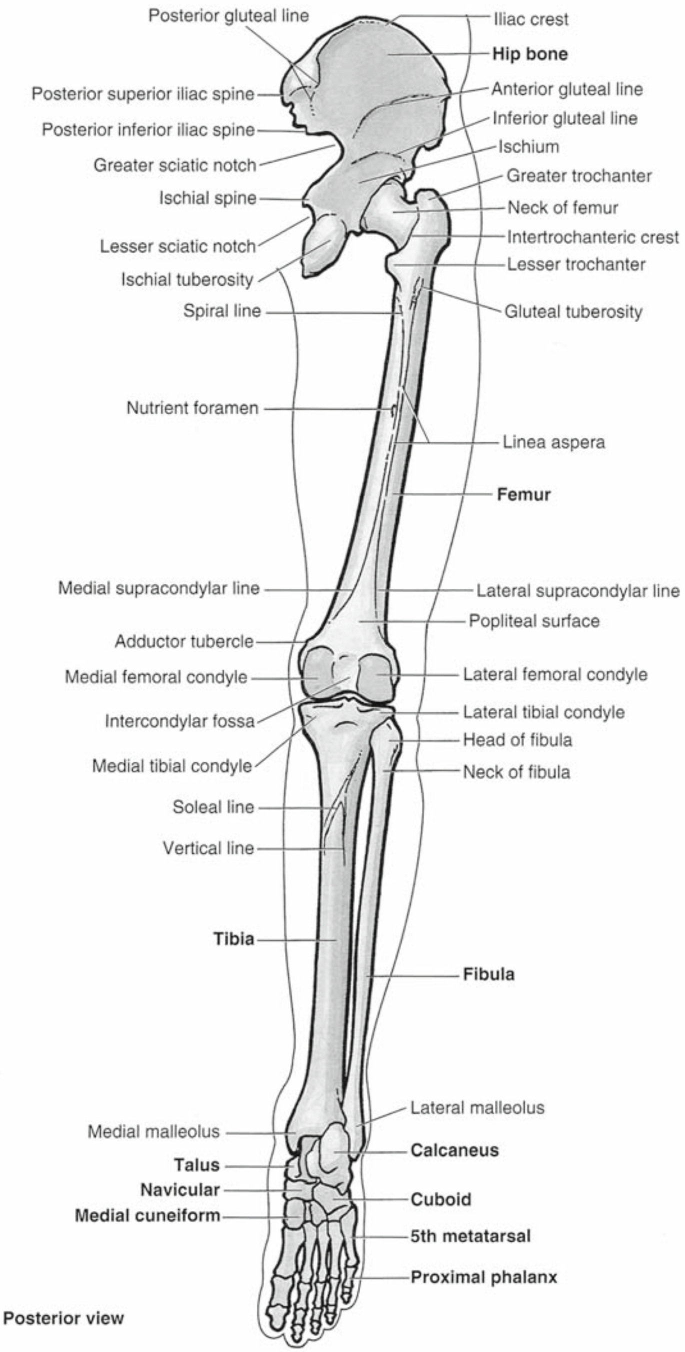

Fig. 2.15. Bones of the leg and hip, posterior view, with names of bones in bold. (From [79]. Used with permission)

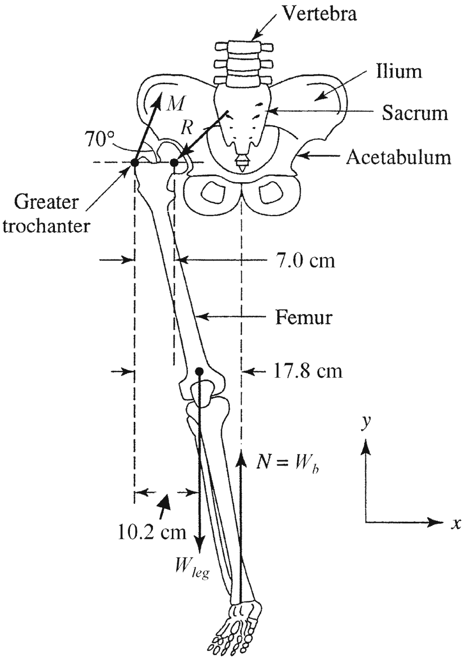

Fig. 2.16. Anatomical diagram of the leg and hip for someone standing on one leg, or during slow walking, showing the forces on them and relevant dimensions, including the force exerted on the head of the femur by the acetabulum R and the net force exerted by the hip abductor muscles. (From [65])

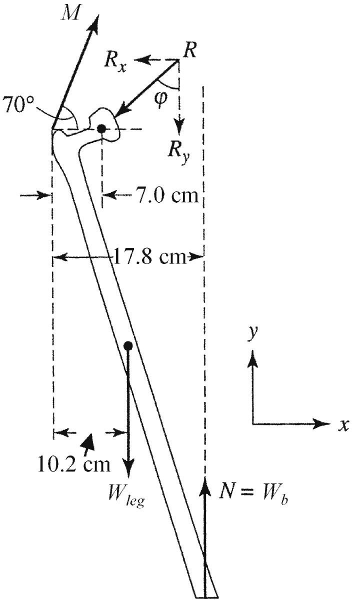

Fig. 2.17. Force diagram for a leg for someone standing on one foot, using Fig. 2.16. (From [65])

(b) W_{leg} is the weight of the leg. Table 1.7 shows that this is about $0.16W_{\mathrm{b}}$. It acts as if it were applied at the center of mass of the leg, which is approximately halfway down the leg.

(c) **R** is the reaction force on the leg from the hip, and it is normal to the hip socket. We will define the x and y components R_x and R_y so they are positive (Fig. 2.17). Equivalently, we could define the magnitude of R and the angle it makes with the vertical, φ.

(d) **M** is the force (of magnitude M) due to the hip abductor muscles. There are actually three muscles involved here: the tensor fascia (fash-ee'-uh) latae (la-tuh) (see Fig. 3.2a), gluteus (gloo'-tee-us) minimus (see Fig. 3.3c), and the gluteus medius (see Fig. 3.3a, b). (The gluteus maximus muscle is what the author is sitting on as he is typing this.) These three muscles have a mass ratio of about 1:2:4 and, as we will see, this is roughly the ratio of the forces each can exert. The hip abductor muscle structure we consider is a composite of the three muscles. It has been shown that the effective action of this system is $\simeq 70°$ to the horizontal, acting on the greater trochanter.

We have three equations with three unknowns: R_x, R_y, and M. The two force equations are

$$\sum F_x = M\cos 70° - R_x = 0, \tag{2.25}$$

$$\sum F_y = M\sin 70° - R_y - 0.16W_{\mathrm{b}} + W_{\mathrm{b}} = 0, \tag{2.26}$$

where W_{leg} has been replaced by $0.16W_{\mathrm{b}}$.

We will choose the rotation axis to emanate from the center of the head of the femur because the reaction force from the acetabulum passes through this point. This makes the analysis easier, but, of course, the solution would be the same if we chose any other parallel axis. The relevant distances of interest are shown in the diagram (obtained from anatomical dimensions and geometry), as needed for torque analysis.

(a) The component of the distance vector perpendicular to the normal force (r') is 10.8 cm, so the normal force causes a torque of $(10.8\,\mathrm{cm})W_{\mathrm{b}}$. This is a positive torque because the normal force induces a counter clockwise rotation about the chosen z-axis (see Fig. 2.2).

(b) The component of the distance vector normal to the force of the weight of the leg is 3.2 cm and this force tends to induce a clockwise rotation, so it contributes a torque of $-(3.2\,\mathrm{cm})W_{\mathrm{leg}} = -(3.2\,\mathrm{cm})(0.16W_{\mathrm{b}}) = -(0.5\,\mathrm{cm})W_{\mathrm{b}}$.

(c) With the choice of the axis, the torque from the reaction force from the hip is zero, because the distance vector and normal force are antiparallel.

(d) The component of the force from the hip abductor muscles normal to the horizontal distance vector (of magnitude 7.0 cm) is $M\sin 70°$. Since this causes a clockwise rotation, the torque is $-(7.0\,\mathrm{cm})M \sin 70°$. So we see that

$$\sum \tau_z = (10.8\,\mathrm{cm})W_\mathrm{b} - (3.2\,\mathrm{cm})(0.16W_\mathrm{b}) + 0 - (7.0\,\mathrm{cm})M \sin 70° = 0, \tag{2.27}$$

$$M = \frac{10.8 - 0.5}{7.0 \sin 70°} W_\mathrm{b} = 1.57 W_\mathrm{b}. \tag{2.28}$$

We see that torque provided by the hip abductor muscles is needed to counter the torques from the normal force from the floor and the weight of the leg. This normal force torque is much more important than that due to the leg, because of the greater magnitude of the force and the larger moment arm.

Using this value for the muscle force, the force balance in the x direction gives $R_x = M \cos 70° = 0.54W_\mathrm{b}$. From the balance in the y direction, $R_y = M \sin 70° + 0.84W_\mathrm{b} = 2.31W_\mathrm{b}$. The magnitude of $R = (R_x^2 + R_y^2)^{1/2} = 2.37W_\mathrm{b}$, and $\tan\theta = R_x/R_y = 0.54/2.31 = 0.23$, so $\theta = 13°$.

Because $M \simeq 1.6W_\mathrm{b}$ and $R \simeq 2.4W_\mathrm{b}$, for $m_\mathrm{b} = 90\,\mathrm{kg}$ we have $W_\mathrm{b} = 880\,\mathrm{N}$ (200 lb), and so $M \simeq 1{,}400\,\mathrm{N}$ (320 lb) and $R \simeq 2{,}100\,\mathrm{N}$ (470 lb). The origin of hip problems is clear: The force from the hip is much greater than the body weight because of the large moment arms.

We next examine a variation of this problem. The person now uses a cane to provide support on the left side while standing on his or her right leg (Fig. 2.18). As shown in Fig. 2.19, the cane is 30.5 cm (1 ft) from the body midline. It is supported by and pushed down by the left arm or shoulder. Consequently, there is a normal force N_c from the floor. We assume $N_\mathrm{c} = W_\mathrm{b}/6$. We will see that this has two immediate consequences. The right foot is no longer directly in the body midline but is displaced a distance L to the right (in the reference of the body) and the normal force felt by the right foot N_f is no longer the body weight.

Whole body equilibrium gives

$$N_\mathrm{f} + N_\mathrm{c} - W_\mathrm{b} = N_\mathrm{f} + W_\mathrm{b}/6 - W_\mathrm{b} = 0 \tag{2.29}$$

or $N_\mathrm{f} = 5W_\mathrm{b}/6$. Using the same axis as before, the torque balance is

$$\sum \tau_z = (30.5\,\mathrm{cm})N_\mathrm{c} - L(N_\mathrm{f}) = (30.5\,\mathrm{cm})(W_\mathrm{b}/6) - L(5W_\mathrm{b}/6) = 0 \tag{2.30}$$

or $L = (N_c/N_\mathrm{f})30.5\,\mathrm{cm} = (1/5)30.5\,\mathrm{cm} = 6.1\,\mathrm{cm}$.

For the same leg as in Fig. 2.17, the corresponding distances are different because the angle of the leg is now different (as obtained from anatomical dimensions and the new geometry) (Fig. 2.20). We will examine the whole leg equilibrium again. The cane is not explicitly involved, but implicitly through the changes in the leg position and the load borne by the leg.

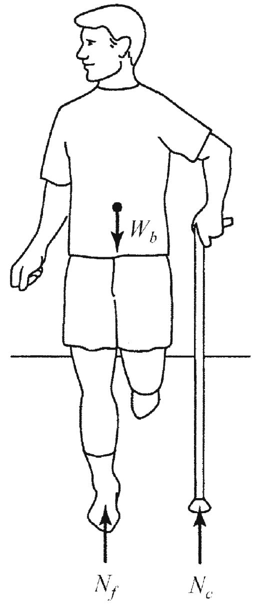

Fig. 2.18. Forces on entire person for someone walking with a cane. (From [65])

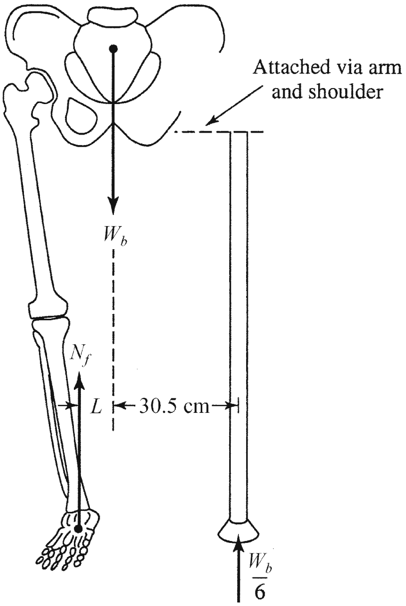

Fig. 2.19. Force diagram for a person using a cane for some support. (From [65])

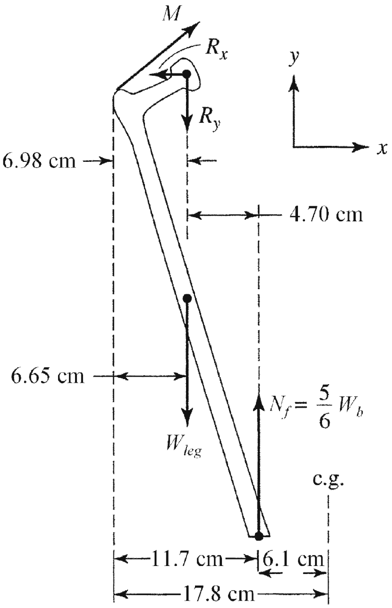

Fig. 2.20. Free-body force diagram of the leg for someone walking with a cane. Note that the center of mass of the leg is now 0.33 cm to the left of the vertical from the center of the head of the femur, whereas without the cane it was 3.2 cm to the right of the vertical. Consequently, the leg center of mass is 6.65 cm from the vertical line from the greater trochanter, whereas the center of the head of the femur is 6.98 cm from it. (From [65])

Now

$$\sum F_x = M\cos 70° - R_x = 0, \tag{2.31}$$

$$\sum F_y = M\sin 70° - R_y - 0.16W_\mathrm{b} + (5/6)W_\mathrm{b} = 0, \tag{2.32}$$

$$\sum \tau_z = +(4.7\ \mathrm{cm})(5/6)W_\mathrm{b} + (0.33\ \mathrm{cm})(0.16W_\mathrm{b}) + 0 - (6.98\ \mathrm{cm})M\sin 70° = 0. \tag{2.33}$$

The first equation is unchanged. (We are assuming that the effective angle of the hip abductor muscles with the x-axis is still 70°, even though this is no longer rigorously true.) The only change in the second equation is the smaller normal force on the leg. There are two changes in the torque equation. The first term, due to the normal force from the floor, is much smaller due to the change in the moment arm and the normal force of the leg – the former effect being much larger than the latter. The second term, due to the weight of the leg, is much smaller in magnitude and is now a positive torque instead of a negative torque, because the center of mass of the leg is now to the left of the vertical drawn down from the axis (Figs. 2.2 and 2.20).

Table 2.2. Analytic estimates of peak hip forces. (Using data from [82])

activity	magnitude/body weight, W_b
walking	4.8–5.5
walking slowly with/without a cane	2.2/3.4
stair ascending/climbing	7.2–7.4
stair descending	7.1
chair raising	3.3

Now $M = 0.61W_\mathrm{b}$, $R_x = 0.21W_\mathrm{b}$ and $R_y = 1.24W_\mathrm{b}$, with $R = 1.26W_\mathrm{b}$ and $\varphi = 9.5°$. With W_b still 880 N (200 lb), the muscle force M is now 540 N (120 lb) (instead of 1,400 N (320 lb) without the cane) and the reaction force at the hip R is now 1,100 N (250 lb) (instead of 2,100 N (470 lb)). This is a very big effect, considering that only about 145 N (35 lb) ($\approx W_\mathrm{b}/6$) is resting on the cane. By far the major consequence of the cane is the change of the moment arm of the normal force from the floor from 10.8 to 4.7 cm, because the foot shifted to the right by 6.1 cm.

Another variation of this situation is presented in Problems 2.8 and 2.9. Instead of using a cane with the left hand, you will examine the consequences of carrying a weight on the left side. This greatly increases the moment arms, thereby increasing M and R. Table 2.2 shows estimates for peak hip forces for several activities from more detailed analyses.

Excessive hip forces for extended periods of time can thin cartilage and cause other damage; this leads to pain during walking that can sometimes only be alleviated with a hip replacement. Total hip replacements entail replacing the ball of the femur and the acetabulum. The femoral component is a highly polished ball of a high-strength alloy, such as cobalt–chromium or titanium, with a step that is placed in the canal of the femur. It is often fixed with an acrylic plastic, such as poly(methyl methacrylate), PMMA, "bone cement." The acetabulum component is a socket made from ultrahigh molecular weight polyethylene, and may have a metal backing.

2.3.3 Statics of Other Synovial Joints

Other synovial joints can be examined in similar ways by using models of the muscles and tendons. They are briefly outlined here; see [86] for more details.

Shoulder

The anatomy and musculature of the shoulder are shown in Figs. 1.2 and 1.8. Figure 2.21 shows a model of the static equilibrium of a horizontal arm that holds a weight by means of the deltoid muscle. This weight and muscle

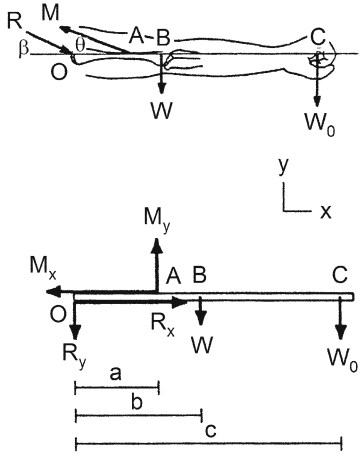

Fig. 2.21. Forces on the arm and shoulder, when the arm is abducted to the horizontal position and the hand holds a weight, along with the force diagram. (From [86])

are in equilibrium with the weight of the arm and the reaction force on the joint. Problems 2.15–2.17 address this with and without a weight held in the hand. For reasonable parameters, the component of the muscle force in the x direction is about 4× that in the y direction, suggesting that holding the arm horizontally is not stable – as we all know.

Dislocation of the shoulder (glenohumeral) joint is common because it is shallow. The large degree of rotational motion of the head of the humerus about the articulating surface of the glenoid fossa enables this large motion, albeit with little stability. The shoulder joint angles range by ~249° during flexion/extension, ~182° in abduction/adduction, and ~131° for inward/outward rotation (Table 1.10). Fracture of the humerus is also relatively common.

Knee

The knee joint is really two joints (Fig. 1.3). The tibiofemoral joint is located between the medial and lateral condyles of the femur and tibia, which are separated by cartilaginous regions called menisci. (A *condyle* is the rounded prominence at the end of a bone, often at an articulation joint.) The second is the patellofemoral joint between the kneecap (patella) and the anterior end of the femoral condyles. Muscle control of the knee is mostly through the quadriceps and hamstring muscles. The quadriceps attach to the quadriceps tendon, which attaches to the kneecap, which attaches to the patellar tendon, which attaches to the tibia.

The forces on the static lower leg loaded with an ankle weight, as during exercise (Fig. 2.22), are shown in Figs. 2.23 and 2.24. The forces shown are

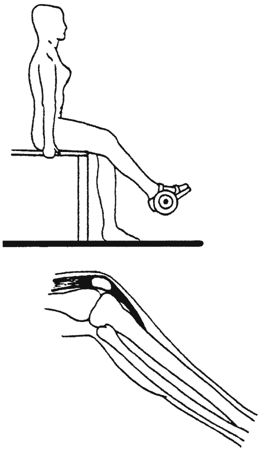

Fig. 2.22. Exercising muscles near and at the knee. (From [86])

due to this added weight, the weight of the lower leg, the quadriceps muscle force transmitted by the patellar tendon **M** (of magnitude M), and the joint reaction force **R** (of magnitude R), while the angle between the horizontal and the leg is β. In equilibrium, the muscle force and the x and y components

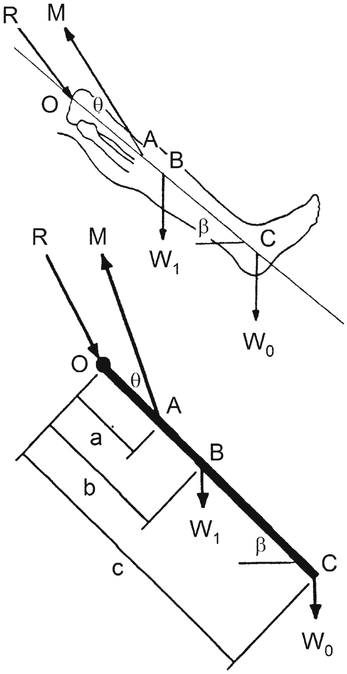

Fig. 2.23. Forces on the lower leg, while exercising the muscle around the knee. (From [86])

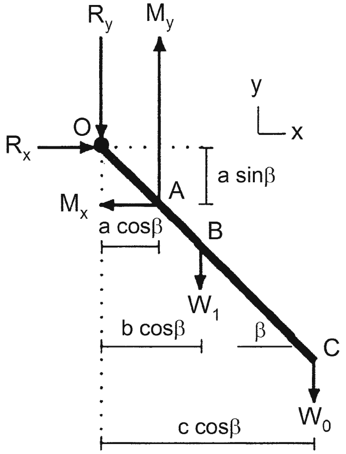

Fig. 2.24. Resolution of the forces on the lower leg in Fig. 2.23. (From [86])

of the joint force are

$$M = \frac{(bW_1 + cW_0)\cos\beta}{a\sin\theta}, \tag{2.34}$$

$$R_x = M\cos(\theta + \beta), \tag{2.35}$$

$$R_y = M\sin(\theta + \beta) - W_0 - W_1. \tag{2.36}$$

These forces are very large [84, 86]. For $a = 12\,\mathrm{cm}$, $b = 22\,\mathrm{cm}$, $c = 50\,\mathrm{cm}$, $W_1 = 150\,\mathrm{N}$, $W_0 = 100\,\mathrm{N}$, $\theta = 15°$ and $\beta = 45°$, we see that the muscle force $M = 1{,}381\,\mathrm{N}$ and the joint force $R = 1{,}171\,\mathrm{N}$. This is examined further in Problem 2.18. Problems 2.21–2.24 address a related condition, that of the crouching position, as occurs during ascending and descending stairs or jumping.

One function of the kneecap is to increase the moment arm (Fig. 2.25). We can analyze the equilibrium of the kneecap at the patellofemoral joint between the reaction force on the kneecap from the anterior end of the femoral condyles, the patellar tendon and the quadriceps tendon. This is shown in Fig. 2.26. The compressive force applied on the kneecap is

$$F_\mathrm{P} = \frac{\cos\gamma - \cos\alpha}{\cos\phi} M \tag{2.37}$$

at an angle

$$\phi = \arctan\left(\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}\right). \tag{2.38}$$

Ankle

We now examine the equilibrium of the foot. The anatomy of the ankle and foot are depicted in Figs. 2.14 and 2.15 (also see Figs. 1.8 and 3.4). Figures 2.27

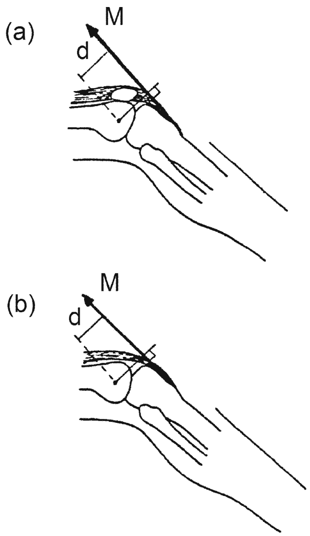

Fig. 2.25. The presence of the kneecap (patella) increases the moment arm in the lever. (From [86])

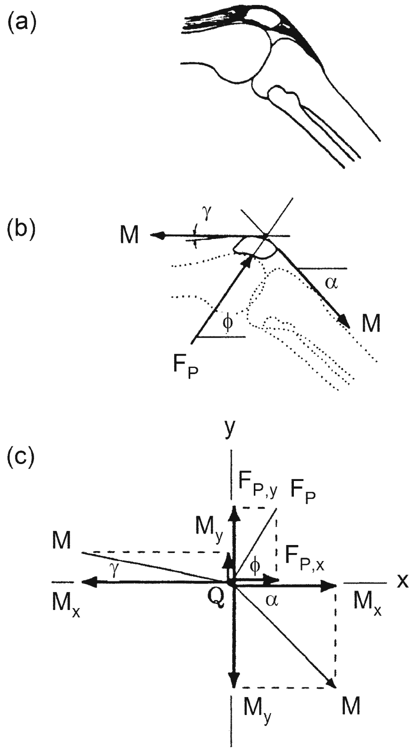

Fig. 2.26. Force diagram of the kneecap (patella) in equilibrium. (From [86])

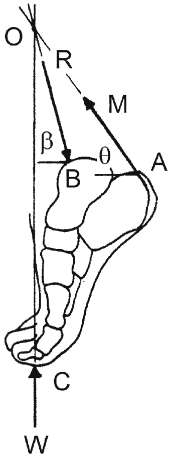

Fig. 2.27. Force diagram of the foot on tiptoe, showing that they form a concurrent system. (From [86])

and 2.28 show the forces when someone stands on tiptoes on one foot. The reaction force on the talus bone of the foot is in balance with the normal force from the floor (equal to the body weight) and the muscle force transmitted by the Achilles tendon on the calcaneus (heel). (The mass of the foot itself is neglected here.) The muscle and reaction forces are

$$M = W_{\mathrm{b}} \frac{\cos\beta}{\cos(\theta+\beta)}, \tag{2.39}$$

$$R = W_{\mathrm{b}} \frac{\cos\theta}{\cos(\theta+\beta)}. \tag{2.40}$$

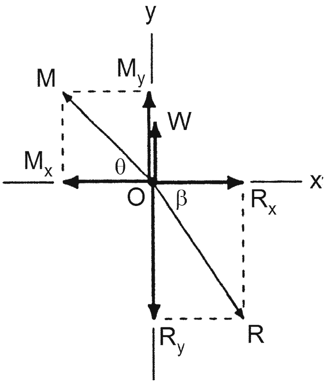

Fig. 2.28. Components of the forces acting on a foot on tiptoe. (From [86])

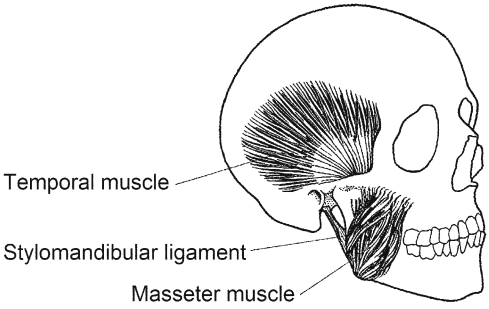

Fig. 2.29. The masseter and temporal muscles in the jaw, about the temporomandibular joint. (From [88])

These are both much greater than the body weight W_b. For $\theta = 45°$ and $\beta = 60°$, we see that $M = 1.93W_\mathrm{b}$ and $R = 2.73W_\mathrm{b}$.

Jaws and Teeth

Forces on teeth arise from several sources [72]. Figure 2.29 shows how the masseter muscles provide the force in the lever system involved in chewing and biting. Lever models can be used to examine the quasistatics of chewing and biting. (See Problem 2.27.)

Orthodontics is the practical application of biomechanics to move teeth using forces applied by *appliances*, such as wires, brackets, and elastics [81, 87, 90]. Each tooth has a center of mass, but since teeth are not free bodies – they are restrained by the periodontium – a more useful position in the tooth is defined, the *center of resistance*. This is the balance point for the tooth. Figure 2.30 shows how forces and torques (moments) applied to the crown of a tooth, can be designed to create a lateral force at the center of resistance, but no torque about it. Appliances can affect several teeth, such as the intrusion

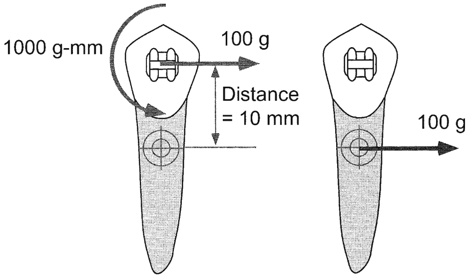

Fig. 2.30. In orthodontics forces and torques are applied to the crown (*left*), leading to forces (and in this case no torques) about the center of resistance. (Reprinted from [81]. Used with permission of Elsevier)

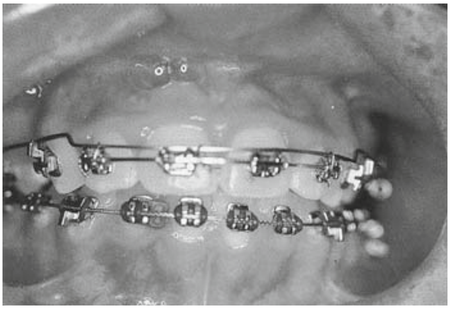

Fig. 2.31. Photo of an intrusion arch used in orthodontics. (Reprinted from [81]. Used with permission of Elsevier)

arch shown in Fig. 2.31, which leads to the application of forces and torques shown in Fig. 2.32.

2.3.4 Lower Back Problems

Most people eventually develop chronic lower back problems. The best way to avoid such a persistent and annoying problem is to try to avoid situations that might trigger your first back problem, such as bending over and lifting heavy objects, or sleeping on very hard beds. We will examine a simple statics model that will show why you should never bend and lift. (You should use the muscles in your legs to lift and not those in your back.) We will examine the force on the fifth lumbar vertebra when you bend and lift.

The spinal cord consists of 33 vertebra with 26 bones, and is classified into five sections. Starting from the top (superior) in Fig. 2.33, there are seven cervical, twelve thoracic – which rhymes with "Jurassic" – and five lumbar vertebra, and then five fused vertebra in the sacrum (sae'krum) and four fused vertebra in the coccyx (koak'-sis) (tail bone). Figure 2.34 shows two lumbar vertebra with the central regions of each, the centrum, separated by the intervertebral disc; more detail about the vertebrae is shown in Fig. 2.35. Note

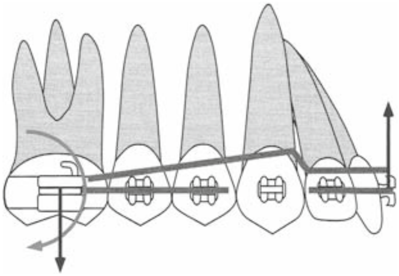

Fig. 2.32. Forces and torques applied by the intrusion arch in Fig. 2.31. (Reprinted from [81]. Used with permission of Elsevier)

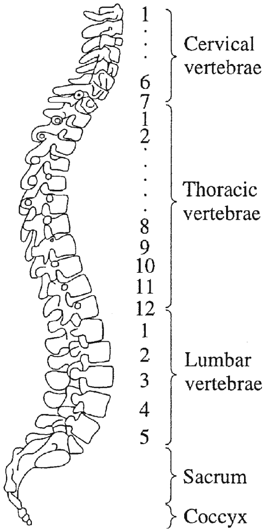

Fig. 2.33. The vertebral column (spine). (From [65].) The thoracic and sacral curves are primary curves, while the cervical and lumbar curves are secondary curves

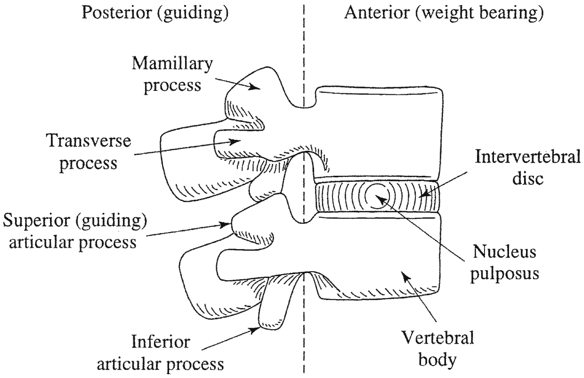

Fig. 2.34. Side view of two vertebrae separated by a vertebral disc. (From [65])

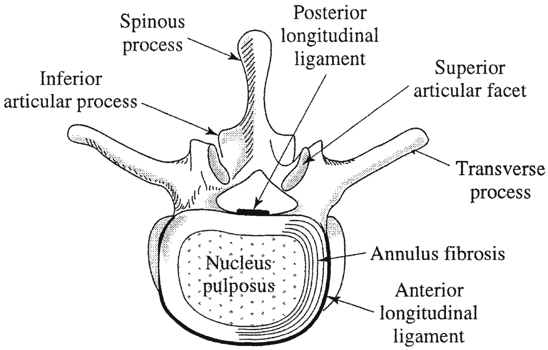

Fig. 2.35. Vertebra viewed from above. (From [65])

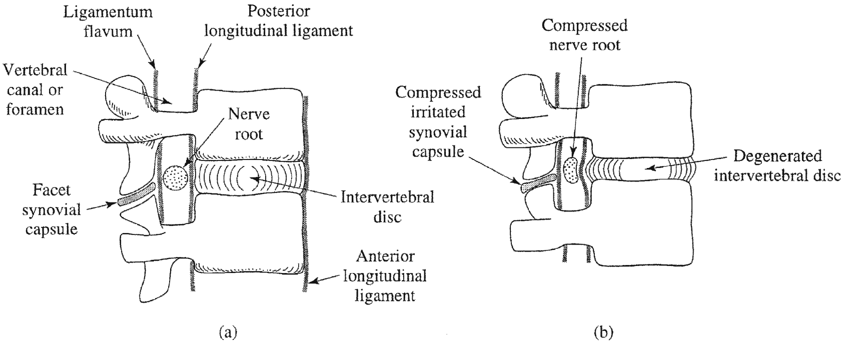

Fig. 2.36. Cross-section of two vertebrae and a vertebral disc with nerve for (**a**) a normal intervertebral disc and (**b**) one that has degenerated and is compressing the nerve root. (From [65])

the spinal nerve root (in Fig. 2.36) (pain region) and the spinous processes. The distinct vertebrae become successively larger down the spinal cord, because of the additional load they bear. This combination of vertebrae and intervertebral discs provides flexibility in the spinal cord, but also causes potential problems.

The spinal cord is not straight; each section is curved. At birth, only the thoracic and sacral curves are developed. These *primary curves* are in the same direction and lead to the "fetal position." At three months, the cervical curve develops, so the baby can hold his/her head up. When the baby learns to stand and walk, the lumbar curve develops. These *secondary curves* have curvature opposite to that of the primary curves (Fig. 2.33). Figure 2.37 shows the lumbosacral angle between the fifth lumbar vertebra and the sacrum.

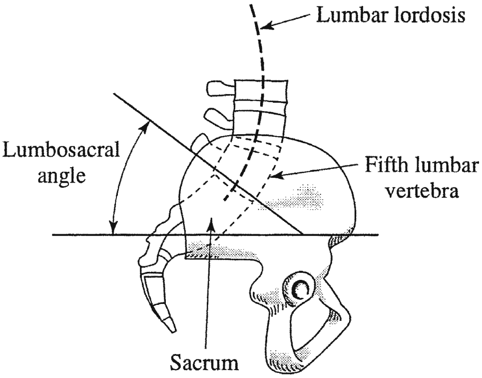

Fig. 2.37. The lumbosacral angle is defined as that between the horizontal and the top surface of the sacrum. (From [65])

Deviations in the angle from ∼30° can lead to lower back pain. We will model the spinal cord as a rigid bar even though this description of the spinal cord curves would suggest a more complex model.

There is a series of massive muscles from the posterior parts of the iliac crest and sacrum to the skull (occipital bone), called the erector spinae (spy'nee) (or sacrospinalis (sae-kro-spy-na'-lis)) (Figs. 2.38 and 2.39). They will be modeled as a single muscle inserted 2/3 from the center of mass of the head and arms, at a 12° angle (Fig. 2.40).

Consider the spinal cord at an angle θ to the horizontal (Fig. 2.40); initially we will take $\theta = 30^\circ$, corresponding to a deep bend. It is hinged at the lumbosacral disc, just below the fifth lumbar vertebra and the sacrum (Figs. 2.37 and 2.40). We will choose the axis for torque analysis right there. There is a reaction force $\mathbf{R}$ (of magnitude R) from the sacrum with components R_x and R_y. The weight of the trunk (above the hips, excluding arms and head) W_1 acts half way down the spinal cord. The weight of the arms, head, and any object lifted, W_2, act at the top of the spinal cord. These are shown in Fig. 2.40 along with the erector spinae force $\mathbf{M}$ (of magnitude M). Using Table 1.7, it is reasonable to approximate $W_1 = 0.4W_\mathrm{b}$ and $W_2 = 0.2W_\mathrm{b}$, with nothing being lifted.

Figure 2.40 shows that $\mathbf{M}$ acts at an angle that is $\theta - 12^\circ$ relative to the horizontal; for our first example $\theta = 30^\circ$ and so this angle is 18°. The force balances are

$$\sum F_x = R_x - M\cos 18^\circ = 0, \tag{2.41}$$

$$\sum F_y = R_y - M\sin 18^\circ - 0.4W_\mathrm{b} - 0.2W_\mathrm{b} = 0. \tag{2.42}$$

With this choice of rotation axis, the torque due to the reaction force is zero. The component of $\mathbf{M}$ normal to the spinal cord is $F' = M\sin 12^\circ$, leading to a torque $(2L/3)(\sin 12^\circ)(M)$. The fraction of each weight force normal to the spinal cord is $\theta = \cos 30^\circ$. Torque balance requires

$$\sum \tau_z = \frac{2L}{3}\sin 12^\circ(M) - \frac{L}{2}\cos 30^\circ(0.4W_\mathrm{b}) - L\cos 30^\circ(0.2W_\mathrm{b}) = 0. \tag{2.43}$$

The torque equation gives $M = 2.5W_\mathrm{b}$ or 2,200 N (500 lb) for the 880 N (200 lb) body weight of a 90 kg body mass. The reaction force parameters are $R_x = M\cos 18^\circ = 2.38W_\mathrm{b}$, $R_y = 1.37W_\mathrm{b}$, $\phi = \arctan(R_y/R_x) = 30^\circ$ (which is the angle the reaction force makes with the horizontal), and $R = 2.74W_\mathrm{b}$, which is 2,400 N (540 lb) here. Clearly, the muscle forces and reaction force on the lower spinal cord are much larger than the body weight. The moment arms of the weights are about the same as that of the muscle. However, the direction of weight forces lead to large torques at large bending angles, while the direction of the muscle force does not lead to a large torque to balance the weights – at any bending angle.

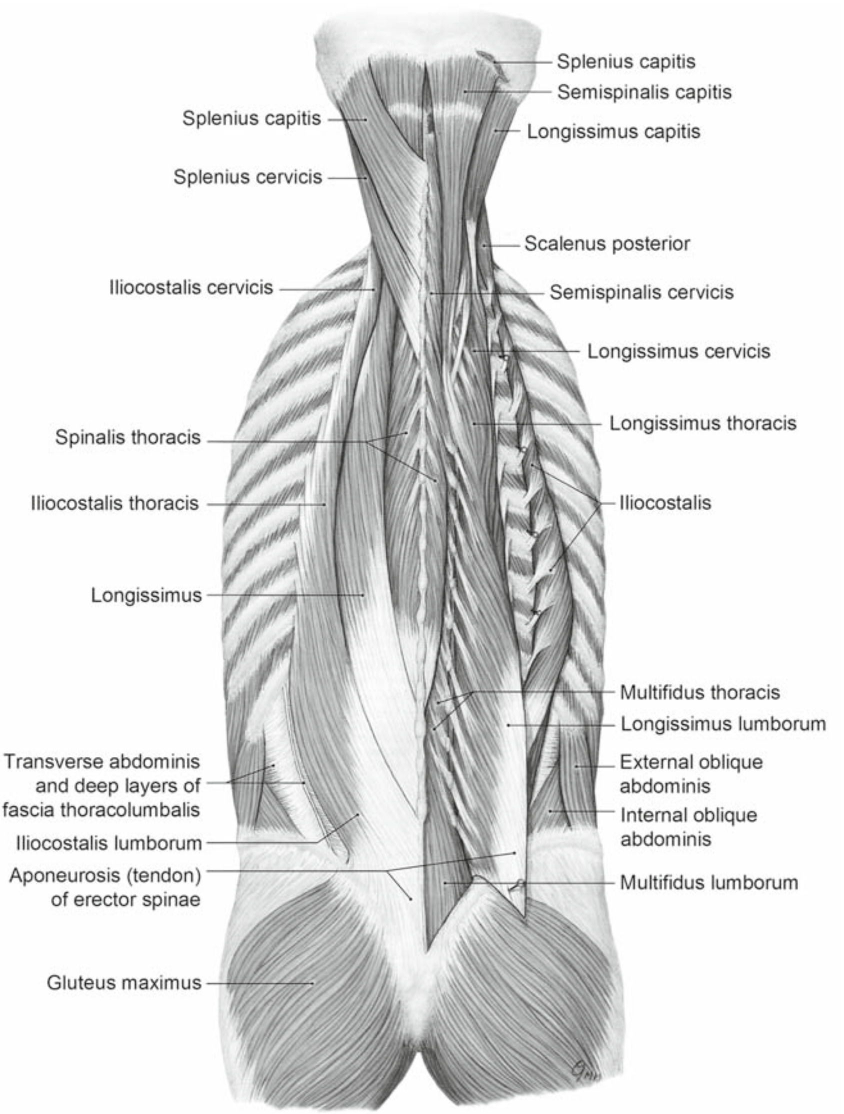

Fig. 2.38. Intermediate (*left*) and deep (*right*) layers of back muscles – showing the erector spinae muscles. The erector spinae consists of lateral columns (the iliocostalis lumborum, thoracis, and cervicis muscles), intermediate columns (the longissimus thoracis, cervicis, and capitis muscles), and a medial column (spinalis thoracis). (From [93])

Matters are even worse if you hold a weight in your arms; this simulates bending and lifting an object of this weight. All this does is to increase W_2. Let us increase W_2 by $0.2W_\mathrm{b}$ (180 N (40 lb) in our example) to $0.4W_\mathrm{b}$. Now

$$\sum F_x = R_x - M\cos 18^\circ = 0, \tag{2.44}$$

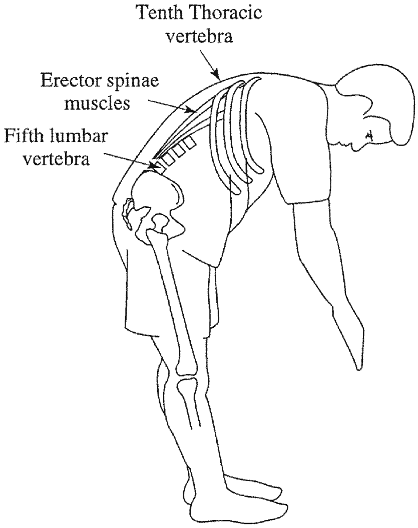

Fig. 2.39. Diagram of the erector spinae muscles used to control the trunk when bending. (From [65])

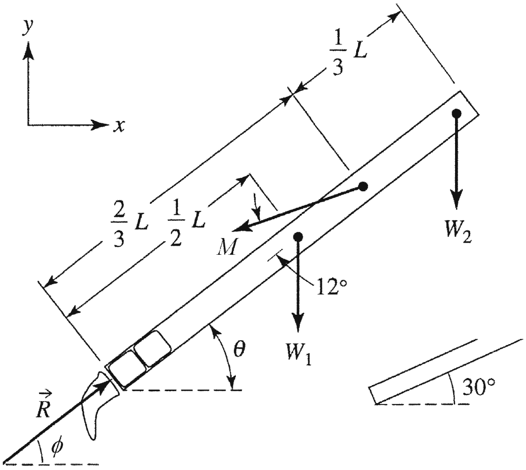

Fig. 2.40. Free-body diagram of the vertebral column while bending, with the spine modeled as a straight bar at an angle θ to the horizontal, which we will take to be ~30° – a bit steeper bend than is depicted here and shown in the inset. The angle of **R** to the horizontal is ϕ. With nothing being lifted, we will take $W_1 = 0.4W_b$ and $W_2 = 0.2W_b$. (From [65])

Table 2.3. Forces in the body during bending and lifting

θ	M (no lifting)	M (extra $0.2W_\mathrm{b}$)	R (no lifting)	R (extra $0.2W_\mathrm{b}$)
30°	2.50	3.74	2.74	4.07
60°	1.44	2.16	1.93	2.81
80°	0.50	0.75	1.08	1.53
90°	0	0	0.60	0.80

All forces are in units of the body weight. For a body mass of 90 kg, multiply each number by 880 N (200 lb)

$$\sum F_y = R_y - M \sin 18^\circ - 0.4W_\mathrm{b} - 0.4W_\mathrm{b} = 0, \tag{2.45}$$

$$\sum \tau_z = \frac{2L}{3} \sin 12^\circ (M) - \frac{L}{2} \cos 30^\circ (0.4W_\mathrm{b}) - L \cos 30^\circ (0.4W_\mathrm{b}) = 0. \tag{2.46}$$

The force the erector spinae muscles need to exert increases to $3.74W_\mathrm{b}$ or 3,300 N (740 lb). The muscle must exert an additional 1,100 N (250 lb) to balance only an additional 180 N (40 lb). We see that $R_x = 3.56W_\mathrm{b}$, $R_y = 1.96W_\mathrm{b}$, and $R = 4.07W_\mathrm{b}$. This is 3,600 N (810 lb), an additional 1,200 N (270 lb) of reaction force on the fifth lumbar vertebra.

Table 2.3 shows these forces for several bending angles. We can see why bending itself, and bending and lifting can lead to problems with the back muscles and the lower vertebra discs.

What does this mean for the lumbosacral (intervertebral) disc? Let us consider our initial example of bending by 60° from the upright position, to 30° from the horizontal, without any lifting. The reaction force on this disc is 2,400 N (540 lb). This force pushes down on the top and up on the bottom of this cylindrical disc. We are assuming a load that is normal to the axis, which is not exactly correct at these angles. Figure 2.41 shows how much the height of the disc decreases (fractionally) with this type of load. The weight corresponding to 250 kg is 2,400 N (540 lb), so by using this figure a 20% contraction is expected parallel to the spine. If the disc is a cylinder of radius r and height H, then its volume is $\pi r^2 H$. If the material in the disc is incompressible, its volume will not change with this compression and if H decreases by 20%, r will increase by about 10%. The bulging disc can press against the nerve, as seen in Fig. 2.36b. With lifting, the reaction force goes up more, the compression of the disc increases, and there is more bulging and more irritation of the nerves. (We have compared a 245 kg (2,400 N, 540 lb) load on the disc to a situation with no force on this disc. In the upright position, there is 0.6× the body mass (and weight), 54 kg (530 N, 120 lb) already supported by this disc due to body weight, so the load is really changing from 530 N (120 lb) to 2,400 N (540 lb). (How does this affect our numerical results and conclusions?) Also the loading is not uniform when you lift or bend, and there can be tension and compression and bulging due to this, as in Fig. 2.42.)

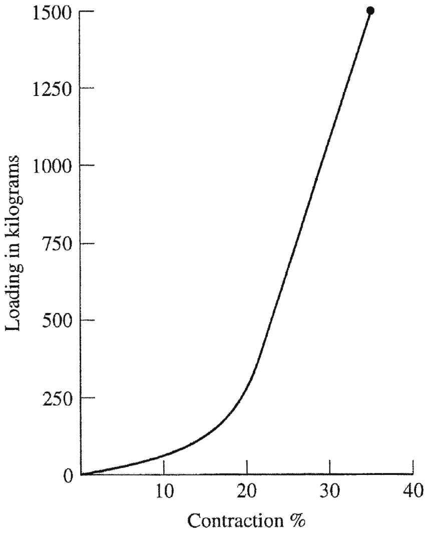

Fig. 2.41. Loading of wet lumbar vertebral discs of persons 40–59 years of age vs. percent compression. For the loading in N, the ordinate scale needs to be multiplied by 9.8. (From [65]. Based on [96])

This teaches us two things. (1) Understanding the mechanical properties of the parts of the body is essential to understand the implications of forces on the body. (2) You should never bend and lift. Problems 2.33–2.35 examine the torques during lifting of objects of different sizes, with different upper body positions (as above), and with different techniques.

These lower back forces depend not only on spine angle and load, but on knee bending during lifting and where the load is positioned relative to the body. During lying the lower back forces are 20–50% of those while standing upright relaxed (Table 2.4), and reach values over double the weight and over

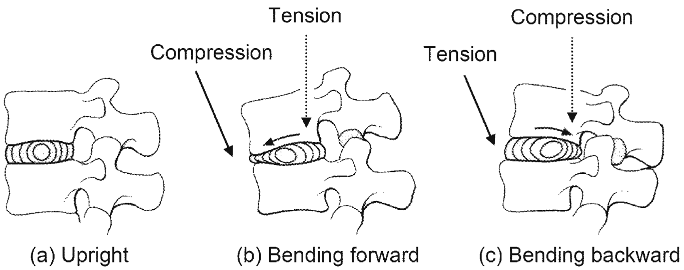

Fig. 2.42. Disc compression for a person who is (**a**) vertical, (**b**) bending forward, and (**c**) bending backward. Bending forward leads to disc compression anteriorly and tension posteriorly, bulging on the compressive side, and the shifting of the disc nucleus posteriorly. (From [88])

Table 2.4. Values of intradiscal pressure for different positions and exercise, relative to that during relaxed standing. (Using data from [80, 83, 92])

position or activity	%
still	
lying supine	20
side-lying	24
lying prone	22
lying prone, extended back, supporting elbows	50
relaxed standing	100
standing, bent forward	220
sitting relaxed, no back rest	92
sitting actively straightening back	110
sitting with maximum flexion (bent forward)	166
sitting bent forward, thigh supporting the elbows	86
sitting slouched in a chair	54
motion	
standing up from chair	220
walking barefoot or in tennis shoes	106–130
jogging with shoes	70–180
climbing stairs, one at a time	100–140
climbing stairs, two at a time	60–240
walking down stairs, one at a time	76–120
walking down stairs, two at a time	60–180
lifting	
lifting 20 kg, no bent knees	460
lifting 20 kg, bent knees, weight near body	340
holding 20 kg near body	220
holding 20 kg, 60 cm from chest	360

3× the torso weight during fast walking (Fig. 2.43). Table 2.4 shows that the pressure between vertebral discs for people sitting is minimized when they slouch, so when your parents tell you to stop slouching you can respond that you are trying to minimize intradisc pressure for long-term care of your spinal cord.

How can you maintain your back and relieve lower back pain if you have relatively minor lower back damage? Stretch your back muscles. Always bend your knees and use your leg muscles (and not your back) when you lift. Make sure that you maintain the curvature of your lower back at all times. You should sit on chairs with lower back support (see Fig. 2.44 and [91]), but when you must sit on flat back chairs, you may want to use a rolled-up towel or a pillow at the bottom of your back to help maintain the curvature. When you sleep, maintain one of two positions: on your back with your knees bent

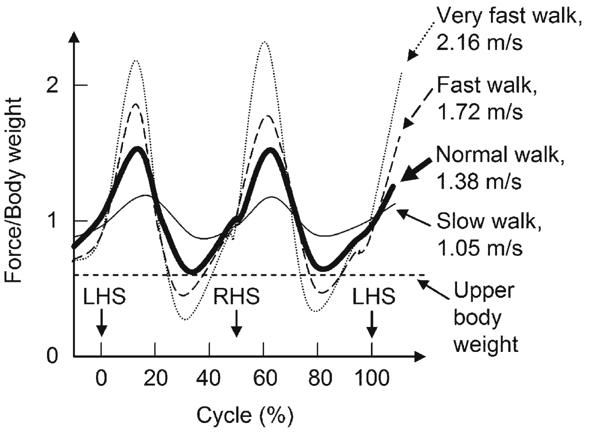

Fig. 2.43. Axial load on the disc between the L3 and L4 vertebra while walking at different speeds. LHS and RHS are left and right heel strike, respectively. (Based on [68, 83])

and feet pulled up (bent hips and knees) or the fetal position; never sleep on your front. (One reason for this is that in the supine position with straight legs, the vertebral portion of the psoas muscle puts a load on the lumbar spine, whereas this muscle relaxes and this load decreases when the hips and knees are bent and supported [83].) Only sleep on beds that allow your back

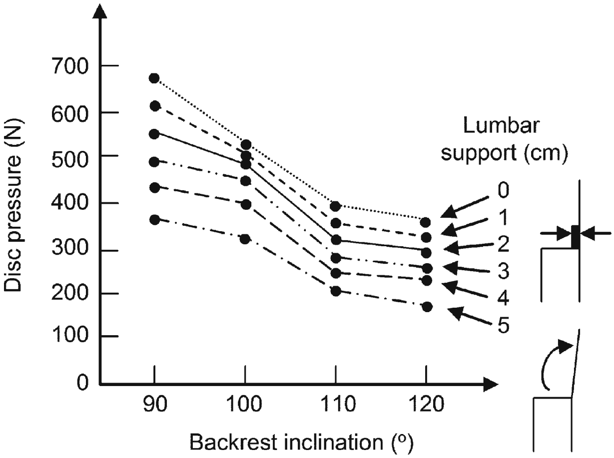

Fig. 2.44. The pressure on the third lumbar disc is decreased with backward backrest inclination and with lumbar support. Also, it is increased with support in the thoracic region, which is not shown here. Chairs with some backward inclination and lumbar support provide needed support, while those with an upright flat back or that curve toward the body can cause painful pressure. (Based on [64, 69])

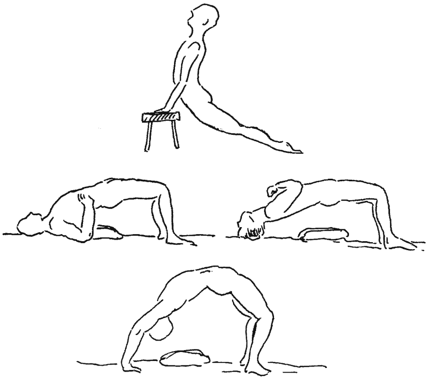

Fig. 2.45. Hyperextension exercises recommended (at the Tientzin Hospital in China) to strengthen the back of lumbago patients (i.e., those with mild to severe pain or discomfort in the lower back); each is performed with the patient's back kept hollow. Several other exercises are also recommended for those with lower back pain. If you have lower back pain, please consult your physician before attempting any of these exercises. (From [70])

to maintain its natural curvature; beds should not be too firm or too soft. Some hyperextension exercises that can strengthen muscles to help your back are shown in Fig. 2.45. (Guess, who has a lower back problem?)

2.3.5 Three-Force Rule

Consider the foot–lower leg combination shown in Fig. 2.46a, which is a model for a person walking upstairs. There are three forces acting on this isolated

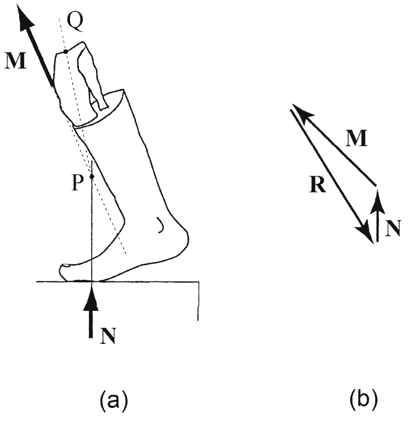

Fig. 2.46. (a) Illustrating the three-force rule on the free lower leg, with reaction force **N**, patellar tendon force **M**, point where these extended vector lines meet P, and the point where the joint force acts on the tibia Q. (b) Vector diagram of three-force rule with **R** being the joint reaction force at Q. (From [76])

system. The normal force **N** acts vertically at the front of the foot and for slow stair climbing has a magnitude equal to the body weight W_b. This normal force tries to rotate the lower leg clockwise about the knee. The patellar tendon transmits the force of the thigh quadriceps muscle **M**, as shown, and tries to counter this rotation. The direction of this force is along this tendon and the direction of this tendon is known. There is also a reaction force **R** exerted by the femoral condyles on the top of the tibia, acting as shown at point Q. In this two-dimensional problem, there are three equations (two for force balance and one for torque balance) and three unknowns: the magnitudes of **M** and **R**, and the direction of **R**. As such, this problem can be solved in the straightforward way illustrated earlier. However, there is a graphical procedure called the three-force rule that offers some additional insight and, of course, the same answers as the straightforward method. (For simplicity, we are ignoring the mass of the leg and foot here.)

Extending the **N** and **M** vectors, we see that they intersect at point P. If we choose the torque rotation axis perpendicular to this point, we see that these two forces contribute nothing to the torque since their **r** and **F** vectors are, respectively, antiparallel and parallel to each other. Since in static equilibrium (or the quasistatic situation here) the net torque is zero, the reaction force vector **R** must also pass through P when it is extended, and so **R** is parallel to the QP line segment. Because the directions of **M** and **R** are now known, we can place them at the head and tail of the **N**, respectively, as shown in Fig. 2.46b (always with the head of one vector to the tail of the other, as in adding vectors). The lengths of both vectors are now determined by lengthening them until they hit each other. Then the sum of the force in the x and y directions is zero (because of the closed triangle), and we have obtained all of the needed information.

By the way, the reaction forces at the knee are $\sim$3–4W_b during walking [82] and are much greater during stair climbing.

2.3.6 Multisegment Modeling

Several parts of the body are often important in modeling the body, both in static situations and for those involving motion. As an example, Fig. 2.47 shows how an anatomical model of leg can be modeled by three segments, describing the upper leg, lower leg, and the foot. Each segment is labeled by its mass (m), center of mass (dot), and moment of inertia (I). In this example there are also three joints (freely rotating hinges or pins), between the hip and upper leg, the upper leg and lower leg (the knee), and the lower leg and foot (the ankle), each denoted by an open circle. The forces by muscles and normal forces at the joints and other places (such as the floor) can be added to this link segment model, and the forces on torques on the entire object can be analyzed or those on each segment can be analyzed by itself. (This is

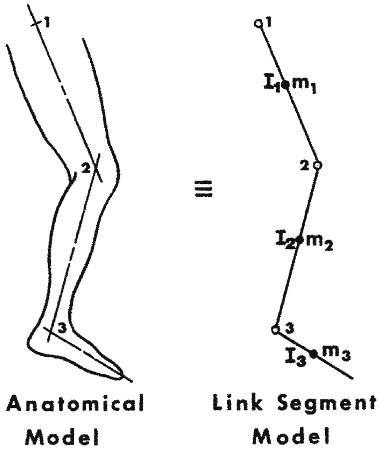

Fig. 2.47. Relationship between an anatomical model of the leg and a link segment model of the upper leg, lower leg, and foot. (From [95]. Reprinted with permission of Wiley)

somewhat similar to examining the whole body and then the leg by itself in the above analysis of the hip.)

In analyzing each segment in Fig. 2.47 by itself, you arrive at a free body diagram for each segment, as in Fig. 2.48. In this example, the reaction forces

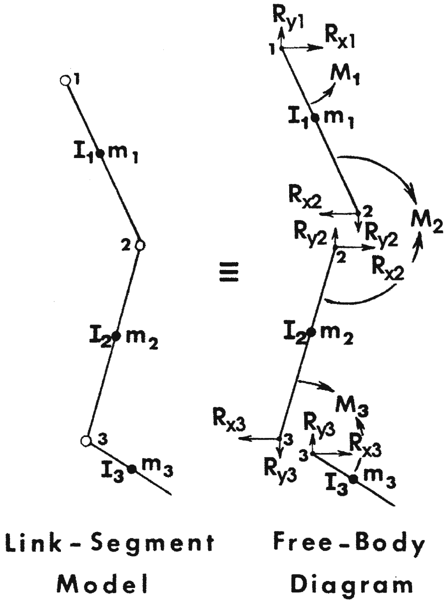

Fig. 2.48. Relationship between the link segment of the leg with a free-body diagram, with individual upper leg, lower leg, and foot. (From [95]. Reprinted with permission of Wiley)

at every body joint are shown (**R**), and at each particular joint the forces are equal in magnitude and opposite in direction due to Newton's Third Law, such as $\mathbf{R_2}$ shown for the knee. The forces due to each muscle (or effective set of muscles) could be added, acting with equal magnitudes but opposite directions at the origin and point of insertion (again Newton's Third Law). (This is examined in Problem 2.37.) In the free-body diagram shown in Fig. 2.48, the total torque (which is called moment M in the figure) about each joint is shown.

2.4 The Sense of Touch

The body also "feels" force. This is the sense of touch. The *somatic senses*, include the sense of touch by the skin (cutaneous sensations), the sense of position of the limbs (proprioception), and the sense of movement of the limbs (kinesthesis). There are sensors in the skin for tactile perception, perception of temperature, and the perception of pain [73].

There are four types of tactile receptors in glabrous (hair-free) skin (Fig. 2.49), each sensitive to stresses and displacements at different frequencies. Merkel receptors (or disks), located near the border between the epidermis (outermost skin layer) and dermis (the underlying layer), are most sensitive to pressure disturbances which vary in the 0.3–3 Hz range. The other receptors are deeper within the dermis. Meissner corpuscles sense light

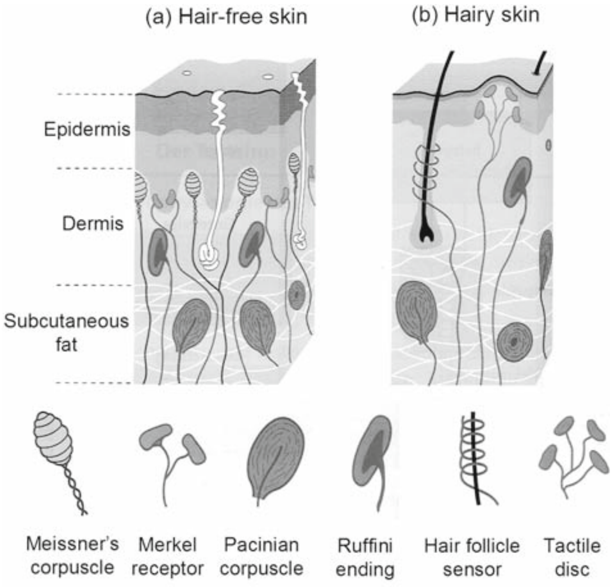

Fig. 2.49. Cross-section of (**a**) hair-free and (**b**) hairy skin, showing tactile sensors. (From [89])

tapping, which is characterized by 3–40 Hz variations, and sensed as flutter. Ruffini cylinders (or endings) sense faster vibrations, 15–400 Hz, such as those caused by stretching of the skin or joint movement, and sensed as buzzing. Pacinian corpuscles sense a range of rapid vibrations, from 10 to 500 Hz. Motion of hair also stimulates these receptors.

The spatial density of mechanical receptors on the skin varies from $25/\mathrm{cm}^2$ at the tip of the tongue to $0.02/\mathrm{cm}^2$ on the back [66]. The movement of hair on hairy skin contributes to the sense of touch, along with sensing by these four receptors. The mechanism for converting these impulses into electrical signals is discussed in Chap. 12.

The skin also has two types of receptors that sense temperature. Warm fiber thermoreceptors increase their firing rate only when it is warmer and cold fibers only when it is colder; they are not sensitive to mechanical stimulation. Warm fibers respond from 30 to 48°C, and best at about 44°C, while cold fibers respond from 20 to 45°C, and best at 30°C. There are on the order of 5–10 thermal receptors per cm^2 over most of the skin. This sense of temperature is important in the body's drive for temperature control (Chap. 13).

Nociceptors in the skin sense excessive pressure, extreme temperature, and corrosive chemicals, each which can damage the skin. Some pain receptors transmit signals very rapidly to the brain, on myelinated axons in neurons with conduction speeds up to 30 m/s (see Chap. 12). Some pain receptors indicate persistent pain and have signals that are transmitted on very slow unmyelinated axons with speeds of 2 m/s and slower.

As characterized by Stevens' Law ((1.6), Table 1.15), the variation of the perceived stimulus is slightly sublinear with stimulus strength for vibration, somewhat superlinear for sensing pressure on the palm and the heaviness of lifted objects, linear for coldness, and superlinear for warmth.

A problem in Chap. 7 (Problem 7.14, Fig. 7.24) addresses how to use the sense of touch to estimate the internal pressure in an elastic vessel. This is an example of *palpation*, which is the use of a physician's hands to examine parts of the body during a medical examination. (A physician also taps the body in the diagnostic known as *percussion*. A third physical diagnostic is listening to sounds emanating from the body, which is called *auscultation*; it is discussed in Chap. 10.)

2.5 Diversion into the Units of Force and Pressure

2.5.1 Force

We should be clear on the correct units of force, as in Table 2.5. In the English (or FPS) system it is pounds (lb). In the metric MKS-SI system it is newtons (N) and in the CGS systems it is dynes. Most technical work is in MKS-SI; however, much medical work is done using the English system (in the US). We

Table 2.5. Units of force

$1\,\mathrm{N} = 1\,\mathrm{kg\text{-}m/s^2}$
$1\,\mathrm{N} = 10^5\,\mathrm{dyne}$
$1\,\mathrm{N} = 0.225\,\mathrm{lb}$ ($\sim$2/9 lb)
$1\,\mathrm{lb} = 4.45\,\mathrm{N}$

will usually use lb or N, with lb used when referring to body weights. (Also see Appendix A.)

One kilogram (kg) is a mass (m) that at sea level (on the planet Earth) has a weight of 9.8 N (or 2.2 lb). This is a consequence of gravitational acceleration on earth and Newton's Second Law of motion

$$F = mg, \tag{2.47}$$

where $g = 9.8\,\mathrm{m/s^2}$. "g" is the acceleration of any freely falling object due to gravity. In CGS units, a mass of $1\,g$ has a weight of 980.7 dynes because $g = 980.7\,\mathrm{cm/s^2}$. In the English system $g = 32.2\,\mathrm{ft/s^2}$. The unit of mass in the English system is the slug; it is rarely used in the US. One slug has a weight of 32.2 lb. (Sometimes in the biomechanics literature the loading mass is given, and it is possible for the unit "kg" to be mistakenly referred to as a force. What is really meant is the weight of an object with a mass of 1 kg. To lessen confusion, this should be expressed as 1 kg (force). Even better, when 1 kg (force) is seen it should be replaced by 9.8 N ($\sim$10 N).)

2.5.2 Pressure

We will encounter the concept of pressure later. Pressure (P) is simply a force per unit area. (More generally, a force per unit area is a stress (Table 2.6). Pressure or more precisely hydrostatic pressure is a stress that is the same in all directions.) The units of pressure (or stress σ) are the units of force divided by the units of area. Because work (or energy) is force × distance, the units of pressure are the same as those of energy/volume. (Also see Appendix A.)

The standard MKS-SI unit of pressure is a pascal (Pa), with $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$. We will find that often the numbers we will encounter are simpler in units of $\mathrm{N/mm^2}$, which are the same as the units MPa. Common units in English units are pounds per square inch (psi).

These units are independent of the planet we happen to be on. Some common units of pressure are specific to Earth (at sea level, at 0°C temperature). A column of a liquid of mass density ρ and height h exerts a pressure

$$P = \rho g h. \tag{2.48}$$

This is the same as (2.47), after both sides have been divided by the column area A, because $P = F/A$ and $m = \rho \times \text{volume} = \rho A h$. At sea level on

Table 2.6. Units of pressure

$1\,\text{N/m}^2 = 1\,\text{Pa} = 9.87 \times 10^{-6}\,\text{atm.} = 0.0075\,\text{mmHg} = 0.102\,\text{mmH}_2\text{O}$

$1\,\text{N/mm}^2 = 10^6\,\text{N/m}^2 = 10^6\,\text{Pa} = 1\,\text{MPa} = 145\,\text{psi} = 9.87\,\text{atm.} \simeq 10\,\text{atm.}$

$1\,\text{psi} = 0.0069\,\text{N/mm}^2 = 6{,}894.8\,\text{N/m}^2$ (or Pa) $= 1/14.7\,\text{atm.} = 0.068\,\text{atm.}$

$1\,\text{bar} = 10^5\,\text{N/m}^2$ (or Pa) $= 0.1\,\text{N/mm}^2 \simeq 1\,\text{atm.}$

$1\,\text{atm.} = 1.013 \times 10^5\,\text{Pa} = 1.013\,\text{bar} = 0.103\,\text{N/mm}^2$ (or MPa) $= 14.7\,\text{psi}$
$= 760\,\text{mmHg} = 29.9\,\text{inchHg} = 1{,}033\,\text{cmH}_2\text{O} = 407\,\text{inchH}_2\text{O}$

$1\,\text{mmHg} = 0.00132\,\text{atm.} = 133\,\text{N/m}^2 = 13.6\,\text{mmH}_2\text{O}$

$1\,\text{mmH}_2\text{O} = 0.1\,\text{cmH}_2\text{O} = 9.68 \times 10^{-5}\,\text{atm.} = 9.81\,\text{N/m}^2 = 0.0735\,\text{mmHg}$

$1\,\text{kg (force)/cm}^2 = 9.8\,\text{N/cm}^2$

We will often use MPa ($= \text{N/mm}^2$), but occasionally use other units, such as mmHg when discussing blood pressure

Earth (0°C) the air pressure is 1 atmosphere (1 atm.), which is the pressure exerted by the air column above it. The same pressure is exerted by 760 mm of Hg (mercury) or 1,033 cm of water. The units of mmHg and cmH_2O (or inches of H_2O) are very commonly used even though they really refer to h in (2.48) and not pressure. (In a calculation, they need to be multiplied by $\rho_{\text{Hg}} = 13.6\,\text{g/cm}^3$ or $\rho_{\text{H}_2\text{O}} = 1.0\,\text{g/cm}^3$ and then by g. Note that $\rho_{\text{blood}} = 1.0\,\text{g/cm}^3$.) In the US, air pressure is commonly expressed in weather reports in units of inches of Hg, as in "the air pressure is 29.8 in and dropping." Blood pressure is commonly reported in mmHg, as in 120/80, which means that the systolic and diastolic pressures are, respectively, 120 mmHg and 80 mmHg. Air pressures in the body, such as in the lungs, are sometimes expressed in terms of cm or inches of water, because it is a smaller and much more convenient unit. Pressure is often referenced to atmospheric pressure, so a blood pressure of 120 mmHg really means an absolute pressure that is 120 mmHg above atmospheric pressure (with 1 atm. = 760 mmHg), and as such is called a gauge pressure.

It is an amazing coincidence that the Earth-based unit of atmospheric pressure (1 atm.) is within 1% of the Earth-independent MKS-SI-based unit of a "bar" ($1\,\text{bar} = 10^5\,\text{N/m}^2$ (or Pa)), with 1 atm. = 1.013 bar.

2.6 Summary

Force and torque balance can often be analyzed in terms of levers, such as for the lower arm. The equilibrium of a part of the body can be analyzed by examining the forces due to gravity, muscles, and reaction forces in the whole body and that body part separately. Muscle and internal reaction forces can exceed the body weight, as was seen in analyzing the forces in the hip and lower back, and this can have serious consequences.

Problems

Statics of the Arm and Levers

2.1. A person holds a weight with her arm extended horizontally, using her deltoid muscles to balance the weight about the shoulder joint. What type of lever is this? Sketch and label this lever.

2.2. A person holds himself up in a pushup position. Consider one of the person's arms, with the pivot point being the hand on the ground. Are the biceps brachii or triceps brachii involved? What type of lever is this? Sketch and label this lever.

2.3. For each of the following, identify the type of lever, and show in a sketch the locations of the applied force, the fulcrum, and the load being applied:
(a) cutting with a pair of scissors
(b) lifting a wheelbarrow
(c) picking up something with a pair of tweezers.

2.4. Analyze the force balance in the x and y directions and the torque balance in the z-direction for the lower arm, with a vertical upper arm and a horizontal lower arm (length 35 cm) (Fig. 2.9). As with Case 1 ignore the forearm weight and other muscles. Now the effect of the reaction force $\mathbf{N}$ on the joint (with components N_x and N_y) is explicitly included (because it contributes no torque it did not have to be included in the earlier Case 1 analysis) and assume the biceps are attached 4 cm from the pivot at an angle θ to the lower arm. With $\theta = 75°$, find the muscle force M and the magnitude of the reaction force in terms of the weight W and the angle of this reaction force relative to the x (horizontal) axis.

Statics of the Hip and Leg

2.5. Using Fig. 1.15, what can you say about the height of the person whose leg is depicted in Fig. 2.17?

2.6. Show how the dimensions given in Fig. 2.16 change to those in Fig. 2.20 when the person holds a cane.

2.7. In examining the hip forces during the equilibrium of a man standing on one foot, we assumed that the effective angle of the hip abductor muscles to the x-axis was the same, 70°, without or with a cane. Would using more realistic (and different) values of this angle without or with a cane, significantly affect the conclusions concerning the effect of using a cane?

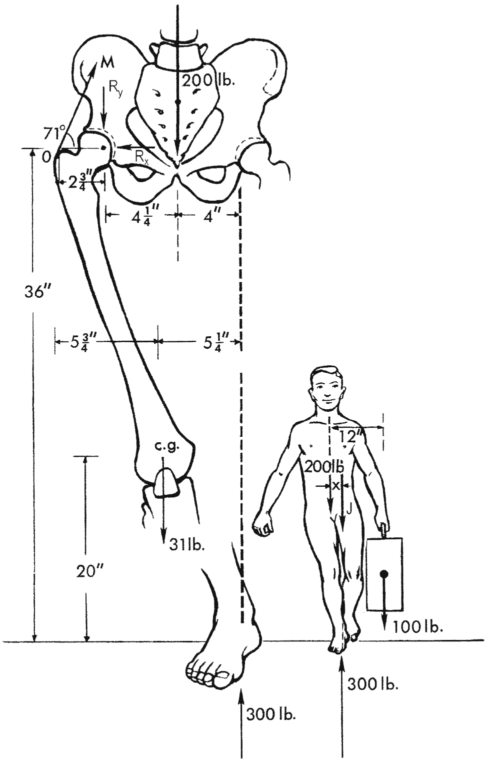

Fig. 2.50. Forces on hip and femoral head while standing on one leg and lifting a weight with the opposite hand. (Reprinted from [75]. Used with permission of Elsevier.) For Problem 2.8

2.8. A 200 lb man stands on his right foot while carrying a 100 lb bag in his left hand. The center of mass of the bag is 12 in from his center of mass (see Fig. 2.50).
(a) Show that the placement of the foot (as shown) leads to no net torque in the body.
(b) Find the force (its magnitude and direction) on the head of the support femur and the force in the hip abductor muscle by examining the right leg.
(c) Compare your answers in parts (a) and (b) with what was found for the man holding no mass – without and with a cane (for $W_b = 880\,\mathrm{N} = 200\,\mathrm{lb}$). Are the forces here greater than for a (200 lb + 100 lb =) 300 lb man (with no cane). Why? (The muscle angle and leg mass are only trivially different for the problem given here and those analyzed above.)

2.9. Redraw Fig. 2.50, changing all distances and forces into metric units and then do Problem 2.8.

2.10. Calculate the force on the hip abductor muscles for a person standing symmetrically on two feet, as a function of foot separation. For what position is this force zero?

2.11. In the arabesque position an initially upright gymnast kicks one leg backward and upward while keeping it straight (mass m_{leg}), pushes her torso and head forward (mass $m_{\text{torso+head}}$), and propels her arms backward and upward while keeping them straight (each m_{arm}). They, respectively, make acute angles θ_{leg}, $\theta_{\text{torso+head}}$, and θ_{arms}, to the horizontal. The centers of mass of the extended leg and vertical balancing leg are $x_{\text{extended leg}}$ and $x_{\text{balancing leg}}$ behind the vertical from the center of mass (in the midsagittal plane), and that of her upper body (torso/head/arms combination) is $x_{\text{upper body}}$ in front of this vertical (so all of these distances are defined as positive). In achieving this arabesque position her center of mass drops vertically from height y_{before} to y_{after}.
(a) Draw a diagram showing the gymnast before and during this maneuver.
(b) Find the equilibrium condition in terms of these masses and distances. (Hint: Analyze the torques about her center of mass in the arabesque position. You can ignore the contribution from her arms. Why? You may not need all of the information that is presented.)
(c) Assume the gymnast is 1.49 m (4 ft 11 in) tall and has a mass of 38 kg (weight 84 lb) and her free leg and arms make a 30° angle with the horizontal. What angle does her upper body make with the horizontal? (Assume the anthropometric relations for a standard human.)

2.12. Redo Problem 2.11c if the free leg of the gymnast is horizontal and her arms are vertical. (This is the cheerleading position.)

2.13. (a) Calculate the torque of the diver of mass m_{b} about the axis through her toes (normal to a sagittal plane) when she is on a diving board and leaning over and about to dive, so the vertical axis through her center of mass is a distance x in front of her toes.
(b) Now say that the diver has a height H and that her body is proportioned as per the data given in Chap. 1. Calculate the torque in terms of H, with her body straight with arms stretched parallel to her torso. Assume that her body can be straight (and so everything in her body can be approximated as being in one plane) and ignore the change in position because she is on her toes.
(c) Redo this if her arms are instead along her sides. How does this torque differ from that in part (b)?

2.14. The split Russel traction device is used to stabilize the leg, as depicted in Fig. 2.51, along with the relevant force diagram [86]. The leg is stabilized by two weights, W_1 and W_2, attached to the leg by two cables. The leg and cast have a combined weight of $W_1 = 300\,\text{N}$ and a center of mass 2/3 of the way from the left, as shown. The cable for W_2 makes an angle $\beta = 45°$ with

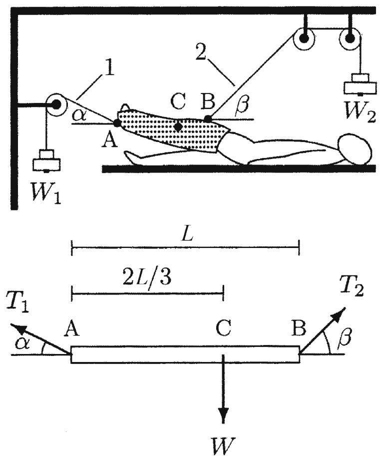

Fig. 2.51. The split Russel traction device. (From [86].) For Problem 2.14

the horizontal. For equilibrium, find the tension in the cables T_1 and T_2 and angle the cable for W_1 makes with the horizontal, α.

Statics of the Shoulder, Knee, Ankle, and Jaw

2.15. You are able to hold your arm in an outstretched position because of the deltoid muscle (Fig. 2.52). The force diagram for this is shown in Fig. 2.53. Use the three equilibrium conditions to determine the tension T in the deltoid muscle needed to achieve this equilibrium, and the vertical and horizontal components of the force exerted by the scapula (shoulder blade) on the humerus. Assume the weight of the humerus is $mg = 8\,\text{lb}$ and the deltoid muscle make an angle of $\alpha = 17°$ to the humerus. (From [65].)

2.16. Solve the more general shoulder problem with a weight in the hand, depicted in Fig. 2.21, by finding M, R, and β. Now evaluate the x and y components of the muscle force, the magnitude of the joint reaction force, and its angle for the following parameters: $a = 15\,\text{cm}$, $b = 30\,\text{cm}$, $c = 60\,\text{cm}$, $\theta = 15°$, $W = 40\,\text{N}$, and $W_0 = 60\,\text{N}$.

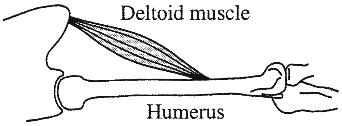

Fig. 2.52. Deltoid muscle during lifting with an outstretched arm. (From [65].) For Problem 2.15

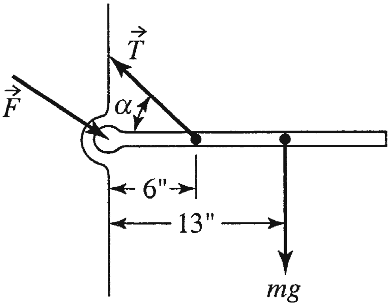

Fig. 2.53. Force diagram for the deltoid muscle and reaction forces during lifting with an outstretched arm. (This is a simpler version of Fig. 2.21.) (From [65].) For Problem 2.15

2.17. (a) A gymnast of mass m_b suspends himself on the rings with his body upright and straight arms that are horizontal with which he clutches the rings. Each ring is suspended by a rope with tension T that makes an acute angle θ with his arms, and the rings are separated by a distance d. Solve for the T and θ. Assume symmetry.
(b) If the gymnast weighs 600 N, $\theta = 75°$, and $d = 1.8$ m, find T. (From [74].)

2.18. Derive (2.34)–(2.36) for the equilibrium of the lower leg with an ankle weight.

2.19. Determine the angle between the leg and the reaction force at the knee for the conditions given in the text for Fig. 2.23.

2.20. Analyze how the patellar tendon and reaction forces depend on the ankle weight (for a fixed leg weight) and leg angle for Fig. 2.23. During exercise, what are the advantages of varying this weight vs. this angle?

2.21. Consider the equilibrium of the foot during crouching for a 200-lb person, with the force through the Achilles tendon, the reaction force of the tibia, and the normal force from the floor in balance, as in Fig. 2.54 – neglecting the weight of the foot for simplicity. Take the angle $\alpha = 38°$.
(a) Why is the normal force from the floor 100 lb?
(b) Find the magnitude of the Achilles tendon tension T and the magnitude and direction of the reaction force **F**.

2.22. The topic of Problem 2.21 is similar to the discussion of forces on the foot in Chap. 3, where we will assume that all forces are parallel or antiparallel to each other and normal to a bar, as in a lever. Is this totally valid? Why or why not?

2.23. Redo Problem 2.21, now including the mass of the foot. If the distance from the bottom of the tibia (where the normal force emanates) is 4 in and the center of mass of the foot is halfway between it and the ground. Use the data for the mass of the foot in Chap. 1.

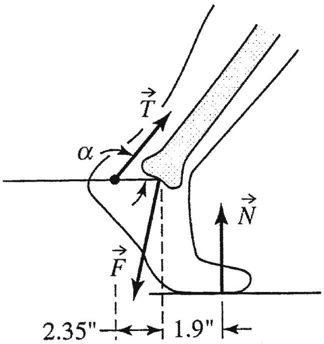

Fig. 2.54. Forces on the foot during crouching. (From [65].) For Problem 2.21

2.24. In the crouching position, the lower leg is held in equilibrium through the action of the patellar ligament, which is attached to the upper tibia and runs over the kneecap. As depicted in Fig. 2.55, the forces acting on the lower leg are **N**, **R**, and **T**. If the lower leg is in equilibrium, determine the magnitude of the tension **T** in the patellar ligament, and the direction and magnitude of **R**. Assume that the tension acts at a point directly below the point of action of **R**. Take the normal force equal to 100 lb (half the body weight), the weight of the leg W_{leg} as 20 lb, and the angle $\alpha = 40°$ (for the leg at a 45° angle). (From [65].)

2.25. Derive (2.37) and (2.38) for the equilibrium of the kneecap.

2.26. Derive (2.39) and (2.40) for the equilibrium of the foot.

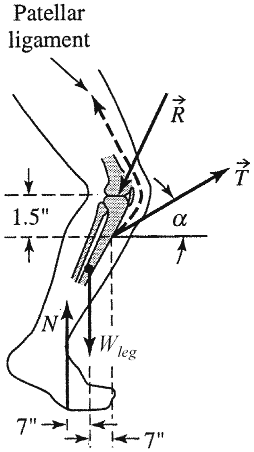

Fig. 2.55. Forces on the lower leg during crouching. (From [65].) For Problem 2.24

2.27. Laterally from the midline in either jaw, we have eight teeth: two incisors, one canine, two bicuspids (or premolars), and three molars (the last molar being a wisdom tooth). (This tooth order can be remembered from its acronym: ICBM – which itself is easy to remember because it is also the acronym for Intercontinental Ballistic Missiles.) The lateral distances from the temporomandibular joint to the insertion of the masseter muscles, the first bicuspids, and the central incisors, are $0.4L$, L, and $1.2L$, respectively.
(a) What type of lever is involved in biting and chewing with the masseter muscles?
(b) For biting in equilibrium with a masseter muscle force of 1,625 N, show that the force on the first bicuspid of 650 N, assuming there is no force on the central incisors [67]. Draw a force diagram for this.
(c) Under these conditions, show that the needed counter force on the central incisors is 540 N, now assuming no force on the first bicuspid. Draw a force diagram for this.

2.28. When you bite an apple with your incisors only, you exert a force of 650 N on it. When you bite an apple with your bicuspids only, you exert a force of 540 N on it. Find the force per unit area (which is called the stress) on the apple for both cases if the effective contact areas of the incisors and bicuspids are $5\,\text{mm}^2$ and $1\,\text{mm}^2$, respectively.

Statics of the Back

2.29. In analyzing bending, we assumed that the weight of the trunk (above the hips, excluding arms and head) is $W_1 = 0.4W_\text{b}$ and the weight of the arms and head is $W_2 = 0.2W_\text{b}$. Is this reasonable. Why?

2.30. Consider a woman of height 1.6 m and mass 50 kg.
(a) Calculate the reaction force on her lower vertebrae and the force in her erector spinae muscle when she is either upright or bent at 60° (and consequently 30° to the horizontal).
(b) Recalculate these forces when she is pregnant. Assume that during pregnancy the mass of her torso increases by 15 kg, but the center of mass of the torso is the same.
(c) The forces in part (b) are equivalent to those for the same nonpregnant woman who lifts a weight of what mass?

2.31. Describe the designs of the back of a chair that could lead to pain in the lumbar vertebrae and those that would give good lumbar support.

2.32. We showed that when the force on the lumbosacral (intervertebral) disc increases from 0 to 2,400 N, the disc height H decreases by 20% and the disc radius r increases by about 10%. When it is recognized that the load on the disc for a vertical person is really 530 N (and not 0), how do the disc dimensions really change when the person bends to an angle of 30° (and then to a load of 550 lb)?

2.33. *Why is it more difficult to lift bulky objects?* A person lifts a package of mass 20 kg in front of her so the back of the package touches her abdomen. The horizontal distance from the person's lumbar-sacral disc to the front of her abdomen in 20 cm. Calculate the bending moments (in N-m) about the center of mass of her disc caused by the lifted loads, assuming the package is alternatively 20 or 40 cm deep [83]. Draw force diagrams for these two cases. The other dimensions of the packages are the same and they both have uniform density. How does this show that the size of the lifted object affects the load on the lumbar spine?

2.34. *Why is it better to standard erect when you hold an object?* A person holds a 20 kg object while either standing erect or bending over. The mass of the person above his lumbar-sacral disc (his torso) is 45 kg. When upright, the center of mass of the torso is (horizontally) 2 cm in front of his disc and that of the object is 30 cm in front of his disc. When bent, the center of mass of the torso is 25 cm in front of his disc and that of the object is 40 cm in front of his disc. Draw force diagrams for these two cases. Calculate the bending moments (in N-m) about the center of mass of his disc caused by holding this load while either being upright or bent over [83]. How does this show that bending when lifting an object affects the load on the lumbar spine?

2.35. *Why is it best to lift an object with bent legs and the object very close to you?* A person lifts a 20 kg object while either bending over with legs straight, with bent knees and the object near to her body, or with bent knees and the object far from her body. The mass of the person above her lumbar-sacral disc (her torso) is 45 kg. When bent over with straight legs, the center of mass of her torso is (horizontally) 25 cm in front of her disc and that of the object is 40 cm in front of her disc. When bent over with bent knees and the object near her body, the center of mass of her torso is (horizontally) 18 cm in front of her disc and that of the object is 35 cm in front of her disc. When bent over with bent knees and the object far from her body, the center of mass of her torso is (horizontally) 25 cm in front of her disc and that of the object is 50 cm in front of her disc. Draw force diagrams for these three cases. Calculate the bending moments (in N-m) about the center of mass of her disc for these three lifting methods [83]. Which position is the worst? How does this show that the position when lifting an object affects the load on the lumbar spine?

2.36. One position during shoveling snow or soil is shown in Fig. 2.56. Assume this is an equilibrium position.
(a) If the shovel and contents have a mass of 10 kg, with a center of mass 1 m from the lumbar vertebra, find the moment about that vertebra.
(b) If the back muscles are 5 cm behind the center of the disc, find the magnitude and direction of the muscle force needed for equilibrium.

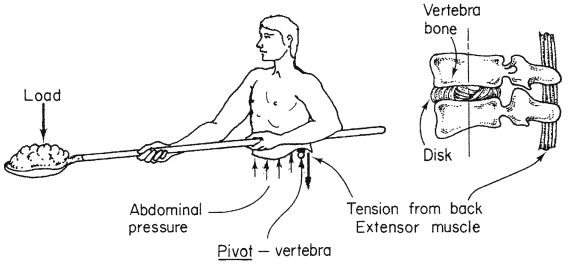

Fig. 2.56. Shoveling. (From [71].) For Problem 2.36

(c) Find the force on the intervertebral disc.
(d) If the abdominal muscles of the person are strong and can provide some upward force, would that help relieve stress to the back muscles and the disc? Why?

Multisegment Modeling

2.37. (a) Sketch an (in-plane) multisegment model of the leg showing the forces on the upper leg, lower leg, and ankle – using the resultant forces in Fig. 2.57. Show the center of mass gravity forces on each segment, along with the normal forces (at each body joint and with the floor).
(b) Label all distances and angles needed to analyze the in-plane forces and torques. For each segment, label the distances starting from the proximal end. Label each angle between muscle and bone; use the angles as shown, with the acute angle when possible.
(c) Write the equilibrium force balance and torque equation for each of the segments.

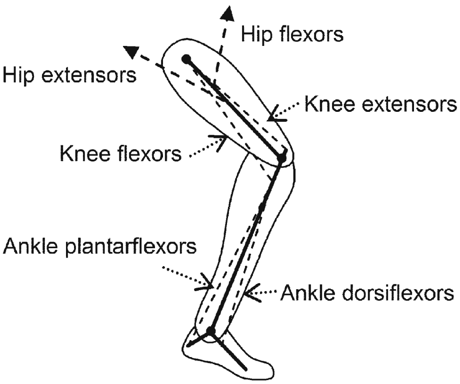

Fig. 2.57. Idealized resultant forces for six muscle groups in the leg. (Based on [77].) For Problem 2.37

(d) For each of these equilibrium conditions, sum the equations – such as the torque equations – for the upper leg, lower leg, and foot. Show that the three resulting balance equations are the correct equations for the entire leg.

Sense of Touch

2.38. (a) A 50 kg person stands on her fingertips. Assuming each finger makes a $1\,\mathrm{cm}^2$ contact area with the ground, find the pressure on each finger tip.
(b) Which tactile sensors are sensing this pressure?