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The simple static model of labor supply

"Economic agents may be taken to reach their decisions in the light of what they want and what they can get" (Arrow and Hahn, 1971, p. 22). Thus, in neoclassical models, labor supply decisions – like consumption decisions, and for that matter all other decisions – are the result of utility maximization ("what agents want") subject to constraints ("what they can get").[1]

In the simplest version of the static labor supply model, the individual's utility or well-being depends on his tastes and on the amount of market (i.e., consumer) goods C and hours of leisure time L that he consumes per period.[2] In maximizing utility, the

[1] As Abbott and Ashenfelter (1976) have stressed, it is natural to analyze labor supply decisions along with consumption decisions, treating leisure time (the complement of labor supply) as one among many consumer goods. For surveys of theoretical and empirical work on consumer behavior as such, see Barten (1977), Brown and Deaton (1972), Deaton and Muellbauer (1981), Goldberger (1967), H. A. J. Green (1971), Houthakker (1961), Katzner (1970), Phlips (1974), and Theil (1975, 1976). Several labor economics texts, such as Ehrenberg and Smith (1982) and Fleisher and Kniesner (1980), discuss the simple labor supply model at length; Abbott and Ashenfelter (1976) and Gilbert and Pfouts (1958) provide mathematical treatments that complement the intuitive account given here.

[2] A few definitional and measurement issues are worth mentioning here. First, "well-being" and "utility" are usually taken to be equivalent, even though, strictly speaking, utility and indifference curves are concerned with desires or preferences rather than with the actual satisfaction of desires. Second, the C of this model refers to consumption of the services of goods rather than to actual expenditures or outlays. Thus, for example, it refers to the stream of services that an individual derives per period from an auto (or other durable good) rather than to the money spent purchasing such

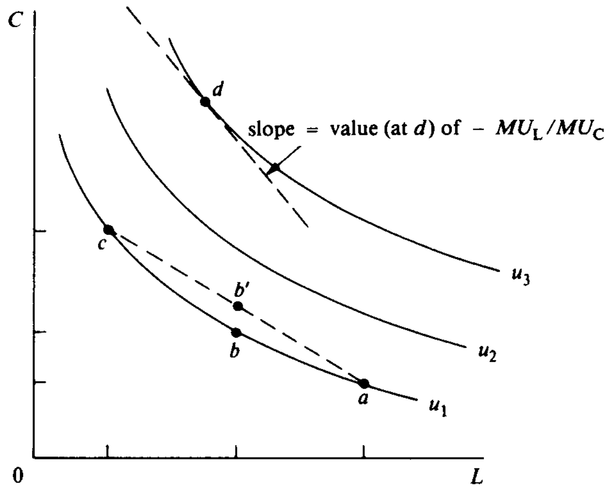

Figure 1.1. Indifference curves

individual faces several constraints. First, the price of a unit of C is P, and the "price" of an hour of L is a fixed amount W, the wage per hour: That is, the individual must forgo or sacrifice W when he devotes an hour to leisure rather than to work. Second, the total amount of time available to the individual per period is fixed at T hours and may be allocated to work hours H and leisure hours L. Finally, in the absence of borrowing, saving, transfer payments to the individual, or tax payments by the individual, spending on market goods PC must equal total income from work WH and other income V derived from sources that are unrelated to work, such as property. In effect, then, the model assumes that the individual acts as if he had neither a past nor a future and were concerned only with the present.

a good. Moreover, an individual may do some consuming at or through the workplace (e.g., drinking "free" coffee, using company recreational facilities, and the like), without paying directly for such C. Thus, some of the C of the model may not appear in any conventional "budget" data for the individual. Third, the L of the model refers to time not spent actually working, and therefore, in principle, includes time spent *at* work that is not devoted *to* work (e.g., time spent on coffee breaks or making personal telephone calls). All this may pose problems for empirical studies, since the C to which the model refers is not necessarily measured by the individual's own outlays for consumption goods, and the L of the model is not necessarily measured by the time the individual spends away from work.

This process of maximization is shown graphically in Figures 1.1–1.3. The individual's tastes or preferences are represented by indifference curves, as depicted in Figure 1.1. Each of these shows different combinations of C and L that give the individual the same level of satisfaction or utility u, where u is given by a utility function $u = u(C, L)$. At any given point – that is, at any given (C, L) combination – the slope of an indifference curve is equal to $-MU_L/MU_C$, the negative of the ratio of the marginal utility of leisure to the marginal utility of consumer goods, at that point.[3] The ratio itself is called the *marginal rate of substitution of consumer goods for leisure*. The individual is assumed always to prefer "more" to "less," so indifference curves that lie farther from the origin entail more utility. Finally, indifference curves are assumed to be convex (that is, bowed away from the origin). This means that if the individual lost successive equal amounts of L (e.g., went from point a to point b and then from point b to point c), then he would require successively larger amounts of C in order to remain at the same level of utility. It also means that the individual prefers any *average* of a "desirable" and a "less desirable" combination of C and L to the "less desirable" combination itself.[4] (For more on

[3] That is, when C and L change by amounts dC and dL, respectively, then, since $u = u(C, L)$, the resulting change in utility, du, may be written as $du = (\partial u/\partial C)dC + (\partial u/\partial L)dL$, where $\partial u/\partial C$ is the rate of change of utility with respect to a change in C (the marginal utility of consumption) and $\partial u/\partial L$ is the rate of change of utility with respect to a change in L (the marginal utility of leisure). Along an indifference curve, utility is constant and so $du = (\partial u/\partial C)dC + (\partial u/\partial L)dL = 0$ as one moves along an indifference curve by changing C and L. Rearrange this expression to obtain the slope of the indifference curve, dC/dL, as $dC/dL = -(\partial u/\partial L)/(\partial u/\partial C) = -MU_L/MU_C$. See Dunn (1978, 1979), MacCrimmon and Toda (1969), Mosteller and Nogee (1951), Rousseas and Hart (1951), Thurstone (1931), and Wallis and Friedman (1942) for discussion of attempts to derive empirical indifference curves using experimental or interview data. It is interesting to note that laboratory experiments appear to suggest that even (?) nonhumans – rats, pigeons, and so forth – seem to have "indifference curves"; for one recent study, see Battalio, Green, and Kagel (1981).

[4] In terms of Figure 1.1, this means that points lying on the dashed line $ab'c$ in between points a and c on indifference curve u_1 are regarded as better than any of the points below them. Note that the points on this dashed line are averages of the (C, L) combinations at point a and at point c: The points on the dashed line that are closer to a have a greater proportion of the a-bundle of C and L, whereas points that are closer to c have a greater proportion of the c-bundle.

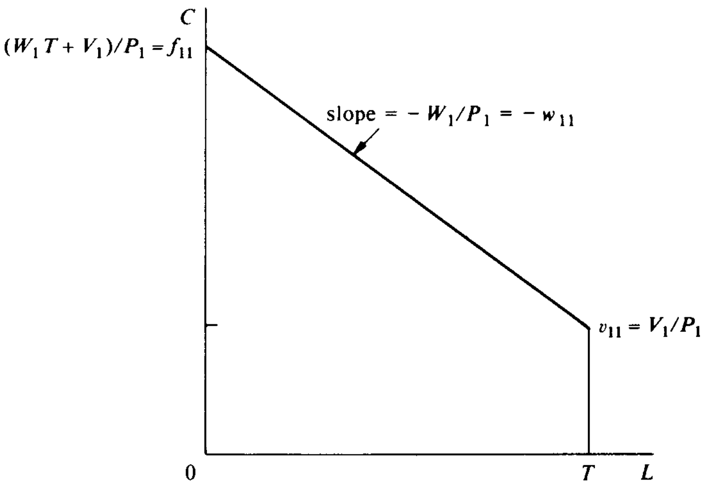

Figure 1.2. Budget line

indifference curves, see H. A. J. Green, 1971, or Henderson and Quandt, 1971.)

The constraints facing the individual are summarized by a budget line, as shown in Figure 1.2. For example, suppose that the individual receives property income of V_1, gets a wage of W_1 for each hour of work, and faces a price level of P_1. If he does not work at all and devotes all available time T to leisure, then he can consume $v_{11} = V_1/P_1$ in consumer goods. If he devotes all available time to work and takes no leisure, then he can earn $W_1 T$ by working and will enjoy a total income of $V_1 + W_1 T$ and hence consume $f_{11} = (V_1 + W_1 T)/P_1$ in consumer goods. Finally, the (C, L) combinations the individual can "purchase" if he divides his time between leisure and work are given by the straight line drawn between v_{11} and f_{11} in Figure 1.2, because in this simple analysis the wage is assumed to be independent of hours of work. (Thus, the slope of the line between v_{11} and f_{11}, equal to $-W_1/P_1 = w_{11}$, where w denotes the real wage, is constant.) In sum, the individual's budget line is $Tv_{11}f_{11}$. Points that lie beyond it are unattainable because – relative to his property income V and the price level P – the individual's wage is too low to permit him to purchase any (C, L) combination that lies beyond it. On the other hand, points lying between the origin and the budget line are all feasible: Given the values of V, P, and W facing him, the individual can purchase any such combination in that region.

In formal terms, the individual's problem is to maximize utility, which is a function of C and L, that is,

(1.1) $$u = u(C, L)$$

by choosing C and L values that give him the highest value of u that is consistent with the budget constraint

(1.2) $$PC = WH + V$$

where total available time per period, T, may be allocated between leisure and work, that is,

(1.3) $$H + L = T$$

Note that (1.2) may be rewritten as an equation for C, that is,

$$C = (W/P)H + (V/P)$$

which indicates that (i) the slope of the line $v_{11}f_{11}$ in Figure 1.2 is indeed W/P, the ''real wage''; (ii) the amount of C the individual can enjoy when $H = 0$ is indeed $V/P = v_{11}$, ''real property income''; and (iii) the amount of C the individual can enjoy when $H = T$ (so that all available time is devoted to work) is indeed $(W/P)T + (V/P) = f_{11}$. Now, f_{11} is sometimes called *full income* (Becker, 1965); it represents maximum attainable real income (since it is the greatest possible amount of the consumer good the individual can have). In particular, note that one can insert (1.3) into (1.2) and rearrange terms to obtain

$$WT + V = WL + PC$$

The left-hand side of this expression is ''full income,'' expressed in nominal terms (e.g., dollars). The first term on the right-hand side is ''expenditure'' on leisure (valued at the wage rate, i.e., at the ''cost'' of leisure), whereas the second is expenditure on the consumer good. Following Becker (1965), one may therefore say that the individual ''spends'' his ''full income'' $V + WT$ – that is, the maximum income attainable when all time is devoted to work – on leisure and on consumer goods so as to maximize utility, where WL and PC represent his ''expenditures'' on leisure and consumer goods, respectively.

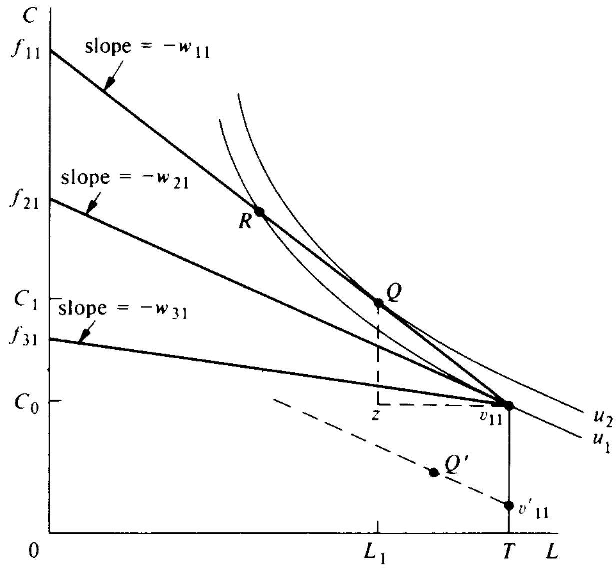

Figure 1.3. Determination of labor supply

The *optimal* (C, L) combination is the one lying on the highest possible indifference curve that is consistent with the requirement that the individual remain on or below the budget line $Tv_{11}f_{11}$. In terms of Figure 1.3, when the wage is W_1, the price level is P_1, and property income is V_1, the optimal point is Q. At Q, the individual consumes $0C_1$ in consumer goods and $0L_1$ in leisure, works $0T - 0L_1 = L_1T = H_1$ hours, and reaches a level of satisfaction given by indifference curve u_2. At Q, the constraints are just satisfied, and the individual gets the greatest possible utility consistent with those constraints. Q just touches the budget line $Tv_{11}f_{11}$; points on any higher indifference curve lie entirely above the budget line (and so are not feasible); and whereas some points on lower indifference curves are feasible, they do not yield as much satisfaction as Q. All available time has been allocated between leisure and labor, so that $0T = 0L_1 + L_1T$; total expenditure on consumption is exactly equal to total income from work and property, so that $0C_1 = w_{11}L_1T + Tv_{11}$.[5]

[5] Note from Figure 1.3 that w_{11} is the absolute value of the slope of the line from Q to v_{11}; $L_1T = zv_{11}$; and $Tv_{11} = 0C_0$. So consumption at Q, $0C_1 = 0C_0 + C_0C_1$, is in turn equal to $Tv_{11} + zQ = Tv_{11} + w_{11}zv_{11}$ = property income + labor income.

Now, at Q, the indifference curve is tangent to the budget line, so at this point the slopes of the budget line and the indifference curve u_2 are equal, that is, $MU_L/MU_C = W_1/P_1 = w_{11}$. In other words, at the optimum, $MU_L/W_1 = MU_C/P_1$. So at Q, the utility that would be gained (lost) from spending one more (less) dollar on C and one less (more) dollar on L would be just offset by the utility that would be lost (gained) by the reduction (increase) in L. Any further reallocation of time from leisure to work, or vice versa, would therefore be pointless. Moreover, if the individual were located at some other point on the budget line, such as R in Figure 1.3, then he would always desire to get back to Q. For example, at R, the individual would have utility of only u_1. Because the slope of the indifference curve u_1 at R is greater than the slope of the budget line $Tv_{11}f_{11}$ at R, the individual would have $MU_L/W_1 > MU_C/P_1$ there. Hence, he could raise his utility by taking a dollar away from consumption (that is, work less and therefore earn a dollar less) and devoting it instead to leisure: Moving from R to Q would be an improvement.

Now, the optimum at Q is called an *interior solution* – one in which C, L, and H are all positive. At lower values of W or higher values of V, a *corner solution*, with $H = 0$, may be optimal. For example, in terms of Figure 1.3, suppose that the wage falls to W_2 (that is, to $w_{21} = W_2/P_1$ in real terms). Then the budget line becomes $Tv_{11}f_{21}$ and the optimal solution is now the point v_{11}, with $MU_L/W_2 = MU_C/P_1$, $L = T$, and $H = 0$. If the wage fell still further – say, to W_3 (that is, to $w_{31} = W_3/P_1$ in real terms) – then the slope of the budget line would be less than the slope (at point v_{11}) of the indifference curve u_1. This is the highest possible curve consistent with the new budget constraint $Tv_{11}f_{31}$, in which the wage is W_3. At this wage, the optimal point is still v_{11} – but note that now $MU_L/W_3 > MU_C/P_1$ at the optimum.

This has a simple and natural economic interpretation: Leisure is so "cheap" (W is so low) relative to the price level P and the individual's property income V and the utility "lost" by transferring time from leisure to work (and hence into market goods C) is so large in relation to what would be gained from work (in the form of additional earnings and hence additional C) that the individual devotes all available time to leisure and none to work. Thus, in the simple model, the "value" of the time of people who do work is given, at the margin, by their wage rate W; but the marginal value of time to nonworkers exceeds the wage they could earn. Note also from Figure 1.3 that if the individual had v_{11} in property income and faced a real wage of w_{21}, then he would face a budget line of $Tv_{11}f_{21}$ and would be indifferent between working and not working. Hence, when his property income is $v_{11} = V_1/P_1$ in real terms, w_{21} is his "reservation wage" in real terms: In other words, w_{21} is the highest wage at which the individual will not work. Thus, when the wage is below the reservation level, *changes* in the wage will not change behavior. (For example, when the real wage changes from w_{31} to w_{21}, labor supply remains at zero.) Of course, if V changes, then the reservation wage will change. For example, other things being equal, people with less property income cannot "afford" to be choosy about working and will have to be prepared to work for a lower wage.[6]

Finally, note that the individual decides *simultaneously* not only how many hours to work but also whether to participate in the labor force. This is because, in the world of perfect certainty and perfect information portrayed by the simple model, the participation decision and the hours-of-work decision are really one and the same.[7]

[6] In other words, the reservation wage is the slope of the line that is tangent at point v_{11} to the indifference curve that just touches the top of the line Tv_{11}; and as v_{11} (= property income in real terms) falls, so does the reservation wage. To see this, assume that C and L are normal goods and that the individual currently faces budget line $Tv_{11}f_{21}$ (and, therefore, is indifferent between working and not working). Now decrease his property income from v_{11} to some lower level v'_{11}. This will shift his budget line to some lower level; a portion of this new budget line appears in Figure 1.3 as the dashed line drawn between v'_{11} and Q'. Because C and L are both normal goods, the individual will react to the drop in property income by consuming less of each and so will move from his old equilibrium at v_{11} to a new point such as Q' in Figure 1.3. At this new equilibrium, the relevant indifference curve will be tangent to the new budget line $Q'v'_{11}$, and the new budget line has the same slope as the old one. In other words, even though he was formerly indifferent about working at the wage w_{21}, he will now work at this wage because his property income has dropped – which means that his reservation wage must have gone down. Consequently, as property income rises or falls, so does the reservation wage, other things being equal.

[7] In a world of imperfect information and risk, however, the individual will first have to look for a job (find a wage offer) before he can decide how many hours to work. In this case, unless he is lucky enough to land an offer immediately, the decision to participate in the job market and the decision about hours of work are separate not only in logic but also in time. For more on such issues, see Burdett and Mortensen (1978), Lippman and McCall (1976a, b), and Pissarides (1976).

For example, if the real wage rose from w_{21} to w_{11}, then the individual would decide both to participate and to supply H_1 hours of work.

The effects of changes in variables such as W, V, or P on the variables L, H, and C are usually discussed in terms of income and substitution effects. First, consider the effect of a wage increase on L. A wage increase makes it possible to earn more income and get greater satisfaction or utility at any given relative price ratio $w = W/P$; it will therefore have an income effect on leisure. By definition, if leisure is a ''normal'' good, then the income effect of the wage increase will cause leisure time to rise (and labor supply to fall): The wage increase makes the individual better off, so he can ''afford'' to consume more leisure time. However, a wage increase also makes an hour of leisure more ''expensive'' – that is, makes an hour of work more remunerative – relative to market goods at any level of income or satisfaction. Thus, a wage increase will also have a negative substitution effect on the consumption of leisure time (and a positive substitution effect on labor supply): the wage increase makes work more attractive, so the individual consumes less leisure time and works longer, thus substituting consumption goods (purchased out of earnings) for leisure.

If leisure is a normal good, as seems likely, then the income and substitution effects on leisure time (and therefore on labor supply) pull the individual in opposite directions. The net effect of this tug-of-war will depend on whether the positive substitution effect on labor supply outweighs the negative income effect, or vice versa. This is shown in Figure 1.4, which shows two different individuals, A and B. These two individuals face the same budget line before the wage increase ($Tv_{11}f_{11}$) and after ($Tv_{11}f_{21}$) but have different tastes: A's indifference curves are u_{A1} and u_{A2} whereas B's are u_{B1} and u_{B2}. Before the wage increase, A and B are at points Q_{A1} and Q_{B1}, respectively; after the increase, they move to Q_{A2} and Q_{B2}, respectively. Hence the wage increase caused A to decrease his labor supply (from $L_{A1}T$ to $L_{A2}T$) but caused B to increase her labor supply (from $L_{B1}T$ to $L_{B2}T$). This is because A's income effect was larger than his substitution effect, whereas for B the reverse was true.

To measure A's income and substitution effects, draw a line AA' that is parallel to the original budget line ($v_{11}f_{11}$) and tangent to the indifference curve u_{A2} at which A reaches his new equilibrium. Let

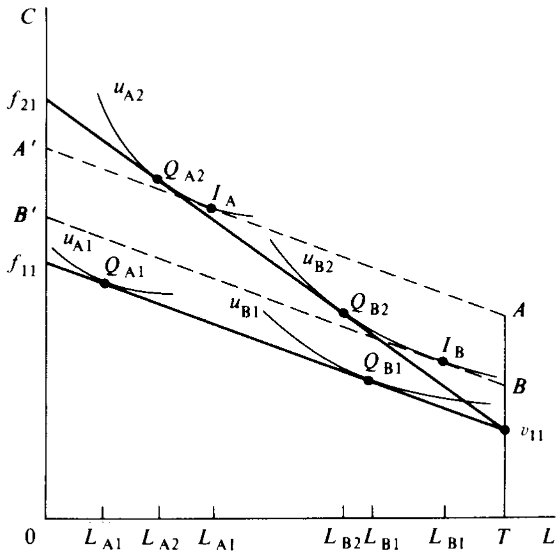

Figure 1.4. Income and substitution effects

the point at which AA' and u_{A2} are tangent be I_A. When the budget line shifts from $v_{11}f_{11}$ to AA', individual A's utility is increased by just as much as when the wage rose, but the relative price ratio $w = W/P$ stays at what it was before the wage increase. Hence, such a shift portrays the income effect (that is, the effect of increasing satisfaction by the amount attributable to the rise in the wage but with no change in relative prices w). Evidently, the income effect of the wage increase on A is the change from Q_{A1} to I_A and led to a rise in A's leisure time from L_{A1} to L_{AI} (see Figure 1.4).

The substitution effect of the wage increase on A may be portrayed as a shift in the budget line from AA' to $v_{11}f_{21}$: This shift keeps utility constant but changes relative prices from their old level (given by the slope of AA', equal to w_{11}) to their new level (given by the slope of $v_{11}f_{21}$, equal to w_{21}). Evidently, the substitution effect on A is the change from I_A to Q_{A2} and led to a reduction in A's leisure time from L_{AI} to L_{A2} (see Figure 1.4).

On balance, the negative substitution effect on A's leisure time (from L_{AI} to L_{A2}) was smaller than the positive income effect (from