

# Chapter 1

## Turbulence, heat and waves

### 1.1 Introduction

Turbulence is the dominant physical process in the transfer of momentum and heat, and in dispersing solutes and small organic or inorganic particles, in the lakes, reservoirs, seas, oceans and fluid mantles of this and other planets. Oceanic turbulence has properties that are shared by turbulence in other naturally occurring fluids and in flows generated in civil, hydraulic and chemical engineering installations and in buildings. The study of turbulence consequently has applications well beyond the particular examples in the ocean<sup>1</sup> that are selected for description below.

Figure 1.1 shows the sea surface in a wind of about  $26\text{ m s}^{-1}$ . It is covered by waves, many of them breaking and injecting their momentum and bubbles of air from the overlying atmosphere into the underlying seawater. Immediately below the surface, and even at great depths, the water is generally in the state of irregular and variable motion that is referred to as ‘turbulence’, although there is no simple and unambiguous definition of the term. Turbulence has, however, characteristics that, as will be explained, can be quantified and which make it of vital importance. Many of the figures in this book illustrate the nature of turbulent motion, the processes that drive turbulence, or the measurements that can be made to determine its effects.

●<sup>2</sup> Turbulence is generally accepted to be an energetic, rotational and eddying state of motion that results in the dispersion of material and the transfer of momentum, heat and solutes at rates far higher than those of molecular processes alone. It disperses,

1 By the ‘ocean’ is meant, here and later, the sum of the major oceans and their connected seas, including the continental shelf seas and those seas, such as the Mediterranean, Black Sea and Baltic, connected by straits to the larger ocean basins.  
2 The symbol ● is used to draw attention to paragraphs of particularly important information or summaries of the earlier text.



**Figure 1.1.** The sea surface in the Bay of Biscay looking upwind in a wind speed of about  $26 \text{ m s}^{-1}$ . Breaking is occurring at the crests of the larger waves, separated by over 100 m, and there are numerous bands of foam aligned in the downwind direction, as well as evidence of short waves with a typical wavelength of 0.2 m. (Photograph taken by Mr J. Bryan from RRS Charles Darwin off northwest Spain and reproduced with his kind permission.)

stresses and strains clusters (or flocs) of sediment or atmospheric dust particles and living organisms within the ocean, and it stirs, spreads and dilutes the chemicals that are dissolved in the seawater or released into the ocean from natural and anthropogenic sources. Perhaps its most important property, and one that is generally used to characterize it, is that by generating relatively large gradients of velocity at small scales, typically 1 mm to 1 cm, turbulence promotes conditions in which, relatively rapidly, viscous dissipation transfers the kinetic energy of turbulent motion into heat, a process of energy transfer and ‘dissipation’.

Since the natural state of the ocean is one of turbulent motion, knowledge of turbulence and its effects is crucial in understanding how the ocean works and in the construction of numerical models to predict how, in the future, the ocean will adjust as the forcing by the atmosphere is altered by changes in the world’s climate. Although estimates of the rate of dissipation of the energy of the tides through turbulence in shallow seas were made as early as 1919, direct measurement of turbulence in the ocean dates back only to the observations of near-bed turbulent stress made in the 1950s and to studies of the spectra of small-scale motions in the tidal Discovery Passage off the west coast of Canada in the early 1960s. In spite of the developments of ingenious

techniques to measure turbulent motions, the geographical variation of turbulence in the ocean is still poorly known, its range of variability is often grossly under-sampled and, in comparison with the atmosphere, there are few sets of data against which to test those models of the ocean that include representation of its turbulent nature. Much is still to be discovered and quantified.

This chapter describes some of the ideas and discoveries that form a basis for understanding of the part played by turbulence in the ocean. Much of this background is derived from studies of turbulence and heat transfer in laboratory experiments, some of which were made well before the first measurements of turbulence in the ocean itself.

1.2 Reynolds’ experiment

The scientific study of turbulence did not begin until late in the nineteenth century. The first substantial step was the publication in 1883 of a paper by Osborne Reynolds. He described how a smooth flow of water through long circular tubes with diameters ranging from about 0.6 to 2.5 cm is disrupted and cannot be sustained when the mean speed of the flow,  $U$ , exceeds a value that is related to the tube diameter,  $d$ , and to the viscosity of water. In his laboratory experiments Reynolds introduced a thin line of dye into the water entering through one end of a horizontal tube from a large tank of stationary water, dye that made the flow visible (Fig. 1.2). He described his observations as follows:

*When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube*

but later

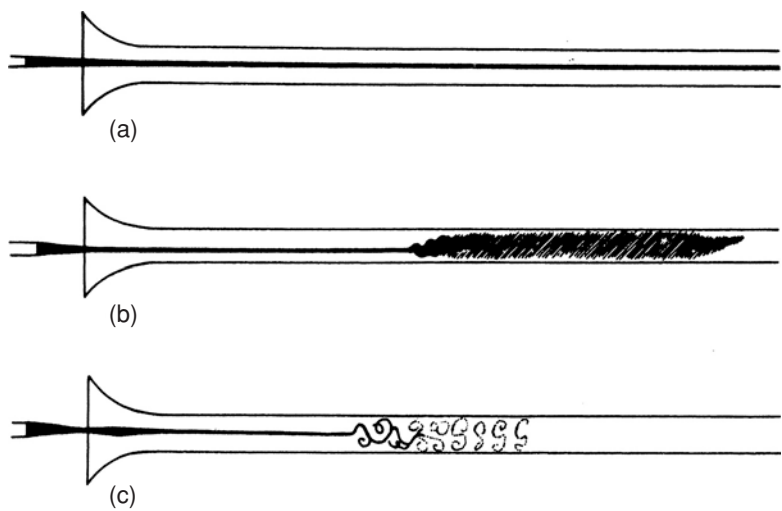
*As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from . . . the intake, the colour band would all at once mix up with the surrounding water, and fill the tube with a mass of coloured water. On viewing the tube by light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies.*

- Reynolds’ remarkable experiments show that the ‘laminar flow’, the smooth flow through the tube at low flow speeds, breaks down into a random eddying ‘turbulent’ motion at higher speeds when a non-dimensional parameter, now known as the Reynolds number,

$$Re = Ud/\nu, \tag{1.1}$$

exceeds a value of about  $1.3 \times 10^4$ .<sup>3</sup> Here  $\nu$  is the kinematic viscosity, which for water has a value of about  $10^{-6} \text{ m}^2 \text{ s}^{-1}$ .

<sup>3</sup> The critical value of  $Re$  is now known to depend on the level of the background disturbances to the flow (sometimes described as ‘noise’), particularly near the entry of flow to the tube, the critical  $Re$  consequently ranging from about  $1 \times 10^3$  for relatively substantial disturbances to about  $4.5 \times 10^4$  in very carefully controlled, low-disturbance, tube flows. (See Further study.)



**Figure 1.2.** Reynolds’ experiment, described in his paper published in 1883. Flow through the tube is from left to right. The shape of the entry to the tube within a large tank of still water on the left is to ensure a smooth flow. (a) A band of dye passes down the tube when the flow is relatively slow, or at a low Reynolds number,  $Re$ . (b) When  $Re > 1.3 \times 10^4$ , the flow becomes turbulent. As observed by eye, the band of dye is dispersed across the width of the tube. (c) An image obtained with a very brief electric spark, showing that the onset of turbulence, and its later form, is associated with eddies of size comparable to the tube diameter.

This provides the first example of a relationship pertaining to turbulence that can be determined on dimensional grounds. A condition for a transition to turbulence can depend only on the independent dimensional quantities that characterize or determine the state of the flow in the tube. These are its mean velocity,  $U$  (with dimensions  $LT^{-1}$ ), its density,  $\rho$  (dimensions  $ML^{-3}$ ), the diameter of the tube,  $d$  (dimension  $L$ ), and the kinematic viscosity,  $\nu$  (dimensions  $L^2T^{-1}$ ), where  $L$  stands for length,  $T$  for time and  $M$  for mass. The velocity varies with distance,  $r$ , from the axis of the tube but in a way determined by  $d$  and  $\nu$ , and the pressure gradient along the tube is directly related to  $U$ ,  $\nu$  and  $\rho$ , and so is not a quantity independent of the four chosen. The tube walls are smooth and so do not introduce a further length scale. If also the tube is ‘very long’, or of very much greater length than either of the two possible length scales,  $d$  and  $\nu/U$ , and so does not introduce a further relevant length scale, the only non-dimensional parameter which can characterize whether the flow may become turbulent is  $Ud/\nu$ , the Reynolds number,  $Re$ . There is no other parameter possible. Although the value of  $Re$  at which a transition from laminar to turbulent flow occurs cannot be determined from the dimensional argument, it serves to identify in a logical way the combination of dimensional quantities that characterize the onset of turbulence. The power of dimensional arguments in characterizing the nature of turbulent flows is demonstrated later by other examples. (For easy reference and comparison they are listed together under ‘dimensional arguments’ in the index.)

Some of the eddies in the turbulent flow in Reynolds’ experiment are of size comparable to the tube radius, but many are smaller, smaller therefore than the distance (about the radius of the tube) over which the mean flow itself varies. As well as dispersing dye, the eddies carry fluid and momentum from the tube walls, where the presence of the boundaries and associated viscosity effects are important constraints, into the interior of the tube.

Reynolds’ experiment underpins many of the concepts that we now have of turbulence. It shows that turbulence may occur as a *transition* from one state of flow to another: even the way in which the mean (time-averaged) flow speed varies with radius in the tube is changed at the onset of turbulence. Turbulence involves eddying motions, some of which are small relative to the characteristic length scale of the flow, namely the tube diameter,  $d$ , in Reynolds’ experiments. It disperses dissolved matter in an irreversible way – the mixed dye cannot be unmixed [P1.1].

- An irregular state of fluid motion, referred to as turbulence, occurs when a *critical* value of the parameter  $Re$ , characterizing the flow, is exceeded, replacing the smooth laminar flow found at lower values of  $Re$ .

The precise value of  $Re$  at which turbulence sets in depends on the particular geometry of the flow and the nature of disturbances to which it is subjected. Geophysical flows in which a value of  $Re$  characterizing the flow exceeds about  $10^4$  are generally found to be turbulent unless affected by other factors (e.g., density stratification, described in Section 1.7) that suppress or delay the onset of turbulence until higher values of  $Re$  are reached. In the ocean, the values of the speed  $U$  and length  $d$  that appear in  $Re$  (1.1) are usually taken to be those characterizing the flow, for example the mean speed of the local flow and water depth (or, in mid-water, a change in mean speed over a vertical distance,  $d$ ). The characteristic  $Re$  commonly exceeds  $10^4$  in the ocean [P1.2].

1.3 Joule’s experiment

Heat is a form of energy contained at a molecular level within a fluid, and temperature is a measure of heat content. Heat is transferred (as a heat flux) by turbulent motion in the ocean, and turbulent energy is largely dissipated by viscosity into heat. The density of seawater depends on temperature (and usually to a lesser extent on salinity and pressure – see Section 1.7.1), and the variation of density with depth in the ocean, normally an increase in density as depth increases, strongly affects or regulates the processes leading to turbulence, as we shall see later. Heat and temperature<sup>4</sup> are consequently important factors in oceanic turbulence.<sup>5</sup>

4 The temperature is measured in degrees Kelvin, K, or °C. Both units are used, selection being made on the basis of which appears most appropriate.  
5 Although variations in heat content or a flux of heat can, through the production of buoyancy forces (Section 1.7.2), lead to convective motions and therefore kinetic energy, the heat energy at molecular level is not available for transfer back to kinetic energy. This is why, although the geothermal energy flux into the ocean may exceed, for example, the flux of tidal energy, it is not as effective in producing mixing, as is explained further in Section 6.9.

- The relation between heat and temperature can be expressed in terms of a change,  $\Delta H$ , in the heat per unit mass (measured in  $\text{J kg}^{-1}$ ) over a period of time and the corresponding change,  $\Delta T$ , in temperature (in  $^{\circ}\text{C}$  or  $\text{K}$ ) by

$$\Delta H = c_p \Delta T, \tag{1.2}$$

where  $c_p$  is the specific heat at constant pressure which, for seawater, has a value now known to be about  $3.99 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ .<sup>6</sup>

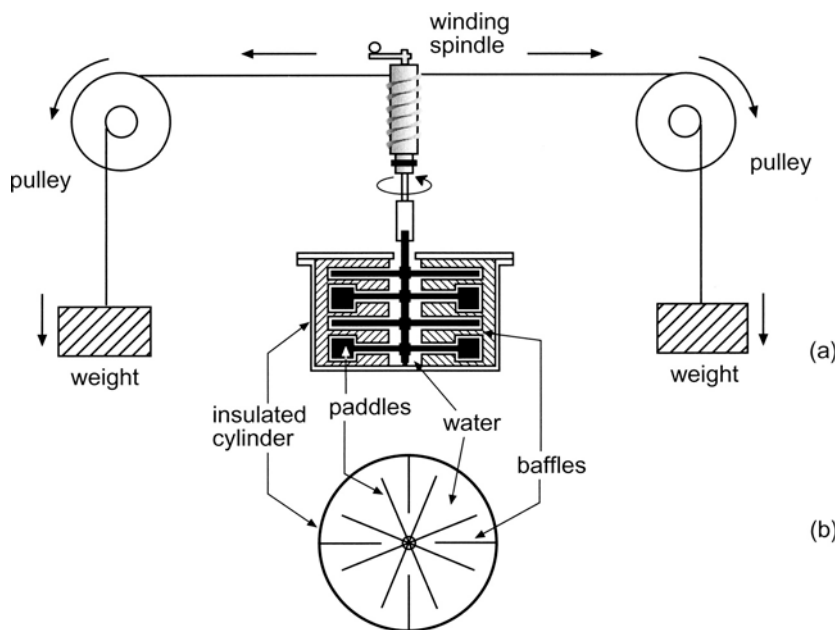
The unit in which energy is measured is named after J. P. Joule.<sup>7</sup> Joule’s most celebrated experiment, an account of which was published in 1850, is that in which he found the relation between changes of energy in its most commonly known, well-described and easily quantified forms (potential or kinetic energy) and the temperature change resulting from the dissipation of that energy. The experiment contains at its heart the substantial effects of turbulent motion, in this case turbulence artificially induced by rotating paddles.

Joule’s apparatus is sketched in Fig. 1.3 and described in the caption. The essence of the experiment is that the falling weights lose a measured amount of potential energy in driving paddles, which churn a fluid (Joule used water and mercury) in the cylinder, leading to its heating. The lost potential energy can be related to gains in two different forms of energy, that of heat and kinetic energy. In the experiment the weights descended through a distance of 1.6 m, reaching a speed of about  $6.1 \text{ cm s}^{-1}$ . They were repeatedly lifted and, over a period of some 35 min in which the weights descended 20 times, the temperature of the water in the cylinder increased by about  $0.31 \text{ }^{\circ}\text{C}$ . This temperature change was carefully measured, the accuracy attained being about 3 mK. (Temperature is now routinely measured at sea to an accuracy of 1 mK and often, with specially designed equipment, with a resolution of 0.1 mK or better, e.g., in studies of mixing in boundary layers where the temperature is relatively uniform.) Joule took great care to minimize heat loss during the period of the experiment by insulating the cylinder, and a wooden screen was erected to avoid effects of radiant heat from the observer. Joule calculated the total potential energy lost by the weights in descending, and, by subtracting their kinetic energy at the end of their descent and accounting for a small unavoidable heat loss to and from the cylinder during the experiment, was able to relate the mechanical energy imparted to the fluid per unit volume through the

6 The specific heat varies with temperature, salinity and pressure. For more precise values of  $c_p$  see Gill (1982, Section A3.4 and Table A3.7). Note that the heat per unit volume is  $\rho \Delta H = \rho c_p \Delta T$ , where  $\rho$  is the density. Equation (1.2) is given incorrectly ( $\rho$  should not be included) in TTO (Thorpe, 2005).

7 James Prescott Joule (1818–1889) was given private lessons in chemistry in his home city of Manchester by John Dalton (1766–1844), now best known as the discoverer of the law of partial pressures of gases. As a young man, Joule observed the aurora borealis and sounded the depth of Lake Windermere in northwest England with his elder brother, Benjamin. The Joule family owned and managed a brewery but to what extent James Joule was actively engaged in its running is unclear; Osborne Reynolds (see Section 1.2), a friend and biographer, asserts that Joule had little to do with the brewery, although he did do experiments within its premises as part of an extensive study of the relationship between different forms of energy. Cardwell’s (1989) biography of Joule provides informative details of his early years and of his contacts with other scientists of the time, including Michael Faraday, who communicated Joule’s paper describing his experiments to the Royal Society, which published his results.

1.3 Joule’s experiment



**Figure 1.3.** A side view of Joule’s experiment, as described in his 1850 paper. The insulated cylinder (shown also in plan view) is filled with water or mercury and stirred with paddles driven by falling weights through the linkage pulley system. By calculating the energy lost by the weights, Joule was able to estimate the energy dissipated by stirring the fluid within the cylinder. Joule carefully measured the rise of the temperature of the fluid, and was then able to determine the relationship between the mechanical energy dissipated and the gain in heat energy of the fluid, proportional to its temperature rise, the constant of proportionality giving the specific heat.

paddles (equal to its change in heat energy) to its rise in temperature and so, from (1.2), to calculate  $c_p$ .

This experiment was later refined to obtain greater accuracy, but, as it is, it contains a major subtlety that involves the motion of the fluid within the cylinder. As Fig. 1.3 shows, there are baffles fixed to the inside of the cylinder. They are important for two reasons. The first is that without them a circulatory flow would be set up, which, containing kinetic energy, would have to be accounted for in the energy balance. (Alternatively, Joule could have waited until the circulation had died out before measuring the temperature, but that would have required a means to ensure that there was no substantial residual motion and would have taken time, during which heat would have been lost from the cylinder to the air.) The second reason is perhaps more important. The rotating paddles drive fluid past the stationary fixed baffles, and this promotes a transfer of kinetic energy from the mean flow to irregular and interacting small-scale eddies, characteristic of turbulence, that are shed by flow separation from the edges of the baffles and paddles. These eddies or ‘turbulence’ enhance the shear within the fluid and greatly increase the rate at which molecular viscosity dissipates the kinetic energy imparted to the fluid, transferring mechanical energy into heat much



more rapidly than can a mean circulation gradually spun down through viscous drag at the cylinder walls. The potential energy of the falling weights that is not transferred to their kinetic energy consequently passes into turbulent energy that, in dissipating through viscosity, results in heating.

- The important factor is that turbulence transfers the energy involved in motion – the kinetic energy – to heat.

The heating caused by turbulent dissipation in the ocean turns out to be generally insignificant (e.g., see P1.6, P2.4 and P6.2), but the energy lost in turbulent motion is very important in the budget of the ocean's energy, and the effect of turbulent motion in mixing the ocean is a vital element in ocean circulation and in climate change.

## 1.4 The surf zone: waves and turbulence

The surf zone on a gently shelving beach provides an example of a turbulent region of the ocean that is familiar to many, and one that has some properties that relate to, and some that contrast with, those of Reynolds' experiment. It is also a region of the ocean in which turbulence is most energetic, and one in which (as in Reynolds' and Joule's experiments) the principal source of turbulent energy – in this case waves – can be identified. It provides an opportunity to introduce several ideas about energy, dispersion and structure relating to turbulent motion.

Figure 1.4 shows the water surface within a surf zone, the shallow and gradually shoaling region at the edge of the sea. It is partly covered by floating bubbles of foam that have been produced by waves as they approach the beach and break, carrying air into the water and producing clouds of subsurface bubbles. The bubbles rise buoyantly, some reaching the surface either to burst, producing tiny droplets in the air, or to float, contributing to the visible foam layer. A few bubbles may completely dissolve before reaching the surface, transporting all their component atmospheric gases into the seawater. The breaking waves also generate motions that disperse both the subsurface bubble clouds and the floating foam. In a casual viewing, the foam layer appears random, without any structure, rather like Reynolds' experiment when viewed without the advantage of the instantaneous spark image to make eddies visible. On more careful inspection some larger, repetitive and regular features with coherent structures can, however, be seen in a foam layer within the surf zone, notably bands or filaments and near-circular holes. How these are produced is described later.

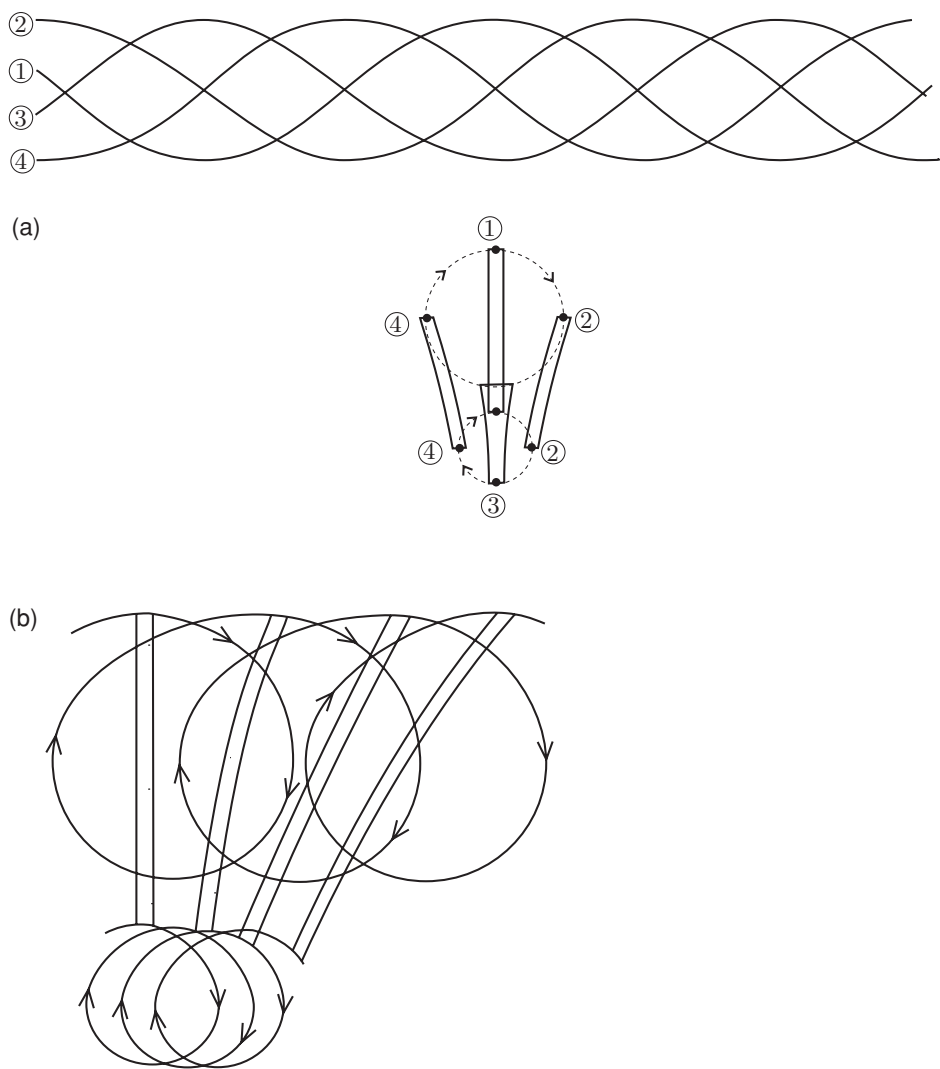
Several distinct 'processes' are associated with the waves. In deep water (before waves break at the edge of the surf zone) the wave-induced motions are relatively regular, benign and, except in high winds when wave breaking becomes frequent, quiescent in comparison with the violent motion within the surf zone. The deep-water waves cause water particles to move in nearly circular orbits. (It may appear at first sight that these motions in the water column beneath surface waves in deep water are like eddies in a turbulent flow, and will result in the overturning and mixing of the water. This is not the case, as explained in Fig. 1.5(a).)





**Figure 1.4.** Plunging breakers, and foam within the surf zone. The plunging breakers are visible at the outer edge of the surf zone. Nearer shore, these become bores (or hydraulic jumps, abrupt changes in water level) advancing towards shore and producing a surface mat of white foam that is broken up by motions within the underlying water.

The waves also produce a mean movement of water called ‘Stokes drift’ in the direction of wave propagation. The drift at the sea surface is typically about 2% of the wind speed and much less than the speed of wave propagation, and it decreases with depth. It carries surface or near-surface floating particles towards shore. Although at this stage the waves contribute very little to the spreading or dispersion of such particles, because of the Stokes drift the particle paths are not exactly circles. This acts like a mean shear, slowly tilting and stretching fluid columns as shown in Fig. 1.5(b).



**Figure 1.5.** A sketch showing motion induced by non-breaking surface gravity waves. (a) Top – the surface of waves propagating to the right at increasing times, 1–4, and, below, the corresponding approximately circular motions of particles at two depths under waves in deep water. For clarity the size of the orbital motions is exaggerated. The radius of the particle orbits decreases approximately as  $\exp(-kz)$ , where  $z$  is their mean depth below the level of the mean water surface and  $k = 2\pi/\text{wavelength}$  is the wavenumber of the waves. Columns of water joining the particles in the two orbits at times 1–4 are shown. These do not overturn, but are periodically slightly stretched, tilted and thinned. In reality, the orbits of particles at the two levels are not exactly closed circles, however, but drift slowly in the direction of the waves as sketched in (b). The Stokes drift decreases with depth below the surface. The stretching of an initially vertical column of water particles (stippled) by the drift is shown as three successive wave crests pass by. The Stokes drift at depth  $z$  is approximately  $a^2 k \sigma \exp(-2kz)$ , provided that the amplitude of the waves,  $a$  (half their height, i.e., half the crest-to-trough distance), is small (so that  $ak \ll 1$ ), where their frequency is  $\sigma$ , equal to  $2\pi/(\text{wave period})$ .