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1
Introduction

The theory of strong interactions,
now that is quite something.

Elementary particles we know are *leptons* e and μ and their corresponding *neutrinos*, ν_e, ν_μ, a photon (γ) and a graviton, and then, hundreds of strongly interacting particles – *hadrons*: proton p and neutron n, pions $\pi^\pm$ and π^0, kaons $K^\pm$, K^0, $\bar{K}^0$, etc., etc.

1.1 Interaction radius and interaction strength

Electromagnetic interaction has two characteristic features. Firstly, it is characterized by a small coupling,

$$\frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}.$$

Secondly, it is a long range force,

$$V = \frac{e_1 e_2}{r},$$

so that there is no typical distance, no characteristic interaction radius.

Gravitation behaves (at large distances!) similarly to the electromagnetic interaction,

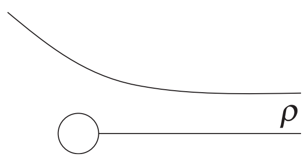
$$V = G_{\mathrm{gr}} \frac{m_1 m_2}{r};$$

thus it has no radius either. To characterize the magnitude of the interaction one needs to construct a dimensionless parameter. Contrary to the case of the electromagnetic charge, mass is not quantized, so that there is no 'unit mass' to choose. Hence one usually takes the mass of the proton, m_p, to quantify the typical interaction strength:

$$G_{\rm gr}\frac{m_p^2}{\hbar c} \simeq 7\cdot 10^{-39}.$$

An important difference with electromagnetism is that here all the 'charges' have the same sign (mass is positive). Therefore the gravitation prevails over the electromagnetic interactions in the macro-world. Moreover, the gravitational interaction grows with energy, making the gravity essential at extremely small distances. This happens solely owing to the existence of the Planck constant $\hbar$, since by confining a system to small distances, Δr, we supply it with a large energy $\Delta E \sim \hbar c(\Delta r)^{-1}$.

Leptons, photons, graviton do not participate in strong interactions. Rutherford was the first to observe (electromagnetic) scattering of strongly interacting particles. By comparing the scattering pattern of α-particles at large angles with classical formulae he concluded that the size of the gold nucleus was about 10^{-13} cm. By the way, to be able to describe the process as classical particle scattering, all the way down to $\rho \sim 10^{-13}$ cm, one has to have



$$kr_0 \gg 1.$$

However, the energies of α-particles in the Rutherford experiment, $E/m_\alpha = \mathcal{O}(\mathrm{keV/GeV}) = 10^{-6}$, correspond to momenta k such that

$$k\cdot r_0 \;=\; \sqrt{2m_\alpha\cdot E}\cdot\frac{1}{\mu} \ll 1, \quad \text{with} \quad (r_0)^{-1}\sim\mu = 140\ \mathrm{MeV},$$

so that the scattering becomes *quantum*, rather than classical, already for the impact parameters much larger than r_0. It was fortunate for Rutherford that the scattering cross section in the Coulomb field happened to be identical to that in the classical theory!

The proton–proton cross section is very small, $\sigma_{pp} \sim 4\cdot 10^{-26}$ cm^2. Why then do we refer to the 'strong interaction' as *strong*? To really evaluate the strength of interaction, one has to take into consideration the existence of the finite *interaction radius* since the interaction cross section is composed of the actual interaction strength and of the probability to hit the target, measured by the transverse area of the hadron $\sim\pi r_0^2$. This being said, if the interaction cross section turns out to be of the order of the geometric cross section,

$$\sigma \sim r_0^2,$$

we call the interaction *strong*; otherwise, if

$$\sigma \ll r_0^2,$$

such an interaction we consider as *weak*.

How can one determine experimentally the interaction radius r_0?

In the *classical theory* it is straightforward: from the angular dependence of the scattering cross section $d\sigma(\mathbf{q})$ one can reconstruct the potential, and, subsequently, extract the characteristic radius r_0.

In *quantum mechanics* we operate with the partial wave expansion of the scattering amplitude,

$$f(k,\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)f_\ell(k)P_\ell(\cos\theta).$$

Guided by quasi-classical considerations, we can define the interaction radius by comparing the magnitudes of the partial wave amplitudes f_ℓ with different orbital momenta ℓ:

$$f_\ell \sim \begin{cases} 1 & \text{for} \quad \ell \lesssim kr_0, \\ 0 & \phantom{\text{for}} \quad \ell \gg kr_0. \end{cases}$$

Assume that the interaction radius is small, $kr_0 \ll 1$. Then, due to the fact that the partial waves with large orbital momenta are suppressed, $f_\ell \propto (kr_0)^{2\ell+1}$ (centrifugal barrier), the S-wave dominates,

$$f(k,\theta) \simeq f_0(k),$$

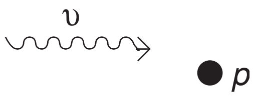
and the scattering pattern is spherically symmetric. Increasing the incident momentum we reach $kr_0 \sim 1$ where a few partial waves will start contributing and the corresponding Legendre polynomials with $\ell \neq 0$ will introduce angular dependence into the scattering distribution. Thus we can determine r_0 by studying at what energies the scattering ceases to be spherically symmetric. Alternatively, at large k, we can extract the interaction radius by measuring the characteristic scattering angle, $\theta_{\mathrm{char}} \sim (kr_0)^{-1} \ll 1$.

Now that we know how to measure r_0 and may compare σ with r_0^2, let us ask ourselves another question: whether the situation when $\sigma \ll r_0^2$ really means that we are dealing with a *weak* interaction.

The answer is, yes and no!

Consider the scattering of a point-like neutrino off a proton, for example, the process $\nu_\mu + p \to \mu + X$. By examining the momentum dependence of the cross section we will extract that very same proton radius

$r_0 \sim 10^{-13}$ cm. At the same time, the interaction cross section is of the order of $\sigma_\nu \sim 10^{-40}$ cm^2. Then, according to our logic we must proclaim the neutrino a weakly interacting particle.



However, imagine that the proton has a tiny core, of the size 10^{-20}, which is smeared over the area of the radius $r_0 = 10^{-13}$. If so, the interaction of the neutrino with the proton actually turns out to be strong: $(10^{-20})^2 \sim \sigma_\nu$. We can only state that ν interacts weakly at the distances *larger* than 10^{-20} cm.

The most important property of the *weak interaction* is its *universality* with respect to hadrons and leptons. They get engaged in the weak interaction in a similar manner and with the same universal *Fermi constant*

$$G_F \simeq \frac{10^{-5}}{m_p^2}, \qquad {m_p}^{-1} \sim 10^{-14}\,\text{cm}.$$

Weak interaction increases with energy. At distances $10^{-3}/m_p \sim 10^{-17}$ cm, corresponding to collision energies of the order of $10^3 m_p \simeq 1000$ GeV, the weak interactions may become *strong*.

The main features of *strong interactions* of hadrons are the following:

(1) probability to interact is $\mathcal{O}(1)$ at the distances $r \lesssim r_0 = 10^{-13}$ cm;

(2) hadrons are intrinsically relativistic objects.

Indeed, to investigate the distances $r_0 = 1/\mu$, momenta $k \sim \mu$ are necessary, which correspond to the proton velocity $v \simeq \mu/m_p \sim 1/6$. (By the way, it is this 1/6, treated as a small parameter, to which the nuclear physics owes its existence.) At the same time, if we substitute for the proton a π-meson (whose mass is $m_\pi = \mu$) we get $v \simeq 1$ and the very possibility of a non-relativistic approach disappears.

1.2 Symmetries of strong interactions

Imagine that we have an unstable particle whose decay time τ is much larger than $r_0/c \sim 10^{-23}$ s. Does it decay due to the strong or weak interaction? The answer lies in the symmetry of the decay process: the *degeneracy* is much larger in the strong interaction; degeneracy means symmetry, and symmetries, as you know, give rise to conservation laws.

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Electric charge Q. The hadrons have to know themselves about the electromagnetic interaction. Each hadron has a definite electric charge, and the strong interactions must respect its conservation, otherwise quantum electrodynamics would be broken.

Baryon charge B. This is another quantum number whose conservation is verified with a fantastic accuracy (stability of the Universe). The baryon charge equals $+1$ for *baryons* like p, n, Λ, Σ, Ξ, $\ldots$ (and -1 for their antiparticles), and 0 for *mesons* (π, K, ρ, ω, φ, $\ldots$).

Isotopic spin I. Phenomenologically, hadrons split in groups of particles with close masses, and can be classified as belonging to *isotopic* $SU(2)$ multiplets. For example, the doublet of the proton and the neutron, p, n ($I = \frac{1}{2}$); the triplet of pions, $\pi^{\pm}$ and π^0 ($I = 1$), etc. The relative mass difference of hadrons in one multiplet is 10^{-2}–10^{-3}, that is, of the order of the electromagnetic 'fine-structure constant':

$$\frac{m_n - m_p}{m_p} \sim \frac{m_{\pi^0} - m_{\pi^+}}{m_{\pi^+}} \sim \alpha \simeq \frac{1}{137}.$$

It looks that if we switched off the electromagnetic interaction, we would arrive at a complete degeneracy in the mass spectrum of strongly interacting particles. Independently of the hypothesis about the nature of this tiny mass splitting, these states can be treated as degenerate in the first approximation and therefore, there must be a symmetry and the corresponding conservation law.

Are the pn and pp scattering cross sections the same, if electromagnetic interactions are switched off? No – in the second case the particles are identical. In order to distinguish p from n, a new quantum number is introduced: the proton is treated as a *nucleon* with the isospin projection $I_3 = +\frac{1}{2}$, and the neutron with $I_3 = -\frac{1}{2}$. Thus, the nucleon wave function depends on coordinates, spin and *isospin* variables, $\psi(\mathbf{r}, \sigma, \tau)$. In strong interactions isospin is conserved.

For example, the lightest stable nuclei – the deuteron and the helium – consist of equal number of protons and neutrons, $D = (pn)$, $\mathrm{He}^4 = (2p2n)$, and both have $I = 0$ (isotopic singlets). Therefore, the fusion reaction

$$D + D \not\to \mathrm{He}^4 + \pi^0$$

is forbidden, since the pion has isospin $I = 1$.

Strangeness S. Any reaction takes place that is allowed by conservation laws. At the same time, it was observed that long-living hadrons like K-mesons, and Λ- and Σ-baryons, cannot be produced *alone* in the interactions of nucleons and pions. They always go *in pairs*, e.g.

$$\pi^- + p \to \Lambda + \bar{K}^0 ,$$

while the reactions

$$\pi^- + p \to n + K^0, \quad \text{or} \quad \pi^- + p \to \Lambda + \pi^0$$

are forbidden.

By prescribing to these hadrons a new quantum number – *strangeness* S,

$$S(K^-, K^0) = S(\Lambda) = -1, \qquad S(\bar{K}^0, K^+) = S(\bar{\Lambda}) = +1,$$

we get the relation between the conserved quantities:

$$Q = I_3 + \frac{B}{2} + \frac{S}{2}.$$

There is one more approximate symmetry which combines strange and non-strange hadrons into of SU(3) multiplets, like *octets* of pseudoscalar mesons,

$$\begin{array}{lcc} S = 1 & (\bar{K}^0, K^+)_{I=\frac{1}{2}}, & \\ S = 0 & (\pi^-, \pi^0, \pi^+)_{I=1}, & \eta_{I=0}, \\ S = -1 & (K^-, K^0)_{I=\frac{1}{2}}, & \end{array}$$

and baryons,

$$\begin{array}{lcc} S = 0 & (n, p)_{I=\frac{1}{2}}, & \\ S = -1 & (\Sigma^-, \Sigma^0, \Sigma^+)_{I=1}, & \Lambda_{I=0}, \\ S = -2 & (\Xi^-, \Xi^0)_{I=\frac{1}{2}}, & \end{array}$$

baryon *decuplet*, etc.

The isospin symmetry is broken by electromagnetic interactions. The weak interaction breaks *everything* except B, Q and, maybe, the lepton charge L. (Apparently, the electron and the muon lepton charges conserve separately, since $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, but $\mu^- \not\to e^- + \gamma$.)

The lightest strange particles are stable under strong interactions. However, K-mesons decay into pions, and the Λ-baryon into $\pi^- p$, due to the weak interaction, disrespecting the strangeness conservation. The weak forces violate spatial parity P, charge parity C, and even the time reflection symmetry T (the later equivalent to the 'combined parity' CP).

1.3 Basic properties of the strong interaction

1.3.1 Interaction radius

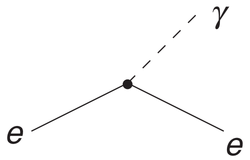
The question arises, what is r_0: is this an *interaction radius* specific for the strong interaction, or rather a real size of an object? This question can be answered using, for example, weak interactions as a short-range probe. It turns out that r_0 is the actual size of the proton that can be extracted, in particular, from the measurement of the spatial distribution of the electric charge inside the proton.

The hadron radius r_0 appears to be equal to the pion Compton wavelength,

$$r_0 \simeq {m_\pi}^{-1} \equiv \mu^{-1} \simeq 10^{-13}\,\text{cm}.$$

Is this coincidence an accident? In the past it was thought to be of fundamental importance; it is not so clear any more that it really is.

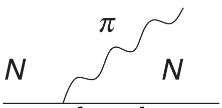
What is the problem with the description of the strong interactions? As we have discussed above, a non-relativistic description does not make sense here. We have just one example which may help us to construct a relativistic theory: electrodynamics. In the quantum electrodynamics, the electron e and the photon γ are point-like, and so is the interaction between them.



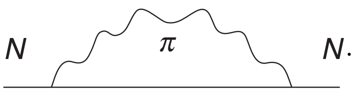
(1.1)

Now we want to describe hadrons: p, n, π. Are these particles point-like? The existence of the finite radius r_0 confirms, apparently, the opposite. There is no way, however, to give a *relativistic* description of a particle of finite radius. So we have to assume that the particles we consider are, in a sense, point-like.

Yukawa suggested that the point-likeness of a hadron does not contradict the existence of a finite interaction radius. Let us draw a pion–nucleon interaction

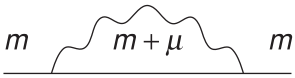


taking (1.1) for a model. The existence of this vertex means that there are processes of virtual emission and absorption of pions by the nucleon,



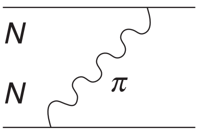
(1.2)

Let us imagine now that this happens quite frequently. What will we see as a result of a scattering of an external particle off such a fluctuating nucleon? Estimating the energy uncertainty as

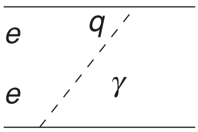


$$\Delta E \simeq (m+\mu) - m = \mu, \tag{1.3}$$

we conclude that the lifetime of the fluctuation is $\Delta t \sim (\Delta E)^{-1} \sim \mu^{-1}$. During this time interval, a pion (with a velocity $v \sim 1$) will cover the distance $\Delta r \sim \mu^{-1}$. Thus, our object, which was point-like in the beginning, is now spread over a distance μ^{-1}, and, in the process of scattering, it will interact with the projectile at impact parameters $\rho \sim \mu^{-1}$. In other words, the scattering of an incident particle with our nucleon can be depicted as a pion exchange between the two nucleons – the process that has a characteristic radius $r_0 \sim \mu^{-1}$!



Without any theory, let us first calculate this amplitude in a naive way, by analogy. What would be the difference between the above process and the scattering of electrons that we have studied in the quantum electrodynamics,



$$= \frac{e^2}{q^2}\big(\bar{u}\gamma^\mu u\big)\big(\bar{u}\gamma_\mu u\big). \tag{1.4}$$

We must replace the photon propagator $1/q^2$ in (1.4) by the Green function of the massive π meson:

$$D_\pi(q) = \frac{1}{\mu^2 - q^2}.$$

The corresponding scattering amplitude will have the form

$$A = \frac{g^2}{\mu^2 - q^2}, \tag{1.5}$$

with g the pion–nucleon interaction constant, replacing the electric charge e in the QED amplitude (1.4).

What does this amplitude correspond to in the case of the non-relativistic scattering? The non-relativistic scattering amplitude reads

$$f = -\frac{2m}{4\pi}\int e^{i\mathbf{k}'\cdot\mathbf{r}}V(r)\,\psi(\mathbf{r})\,d^3r. \tag{1.6}$$

In the Born approximation, replacing the wave function $\psi(\mathbf{r})$ by a plane wave with momentum $\mathbf{k}$, we obtain

$$f_B = -\frac{2m}{4\pi}\int e^{i\mathbf{q}\cdot\mathbf{r}}V(r)\,d^3r, \tag{1.7}$$

where $\mathbf{q}$ is the momentum transfer, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$. For non-relativistic particles, the kinetic energy, $E = \mathbf{k}^2/2m$, is small, and the *energy* transfer component can be neglected:

$$|q_0| \sim \mathbf{q}^2/m \ll |\mathbf{q}|\,, \qquad \text{so that} \quad q^2 = q_0^2 - \mathbf{q}^2 \simeq -\mathbf{q}^2.$$

The scattering amplitude (1.5) becomes

$$A \simeq \frac{g^2}{\mu^2 + \mathbf{q}^2}\,.$$

What is the potential corresponding to this amplitude? Evaluating the inverse Fourier transform of the Born amplitude f_B in (1.7) we obtain the Yukawa potential,

$$V(r) = -\frac{4\pi}{2m}\int \mathrm{e}^{-i\mathbf{q}\mathbf{r}}\frac{g^2}{\mu^2+\mathbf{q}^2}\frac{d^3q}{(2\pi)^3} = \frac{g^2}{2m}\cdot\frac{\mathrm{e}^{-\mu r}}{r}. \tag{1.8}$$

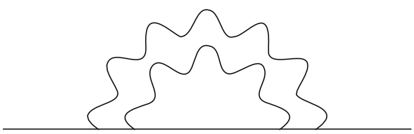
So, indeed, the effective interaction is characterized by a finite radius $r_0 = 1/\mu$.

We conclude that the assumption of the point-like nature of the interaction does not exclude the finiteness of the interaction radius. Moreover, having adopted the point of view that the hadron has no intrinsic size (having no other option), we see that the interaction radius is not an independent quantity but is determined by the *masses* of the particles.

From the point of view of a relativistic theory the π-meson has to exist in nature, otherwise there would be no explanation for such a 'large' value of the proton radius.

1.3.2 Interaction strength

The other side of the strong interactions is their *strength*: once the particles approach each other to the distance r_0, the interaction is inevitable. Since a nucleon is always surrounded by a pion cloud, see (1.2), this means that the coupling constant g^2 (if it exists at all) is obliged to be *large*, $g^2 \sim 1$, contrary to the electromagnetic interaction, characterized by the small coupling $\alpha = 1/137 \ll 1$. Now that is bad indeed, because under these circumstances anything will go. For example, a virtual state with two pions will be there, having a typical lifetime $\Delta t \sim \frac{1}{2}\mu$ and, correspondingly, a spatial spread of the order of $\sim\frac{1}{2}r_0$.



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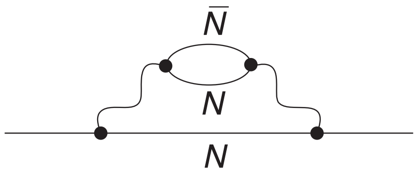
We may even have a three-nucleon state, $N \to N\pi$, $\pi \to N\bar{N}$. Since the nucleon is much heavier than the pion, this fluctuation is short-lived: $\Delta t \sim 1/(2m_N) \ll r_0$. However, we cannot state a priori that such a process does not contribute to the radius of the nucleon since these 'second-order' amplitudes may be actually *larger* than the one-pion emission amplitude (1.2), because the coupling constant is not small.



It is clear that it will be certainly impossible to build a theory like quantum electrodynamics to describe strongly interacting hadrons.

We can, however, introduce initial point-like objects, and then, in fact, observe 'clouds', the radii of which are determined by the masses of the hadrons.

This is the basic idea of the theory of the strong interaction.

We need to construct a framework which would allow us to draw *pictures* representing a formal series for the hadron interaction amplitudes. From these pictures we will extract information without actually calculating the amplitudes, which would be, a priori, impossible. The Feynman diagrams can be considered as a 'laboratory of theoretical physics'.

1.4 Free particles

We start by considering free particle states and their propagation. There is a fantastic variety of hadrons with spins reaching up to $s = \frac{19}{2}$.

1.4.1 Particle states

$s = 0$. A free spinless (scalar) particle with a four-momentum p_μ is described by the wave function

$$s = 0 : \qquad \psi(x) = \frac{1}{\sqrt{2p_0}}\, \mathrm{e}^{-ipx}. \tag{1.9}$$

$s = \frac{1}{2}$. A spin-one-half particle has two states, $\lambda = 1, 2$,

$$s = \tfrac{1}{2} : \qquad \psi_\alpha^\lambda(x) = \frac{u_\alpha^{(\lambda)}}{\sqrt{2p_0}}\, \mathrm{e}^{-ipx}; \tag{1.10}$$

two states with definite parity are selected out of possible four spinors u_α by the Dirac equation, $(\hat{p} - m)u^\lambda$.