Mathematics and War

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Introduction

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Physicists, chemists, and biologists have a tradition of discussing meta-aspects of their subject – among which are the military use and misuse of the knowledge they produce. Concerns of the latter kind are rare among mathematicians.

No rule without exceptions. During the Vietnam war, a number of appeals were circulated among US mathematicians (with reverberations in particular in France and Japan and at the International Congress of Mathematicians in Moscow in 1966 and Nice in 1970) not to engage in war-related work. One such appeal was published in the *Notices of the American Mathematical Society* in January 1968. Alexandre Grothendieck's resigning from mathematics fell in the context of this debate. [Godement 1978], no longer debate but politico-economical analysis, was written from a mathematician's perspective even though it did not deal with mathematical research in particular. [Gross 1978], also written by a mathematician, was shorter but concentrated on mathematics.

In the new context of the euro-missile controversy of the early 1980s, military research came into the focus of debate at universities of Western Germany. [Booß & Høyrup 1984] was an offspring of this new discussion concentrating on mathematics; the broad discussion is reflected in [Tschirner & Göbel (eds) 1990]. The "Forum on Military Funding of Mathematics" published in the *Mathematical Intelligencer* 1987 no. 4 reflects problems arising for the US mathematical community from the "Strategic Defense Initiative" in the same phase.[1] Some more publications followed, mainly with historical emphasis.

As warfare is now again becoming an all-too-obvious aspect of our world and a no less obvious part of "Western" policies, time seemed ripe for taking up the issue anew. Just after the Kosovo War, *Zentralblatt für Didaktik der Mathematik* dedicated an issue to it (vol. 98 no. 3, June 1998); August 29–31, 2002, 42 mathematicians, historians of mathematics, military historians and analysts, and philosophers gathered in the historic Swedish military port of Karlskrona, to discuss a wide variety of questions, each representing its own perspective on or facet of the global topic "mathematics and war". Tentatively, the themes can be grouped as follows:

- To what extent has the military played an active part throughout history, and in particular since World War II, in shaping modern mathematics and the careers of mathematicians?
- Are mathematical thinking, mathematical methods, and mathematically supported technology[2] about to change the character and performance of modern warfare, and if so, how does this influence the public and the military?

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[1] Cf. also [Davis 1989].

– What were, in times of war, the ethical choices of outstanding individuals like the physicist Niels Bohr and the mathematician Alan Turing? To what extent can general ethical discussions provide guidance for working mathematicians?
– What was the role of mathematical thinking in shaping the modern international law of war and peace? Can mathematical arguments support actual conflict solution?

The present volume does not constitute the official proceedings of this meeting, but most contributions were presented there. This introductory essay represents the editors' personal attempt to synthesize the outcome of an inherently (and intentionally) very disparate exchange of information and views. Admittedly, the synthesis also draws on the material presented in [Booß & Høyrup 1984] and on the perspectives developed in that booklet. In order to balance this personal bias we refer in running footnotes to the contributions to the volume. Such references do not necessarily imply that the editors agree with all points of view expressed in the contribution in question, nor that the editors postulate the author's agreement with their interpretation of the subject-matter. In general, no other participants are of course responsible in any way for what we say in this introduction or for what we conclude from what they said; by participating in the endeavour, all of us – authors as well as editors – have agreed implicitly that we find civilized disagreement enlightening.

Perspectives from Mathematics

All mathematicians know the tales, reliable or not, about Archimedes and his defence of Syracuse. They may also have heard about early modern ballistics and fortification mathematics and the importance of trigonometry for navigation. All these cases of mathematics being entangled in conquest, warfare or preparation of war have one thing in common: that which was combined with technical and military knack was almost exclusively *already existing* mathematics. In this respect such examples do not differ from the use of simple accounting mathematics in logistics – which after all is likely to have been much more important from the military point of view. Mathematics served as a toolbox, and military officers may have been the largest group that received some general mathematical training; but the involvement of mathematics as a global endeavour with the military institution was not very intimate, and specifically military applications had no independent role as a shaping force for mathematics. Tartaglia's composition of straight lines and circles in ballistics was clearly inspired from gunnery and the war against the Turks. When Galileo introduced the parabolic law this origin was already left behind, and the theory was linked instead to the philosophical discussion of local motion – and was largely irrelevant for the firing of guns because of the influence of air resistance, as pointed out explicitly by Galileo.

[2] This "broad concept" of mathematics is the one that serves in the rest of this introductory essay; it also embraces computers and computer science.

Figure 1. "Noli turbare circulos meos": When Archimedes's city was conquered in spite of his astounding mathematical engineering, he pretended that he had only made pure mathematics – thus according to an anecdote that has remained popular since Roman Antiquity; the mosaic is an eighteenth-century forgery. [Courtesy: P. C. Bol, Städelsches Kunstinstitut, Frankfurt/M.]

Even to this rule ("the involvement of mathematics as a global endeavour ...") there is an exception. That part of the Sumero-Babylonian legacy which is most spoken of in general histories of mathematics – namely the invention and implementation of the place value system – may be a child of war. In c. 2074 BCE, king Shulgi organized a military reform in the Sumerian Empire, and the next year an administrative reform (seemingly introduced under the pretext of a state of emergency but soon made permanent) enroled the larger part of the working population in quasi-servile labour crews and made overseer scribes accountable for the performance of their crews, calculated in abstract units worth $^1/_{60}$ of a working day (12 minutes) and according to fixed norms. In the ensuing bookkeeping, all work and output therefore had to be calculated precisely and converted into these abstract units, which asked for multiplications and divisions *en masse*. Therefore, a place value system with base 60 was introduced for intermediate calculations.[3] Its functioning presupposed the use of tables of multiplication, reciprocals and technical constants and the training for their use in school; the implementation of a system whose basic idea had been "in the air" for some centuries therefore asked for decisions made at the level of the state and implemented with great force. Then as in many later situations, only war provided the opportunity for such social willpower.

Apart from that the conclusion stands that "the involvement of mathematics as a global endeavour with the military institution was not very intimate, and specifically military applications had no independent role as a shaping force for mathematics" until a century ago. Since around 1500 CE, as already mentioned, the employment of fortification mathematicians and the teaching of naval and artillery officers certainly played a social role for mathematics by providing job opportunities and a market for mathematics text books (copiously decorated with military symbols).

This we may regard as the past. The contemporary situation – the one that is our real interest – can be said to start around the First World War, and to reach full development during the Second World war.[4]

During World War I, two important new military technologies depended on *mathematics in the making*: sonar, and aerodynamics. They were so impressive that Émile Picard, in spite of his own patriotism (which non-French cannot help seeing as pure chauvinism), regretted the perspective that young mathematicians might opt in future for applied mathematics only [*Proc. ... 1920*: xxviii]. In general, however, the immediate role of the pure sciences, mathematical and otherwise, was that of providing manpower that could be converted into first-class creative engineers – not restricted to applying a set of standard rules but able to implement theoretical knowledge and make it function in practice; this was also the role of most of the mathematicians that were actually involved in the war effort (if they were not, as was the case in France, sent into the trenches). Nobody will claim that mathematics was in any way decisive for the outcome of the war, nor that WW I applications of mathematics left important traces in the post-war world (civil aviation still belonged to the future).

Picard's worries proved unfounded. Main-stream mathematics soon reverted to the pre-War model, even more swiftly than the precariously erected organization of planned science was dismantled. Aerodynamics of course survived, but only as one current among others.[5]

[3] Since it was a floating-point system with no indication of absolute place, it could only be used for intermediate calculations – just as the slide rule of engineers in quite recent times. Since intermediate calculations have not survived, the exact dating of the implementation can only be determined from indirect arguments. See, for instance, [Høyrup 2002: 314] for further references.

[4] A broad introduction to the whole period is R. Siegmund-Schultze's survey, this volume, pp. 23–82.

[5] The Polish first application of mathematics (group theory) instead of philological methods in code cracking constitutes another isolated current – or, in fact, an omen of what was to come. See E. Rakus-Andersson, this volume, pp. 83–107.

All of this was different in World War II, both quantitatively and qualitatively: the organization of science intended to support the war effort was a major concern of both Axis and Allied powers;[6] mathematically based technologies (radar, sonar, the decipher computer, *the* bomb) can be argued to have been war-decisive; computers, nuclear energy, jet propulsion – all mathematically constructed and computed for the war – have changed our world beyond recognition after 1945. Admittedly, all of these build on pre-war theoretical insights;[7] some of them (computers, jet motors) were indeed not only "in the air" before the war but functioned as prototypes for the devices which reached completion during the war; but in all cases the war, by making available huge means without counting costs and benefits, made it possible to boost a development that otherwise might have taken decades[8] – and perhaps, in cases like the proliferation of DDT and atomic reactors, might have been stopped at an early moment when the problems they create became visible.

During the war, mathematicians in large numbers were recruited, many of them to teach sailors and air-crew members basic trigonometry (etc.), but many also to serve as best-level creative engineers.[9] Afterwards, the latter have often tended to regard what they did dismissively ("I did not write one line that was publishable", as J. Barkley Rosser [1982: 509*f*] summarizes one reaction to his questionnaire on war work in mathematics), perhaps because puzzle-solving with no further theoretical impact did not look important in the mathematician's hindsight; this assessment notwithstanding, what was done depended critically on mathematical ingenuity and training. A striking example is O. R. Frisch and R. Peierls' mathematical formulation of the essential questions surrounding the construction of a uranium bomb in March 1940 and their "back-of-an-envelope" discovery that its critical mass was so small that military use was feasible.[10]

In some cases, of course, the solving of problems defined by the war *did* have important theoretical impact – well-known examples are the emergence of computer science, information theory, Monte Carlo simulation, operations research, and statistical quality control.

[6] See, beyond Siegmund-Schultze's article, the contributions in this volume of S. Fukutomi (pp. 153–159), A. N. Shiryaev (pp. 103–107), K. B. Williams (pp. 108–125), T. H. Kjeldsen (pp. 126–152), A. Hodges (pp. 312–325) and T. Makino (pp. 326–335).

[7] At times fully detached from every technical application; N. Wiener and E. Hopf had calculated the radiation equilibrium at stellar surfaces, but their theory could be applied to the expanding surface of the exploding bomb [Wiener 1964: 142*f*]. A. A. Markov had investigated his eponymous processes as pure mathematics and illustrated the applicability of the concept on linguistic material [Youschkevitch 1974: 129]; in the Manhattan Project they turned out to be relevant for solving diffusion equations and for describing nuclear branching processes.

[8] The parallel to the invention of the place value system in Sumer is striking. In that case, parallel processes not furthered by a military government indeed asked for much longer time: in China the unfolding took more than a millennium, in India it never really took place before the "Indian" system was brought back from abroad.

[9] Cf. [Morse 1943].

[10] See [Gowing 1964: 40–43, 389–393] and [Dalitz & Peierls 1997: 277–282]. This latter volume presents Peierls as a physicist, but his actual chair was in "applied mathematics"; even a "broad concept" of mathematics does not free us from delimitation problems.

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Figure 2. There is no known picture of Turing during the wartime period, but this photograph shows Alan Turing (at left) with his athletic club in 1946. At this point he was engaged in designing a first digital computer at the National Physical Laboratory, London. This design used his wartime knowledge of electronic technology to put his 1936 theory of the universal machine into a practical form. The codebreaking machinery at Bletchley Park, although very advanced, had never actually used Turing's fundamental idea of the universal machine and the stored program, but as soon as the war ended Turing set to work to bring it to reality. [Courtesy: Turing Archive at King's College, Cambridge]

This time, nothing was dismantled after the war (many mathematicians, of course, hurried away from military research) – the Cold War was already on. In the slightly longer run (a decade or so), civil re-application of the new mathematical war techniques caused profound transformation of these and violent acceleration of their development: only the war effort had allowed the creation of the first costly computers, but only commercial use allowed mass production, open competition, intensive development efforts and reduction of costs (actually, stored-program computers like the ENIAC only reached the working stage after the war though at first in military contexts).[11] We may add, on one hand, that only the freeing from the pressure of immediate applicability ("better a fairly satisfactory answer now than the really good answer two years after defeat") gave space for fruitful interaction between theoretical understanding and applications in for instance computer science.[12] On the other hand we may add that the developments born from civil reapplication were then brought back to the military sector with their now immensely increased efficiency (and further, back-and-forth, back-and-forth,...) – a situation that has given rise to the notion of a "military-industrial" or "military-industrial-scientific complex".[13]

When discussing mathematical research for military purposes, both during World War II and in recent decades, we should differentiate several situations and problems.

- Firstly, we must distinguish the application (sometimes creative, sometimes repetitive) of existing mathematical tools (ballistic computation, modelling, ...) from the creation of new mathematical insights and techniques (sequence analy-

[11] See also T. H. Kjeldsen's discussion of the case of operations research, this volume, pp. 126–152.

[12] Cf. Hodges (pp. 312–325) on Turing's post-war work, and Williams (pp. 108–125) on Grace Hopper, both this volume.

[13] Since this concerns scientifically based industry in general and mathematics only in so far as it enters in all scientific technology, this complex is not much discussed in the present volume (cf. note 18). See, however, for a very specific aspect, Davis, in this volume, pp. 174–179). A broad empirical investigation of related questions is [Godement 1978].

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Figure 3. When listening to a music CD we enjoy and do not think of the military origin of the coding involved. Similarly, mathematicians going to the wonderful Mathematical Research Institute *Oberwolfach* enjoy the ambience and have no reason to worry about the fact that the institute originated as a military research institution in 1944 – apparently well planned for the purpose though too late to become efficient. [Photo: H. Kastenholz]

sis, Monte Carlo simulation, ...). As a rule, but not consistently, the former type is the chore of mathematicians who are paid by or connected to the military institution itself; new insights directed toward military goals are more likely to come from mathematicians who are less closely bound to the military institution but inspired by problems coming from this source.

– Secondly, we should remember that mathematical military research is a complex activity that cannot be understood exclusively as the mere production of theorems of presumed military use. Several institutions (Süss's original planning of the German Oberwolfach Institute in 1944, the American Mathematics Research Center in Wisconsin) exemplify an efficient model, *a two-way chain* (henceforth the "AMRC chain"), which *grosso modo* works as follows:[14] A core group of highly skilled mathematicians familiar with the direct problems of the military employer (efficiency of bombing, controlled spread of bacteriological agents, better radar detection and avoidance of enemy detection, or whatever it may be) find out which of these can be approached mathematically, undertake an initial translation, and direct the translated problems to other experienced

[14] Concerning the Oberwolfach Institute, this structure follows (conventional whitewashing notwithstanding) from analysis of the material presented by H. Gericke [1984], cf. [Høyrup 1986]; on the same institution, see also [Remmert 1999]. For the Wisconsin Institute, see [*The AMRC Papers*].

mathematicians who are well-informed about and centrally located within the whole mathematical milieu; these parcel out the questions into problems which colleagues may take up as mathematically interesting, perhaps even without knowing that they enter into a network of military relevance; once such questions have been answered, the same chain functions backwards, reassembling the answers and channelling the global solution to the employer.

This is only one among several models. We know that it was planned to function in World War II Germany but was implemented too late to become efficient; we know that it has functioned in the US. We know less about the organization of military mathematical research in the late Soviet Union, but it appears that here, as in production and research in general, the civil and military domains were more sharply separated than in the West.

Rounding off what can be said in "the perspective from mathematics" we may make some general observations.

- Mathematical war research has resulted in certain fundamental theoretical innovations. It is striking, however, that all of these appear to have depended critically on an exceptional mathematician. The names of Turing, von Neumann, Shannon, Wald, and Pontryagin may suffice to make the point.[15]
- However, the utility of mathematics *for the treatment of military problems* does not depend critically on the presence of an exceptional mathematician. Mathematicians in large numbers have proved themselves unexpectedly able to function as creative mathematical engineers, in the sense explained above.
- This ability has largely depended on their capacity to become familiar with methods and approaches of various mathematical disciplines and to synthesize them. The still persistent unity of mathematics is thus demonstrated *ad oculos*, if not in the mathematical journals then in technical application.
- It should not be forgotten that the traditional application of the toolbox of already existing mathematics goes on, now at the level created by recent mathematical research.
- In the wake of World War II and as a consequence of the intertwinement of mathematics with advanced technologies (military as well as civil), mathematics as a subject has changed: discrete mathematics and the "mathematization of complexity" have become increasingly important, in some views to such an extent that they now define the actual essence of the field.[16]

[15] Hodges (this volume, pp. 312–325), as mentioned, analyzes Turing's work. R. V. Gamkrelidze (this volume, pp. 160–173) presents Pontryagin's discovery of the maximum principle in control theory. It may also depend on the extraordinary mathematical competence of all five that their theories or techniques were shaped in so mature form that their applicability outside the military domain resulted almost immediately.

[16] This theme has not been explored in the preceding pages, but see [Booß & Høyrup 1984]. [Bleecker & Booss-Bavnbek 2004], to mention one publication among a thousand possibilities, explores one aspect of the "mathematization of complexity".

Military Perspectives

At the conference, the point was strongly made by Colonel Svend Bergstein that actual war cannot be calculated, no more today than in Clausewitz's times;[17] war and fighting not only involve too many unpredictable external factors but also those aspects of human behaviour which are most atavistic and contrary to reason – in Bergstein's view mostly due to the prevalence of stress and sleep deprivation during combat operations.

Nevertheless, and as a matter of fact, mathematics – that is, mathematical thinking, mathematical methods, and mathematics-based technology – has become an integral and even essential part of modern warfare (though often not recognized as such by a general public which only sees the technology and not the underlying mathematics). This does not mean that mathematics has become the major expense of the military apparatus – mathematics and what goes with it is a cheap way to use costly resources more efficiently, and mathematics is used not least for that reason.

Once more, we may list various aspects of this role and utility of mathematics as discussed at the conference and in other contexts.[18]

- Mathematics serves in managing the institution. Purchases of weapons systems are planned, war-games and logistics are calculated.[19]
- Weapons and weapons systems are optimized and their efficiency during action enhanced. This regards munitions (including missiles and bombs provided with guidance systems); delivery systems (including for instance aeroplanes provided with electronic countermeasure circuitry); the reconnaissance, control and communication interface ("to ensure that the right forces are at the right spot at the right moment, and with the right information about the enemy" – Svend Bergstein); and, across all of these, high-speed cryptography.[20] The improvement of data transmission technologies is of general importance for many of these questions, but the creation of data is not only an evident presupposition for having any data to transmit but in itself something which nowadays often asks for the use of even more sophisticated mathematics than the transmission.[21]

[17] See also S. Bergstein, this volume, pp. 183–215.

[18] Evidently it is difficult to find *any* technology that has been created during the last decades which is not somehow driven by mathematics. The list discusses such facets of the matter as go beyond what holds for any practice that involves computers or microelectronics.

[19] See in this volume Kjeldsen (pp. 126–152), S. Clausen (pp. 216–238), and H. Löfstedt (pp. 239–256). Observe that Clausen's title does not speak of the calculation of *war*, as does that of Bergstein, but of the calculation of *warfare* which includes logistics and other managing matters.

[20] Cf. U. Bernhardt and U. Rehmann (this volume, pp. 257–281).

[21] Interestingly, the analysis of the damages of the intestine of a wounded soldier by magnetic resonance imaging (MRI) and the localization of enemy ground forces by synthetic aperture radar (SAR) build on the same mathematics – both, indeed, by cleverly arranged rapid repetition squeeze out of a "short antenna" as much information as could be gained from an extended antenna without advanced mathematics [Schempp 1998: 44 and *passim*].

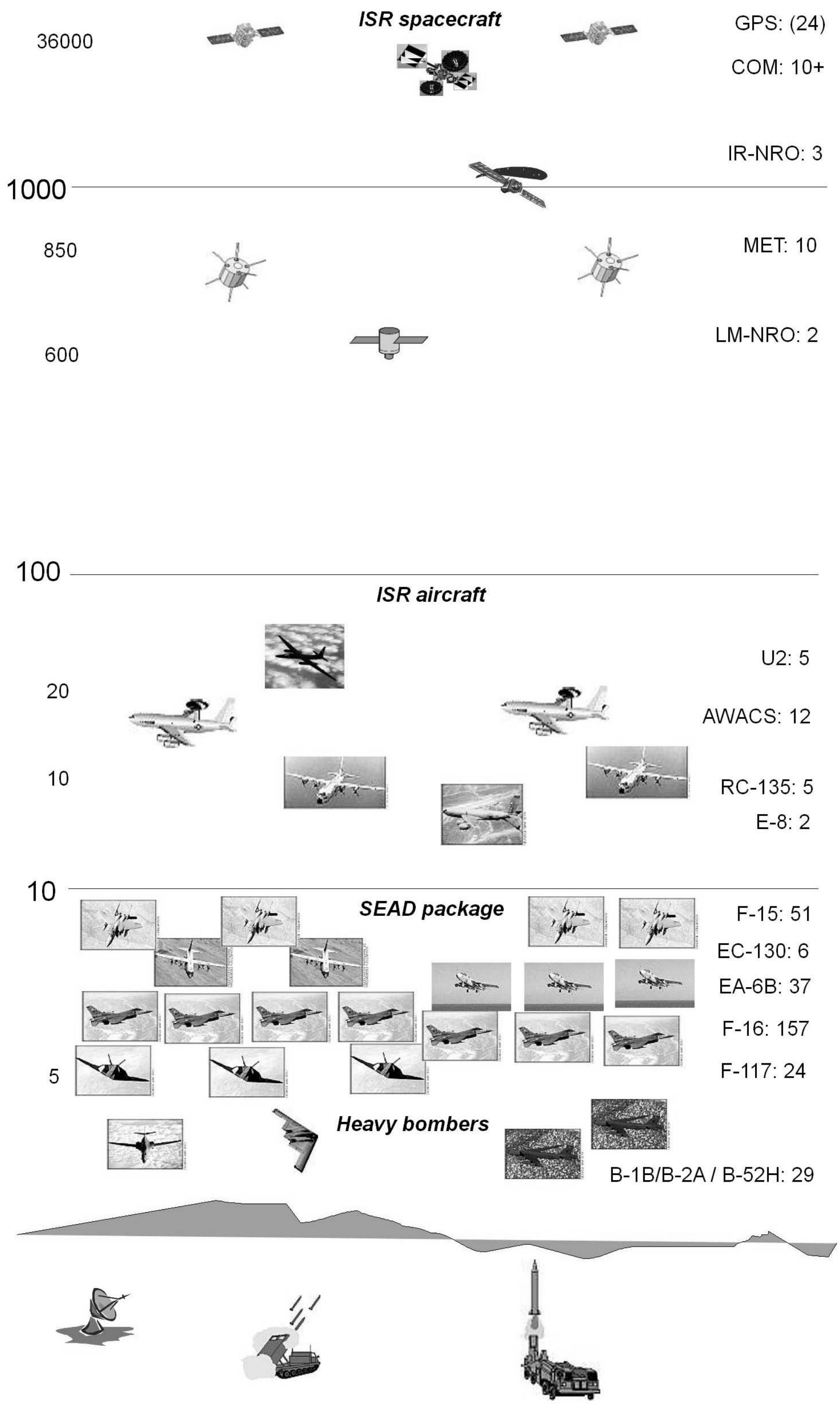
36000
ISR spacecraft
GPS: (24)
COM: 10+
IR-NRO: 3
1000
850
MET: 10
600
LM-NRO: 2
100
ISR aircraft
U2: 5
20
AWACS: 12
10
RC-135: 5
E-8: 2
10
SEAD package
F-15: 51
EC-130: 6
EA-6B: 37
F-16: 157
5
F-117: 24
Heavy bombers
B-1B/B-2A / B-52H: 29

Figure 4 (*opposite*). In World War II, the destruction of a major composite target might ask for the deployment of a thousand bombers. Nowadays a similar task may be effectuated by, say, 29 heavy bombers (lower level of the above schematized front view of an attack – heights are indicated in kilometres). But these have to be supported by another set of 275 fighters and ground attack fighters ("SEAD package") to suppress enemy air defence (bottom). Higher up, 24 "Intelligence-Surveillance-Reconnaissance" (ISR) aircrafts guide the action of the lower levels – and on top, dozens of ISR spacecrafts participate. Much more than informatics is thus involved in the support of the mission itself. So, the dramatic decrease of CEP (see Fig. 5, caption) has to be paid for by dramatic increase of support crafts. However, air raids are still the cheapest way of punitive war (forbidden by international law, but practised), inflicting huge economic losses on the enemy at extremely low operational costs.
[Courtesy: B. Booß-Bavnbek, B. C. Jørgensen, R. O. Rasmussen]

Precision Bombing Acronyms:

AWACS	Airborne warning and control system for air and combat control
B-<>	Long-range bomber with weapon payload of more than 10 tons
COM	Military communication / signals intelligence spacecraft
E-8	Joint surveillance and targeting attack radar system JSTARS
EA-6B	"Prowler" carrier-borne radar jammer
EC-130	"Compass Call" communication jammer
F-<>	Fighter and fighter ground attack aircraft
GPS	Global positioning system navigation satellite
IR-NRO	Infra-red (US) National Reconnaissance Office spacecraft
ISR	Intelligence, surveillance, reconnaissance package
LM-NRO	Imaging radar (US) National Reconnaissance Office spacecraft
MET	Weather satellite
RC-135	"Rivet Joint" signals intelligence gathering aircraft
SEAD	Suppressing enemy air defence package
U-2	Optical spy plane

- Similarly, the strategic planning of the possible use of weapons systems depends on mathematical calculation; even the dismantlement of weapons systems without the risk of destabilizing disequilibrium in the SALT negotiations was analyzed mathematically.[22] Fortunately, nobody implemented the strategy suggested by the naive versions of such planning games – to make a nuclear first strike and promise help to the 80%-annihilated enemy if no counterattack be made, supposing that the enemy would act "rationally" and submit.
- Perhaps unexpected by civilians but emphasized by some military analysts, simple accounting mathematics *performed by mathematically trained independent personnel and not by the active warriors* is mandatory if strategic gains and losses are to be assessed realistically – leading officers, like all of us, are easy victims of self-deceiving optimism and pessimism according to circumstances.[24]
- At the opposite end of the scale, mathematics may also be an indispensable tool. Thus, when the effect of fragmentation bombs on human bodies was to be predicted but humanitarian concerns prohibited testing on pigs, mathematical simulation was put into play.

[22] Cf. J. Scheffran, this volume, pp. 390–412.

[23] This does not disprove the utility of game-theoretical modelling, only the belief that human behaviour is always adequately described by the "rational economic man". Actually, sociobiological models of the same mathematical type indicate that the survival of a species is better guaranteed if egoistic individual sub-optimization is punished. The fear that the enemy might not accept the kind offer but take "irrational" revenge was exactly what made nuclear deterrence work, thus saving our species during the Cold War.

[24] This is the recurrent theme in [Meigs 2002]. Use of the game-theoretical "scenario bundle method" of [Selten 1999] is an advanced analogue.

War	CEP[m]	# bombs
WW II	1100	9140
Korea	330	823
Vietnam	130	128
Gulf	70	38
Kosovo	13	2

Figure 5. The average precision of bombing and firing is commonly characterized by the *Circular Error Probable (CEP)*, that is, the radius of a disc around the goal point within which (on average) 50% of the shots hit while 50% fall outside. (Kolmogorov's approach was more sophisticated). The table shows how CEP has decreased dramatically in aerial bombing over the last 60 years and how the efficiency of a bomber increased correspondingly. The table gives the calculated number of bombs required for "destroying" (i.e., hitting once) a 20 m x 30 m object.

– Ideologically, the waging of war is made more acceptable to the public by the presentation of warfare as precise and hence "more rational and clean". Although that aspect of the matter is not much discussed in the public sphere, this increased precision of weapons (which is real) depends essentially on the application of mathematics. Whereas Hitler preached German invincibility by presenting the Wehrmacht as "Fast as German greyhounds, tough as German *Lederhosen*, hard as Krupp steel", mathematics presents modern warfare as "fast by avionics, precise by GPS, safe by optimized operations planning".
– Similarly, certain mathematical representations of the task to be performed may serve to make the agent see it as a normal manipulation of symbols and thus to eliminate the need for appeals to atavistic instincts – say, seeing a village to be bombed as triangles similar to those of a computer game may facilitate the killing.[25] (Evidently, being at a height of 5 kilometres already has much the same effect).

Utility is one thing, possible backfiring that should be taken into account is another. Firstly, seeing war as "more rational and clean" may deceive (and often appears to deceive) not only the public but also political planners. This is not only devastating for the victims but already pernicious for the planners themselves who may engage recklessly their armed forces in operations and wars that are less easily won than predicted by the machine-rational perception of the character of war.

Less dangerous for planners but just as much for victims is the relative inexpensiveness of present-day mathematically supported asymmetric warfare for the attackers – if the subjugation of Serbia in the Kosovo war cost the subjugator only 7 billion US $, that is, 700 $ per Yugoslav capita, the temptation is great to solve all similar problems in a similar way.[26] (In the moment such a war turns out as things develop to involve the use of ground forces, costs of course explode, and we are brought back to the situation discussed in the previous paragraph).

[25] This issue is touched on by Davis (this volume, pp. 174–179) and by Bernhardt and Rehmann (this volume, pp. 257–281).

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Figure 6. The destruction of the bridges across the important international waterway Danube, some of them in the North of Yugoslavia and hence far away from Kosovo where the Yugoslav (here the Varadin Bridge in Novi Sad) military operational capability should be hindered, was unlawful by The Geneva Protocol I. The counter-argument given is that this kind of warfare is, after all, cost-efficient in human lives, even for the target population – as illustrated by the undamaged blocks of flats standing near to the crushed bridge. The as yet mysterious health problems of Nato soldiers who participated in the Gulf and Kosovo wars and the dramatic increase in cancer rates in Iraq tell us that other damages that do not show up on photographs may turn up in medical statistics. An even greater cost of this high-precision warfare supported by mathematics is its very introduction of the concept of justified risk-less punitive wars without bloodshed. This creates invincibility illusions, lowers the barrier against war and talks people into accepting war. [Source: *NATO Crimes in Yugoslavia*, Ministry of Foreign Affairs, Belgrade, 1999]

Another feature of the mathematization of warfare, also contributing to the ongoing militarization of our world but not restricted to the field of easy asymmetric wars and updated "gun boat diplomacy" – actually less important there than in symmetric conflict – is the transformation of the "Krupp model" into an "infinite Krupp model". War and prepared war is always between two (possibly more) parts – Clausewitz would speak of a *Zweikampf*, a duel, which has now become a "duel of systems".[27] In the nineteenth century, Friedrich Krupp would first develop nickel-steel armour that could resist existing shells, then chrome-steel shells that could pierce this armour, then high-carbon armour plate that resisted these, then cap-shot shells that could break this plate – and that was the end of it. In the duel between surface-air missiles and aeroplanes, no physical limit prevents an ongoing sophistication and ensuing arms race. Cap-shot shells were and remained extremely expensive; so are stealth bombers and fighters – but such measures as depend solely on sophistication of computer soft- and hardware have neither budgetary nor intellectual definitive bounds. Processes depending on physics and

[26] As emerges from E. Schmähling's contribution (this volume, pp. 282–296), what is really at stake here is first of all increased pressure on civil objects and the functioning of society. At least in the Kosovo War, mobile military targets were mostly not detected, and *a fortiori* not destroyed. Tactical warfare being unsuccesful in spite of massive electronic support, mathematics showed its serviceability in strategic bombing of civil objects (legally definable as terrorism).

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chemistry may have definitive natural boundaries. Those depending solely on mathematics seem to have none. The ensuing virtual absence of limits enhances the stress on both sides, and thus the speed and instability of such a race.

Ethics

Mathematics, according to a familiar view, is a neutral tool. As once formulated by the statistician Jerzy Neyman, "I prove theorems, they are published, and after that I don't know what happens to them".

This is certainly an important feature of the mathematical endeavour, and does not only hold for theorems and theorem production. Also the teaching of mathematics, the production of high-level general mathematical competence in the population, is a precondition not only for the waging of modern war but also for the functioning of our whole technological society (quite apart from the cultural value we suppose it to possess).

But the title "mathematics and war" implies ethical dilemmas. In order to avoid having the ethical discussion end up in non-committal "*I* feel..."/"but *I* feel", we may start by looking at the actual ethical choices of some well-known figures.[28]

– Laurent Schwartz used his high academic prestige to make his resistance to the French and American wars in Algeria and Vietnam more efficacious; he saw no connection between his work in mathematics and his political commitment.[29]
– Niels Bohr, when becoming aware of the German nuclear bomb project, supported the competing Anglo-American project; when discovering the dangers that were to arise from the success of the latter, he issued warnings to responsible politicians (Churchill, Roosevelt) and to the public (the "Open letter") – using his prestige as an originator of the underlying theory and as a collaborator in the project (and arguably overrating the impact his interventions might have).[30]
– Alan Turing, quite sceptical of British society (for political as well as personal reasons), put his outstanding abilities in the service of war with total loyalty when he felt it was needed; unlike Bohr, he did so without ever putting himself into focus.[31]
– Kinnosuke Ogura had been a strong promoter of (Marxist-inspired) democratic modernization of Japan, and had opposed Japanese policies as being parallel to German and Italian Fascism. After the beginning of the aggression against China in 1937, however, patriotism and the prospect of using war as a way to

[27] Cf. Löfstedt, this volume, pp. 239–256.

[28] J. Ryberg (this volume, pp. 352–364) discusses the ethical problem in general terms. The reader may think the following examples into the framework he presents.

[29] See [Schwartz 2001].

[30] See F. Aaserud, this volme, pp. 299–311.

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modernization urged him to play a central role in the organization of Japanese mathematics in the service of the military state. After the war he regretted, without specifying too directly what he had done.[32]

- John von Neumann, like Turing, applied his outstanding abilities in war research. Von Neumann did so both during World War II and in the early Cold War; whereas Turing had been a loyal participant about whose personal attitudes in the matter we know nothing, von Neumann made the creation of the H-bomb a personal project which (well served by Stanislaw Ulam and Edward Teller) he did all he could to promote – his aim being to make possible a preemptive first strike.[33]
- Lev S. Pontryagin gave up an extremely fruitful research line in algebraic topology and created control theory. In hindsight this appears to have been caused by a will to serve his socialist country by solving the problems of guiding intercontinental ballistic missiles – thus making *im*possible the same first strike.[34]
- Decades before, G. H. Hardy had tried to avoid that usefulness of his science which consists in "accentuat[ing] the existing inequalities in the distribution of wealth, or more directly promot[ing] the destruction of human life" by concentrating on supposedly useless number theory. Ironically, he repeated this phrase in 1940, when number theory was about to become a cryptographic resource.[35]
- The radical pacifist Lewis Fry Richardson published his path-breaking *Weather Prediction by Numerical Process* in 1922 after having made sure that no less than 64000 "computers" (human beings furnished with desk calculators) would be needed to predict in one day the weather one day ahead. This he saw as a guarantee that numerical weather prediction could not be put to military use.[36]

To what extent can these serve as exemplars and role models? Firstly, they show that two fundamentally different situations must be distinguished. One is that of Schwartz, Hardy and Richardson: deep scepticism towards their own society, or toward aspects of that society as a warring power. The other is that of the remaining examples: they accepted their own society and its warfare or armament policies, either in general or under actual circumstances – certainly with different degrees of identification.

In the second situation, the ethical dilemmas are few. Obviously, one will see no objections to doing his best. Dilemmas, it is true, are not totally absent: one may still, like von Neumann, give an extra push; one may, like Turing, be fully loyal but leave the political decisions to those who are officially entitled to take them (whether politicians, voting citizens in general, or military men); or one may, like Bohr, use one's particular standing and insight and moderate, warn, or point to alternative options.

[31] See Hodges, this volume, pp. 312–325.
[32] See T. Makino, this volume, pp. 327–336.
[33] See, e.g., [Heims 1980].
[34] See Gamkrelidze, this volume, pp. 160–173.
[35] This repetition is found in [Hardy 1967: 120].

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The situation of the sceptic is less clear-cut. Very few of us are in a situation (the situation, say, of von Neumann and Pontryagin) where nobody else could do what we are doing; these few may influence matters directly by deciding to cooperate or not to cooperate, but they remain exceptions.

Most mathematicians, if they choose not to cooperate with the military in mathematics research and teaching, will have little effect, and little of what most mathematicians do in research as well as teaching is directed toward a specific application. Deciding to abstain from working with a particular discipline because it seems "corrupt" is mostly futile. Giving up mathematics is giving up not only military applications but anything mathematics can be used for – and whatever cultural value we may ascribe to mathematics.

However, the practice of the mathematician consists in more than the abstract production and dissemination of theorems.[37] Any mathematician is in a *particular* situation, and in any particular situation there are specific conditions and a specific room for decisions.[38] One may, for instance, widen one's own insight and global understanding of the role of mathematics, and try to share it with students, colleagues and the public – or one may choose to remain (and leave others) as blind as comfortable. One may be a teacher in one or the other position, teaching within a highly stratified or a more egalitarian education system; one may organize the research of an institution, one may be a prestigious researcher, or one may be the newly appointed young colleague. One may be in the top of the "AMRC chain" (cf. above, p. 7), one may be in its periphery knowing or not knowing to belong there, or perhaps be wholly outside it. One may belong in a national or institutional context where political choices of consequence are currently accepted by colleagues or in one where they are unwelcome and considered bad taste and lead to social isolation.[39] In each situation, the scope of ethical choices is different, and no general ethical rules or advice can be issued.[40] What can be said in general is that the supposed neutrality of mathematics per se does not entail the neutrality of these ethical choices.

[36] See [Ashford 1985].

[37] How much more is illustrated by P. Davis's probing (this volume, pp. 174–179) of the relations between mathematics, entertainment, and war. Not only mathematical algorithms but also the creation of mentalities and ideologies are at stake here.

[38] W. Göhring (this volume, pp. 336–351) tells the story of an ethically conscious ordinary research mathematician in a particular real-world situation involving such specific down-to-earth conditions as labour legislation, institutional obligations, etc. Fukutomi (this volume, pp. 153–159) relates how a handful of famous university mathematicians from different countries were able to organize a protest movement against the Vietnam War spreading over the globe in a few months of 1966.

[39] As Fukutomi points out, the anti-war movement gained exceptional strength among Japanese mathematicians, both compared to mathematicians elsewhere and to scientists from other disciplines in Japan. In contrast, mathematicians of the US were much more cautious and conservative than US scientists in general during the same years – see [Ladd & Lipset 1972].

An Enlightenment Perspective?

The Enlightenment believed that reason *might* serve general progress; Rousseau and Swift pointed out that too often reason is used in the service of purely technical rationality and for purposes of sub-optimization, with morally and physically disfiguring effects. According to Defoe's Robinson Crusoe, "Reason is the Substance and Original of the Mathematicks". Hugo Grotius the founder of international law, when referring to the method of mathematicians as his inspiration, appears to have had the same identification in mind.[41] Where does that leave mathematics with regard to disfigurement and progress today?

Much of what was said above concerning the utility of mathematics for the military rather points to disfigurement. Most alarming of all are probably not the actual uses but the ideological veil of rationality, cleanness and surgical accuracy which is derived from the mathematization of warfare. By generalization one might claim that this applies not only to the military aspects of our modern technical society but to technically rational society as a whole.

However, one of the ways in which mathematics serves the military points in the opposite direction: that sober-minded elimination of self-deceiving optimism and pessimism which can be provided by mathematical reasoning and calculation. Mathematics-based reason at its best *should* allow us also in larger scale to unlearn conventional wisdom, to undermine facile indoctrination, to distinguish the possible from glib promises. It *might* help us, if not to find any absolutely best way – this is too much to expect from reasoned analysis – then at least to evade the worst. If reason is really "the Substance and Original of the Mathematicks", mathematics might serve to make clear to us that war is fundamentally irrational and unreasonable *not only in commonplace ideological generality but in specific detail*. If mathematics is *not* able to do such things, then its presumed cultural value might be nothing but a convenient excuse for ruthless technical sub-optimization.

Admittedly, technical rationality prevails over reason for the moment, both concerning the general political situation and the uses to which mathematics is put. Mathematical theories are ethically neutral, it has been argued. Mathematics *as a general undertaking*, instead, is ethically ambiguous: responsibility, whether they like to remember it or not, remains with its practitioners, disseminators, and users.

[40] A somewhat less abstract discussion of the matter can be found, however, in [Booß & Høyrup 1984].

[41] See I. M. Jarvad, this volume, pp. 367–389. Grotius goes somewhat further and tries to reshape legal thought as a formal science modelled after mathematics, pointed out by him to deal with its objects in abstraction from their concrete physical character. Similarly, Grotius requires that international relations should be governed by general rules that are independent of sympathy, religion and of the character of states. That principle was the basis for the Peace of Westphalia of 1648 and was mostly respected since then among states that recognized each other as "civilized". The UN Charter proclaimed the universality of the principle; as these lines are being read the reader will know whether the principle still exists or has been reduced to a mere phrase.

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