# Mathematics and War

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# Military Work in Mathematics 1914–1945: an Attempt at an International Perspective

**Reinhard Siegmund-Schultze\*** 

After discussing some general methodological points this paper investigates in some detail the use of mathematics (not necessarily confined to academic mathematicians) in military work in the German and American cases, where as of now most knowledge is available. With respect to five other countries (Soviet Union, Great Britain, Italy, France, Japan) the discussion has to be restricted to a rough outline of problems and a collection of the rather scattered sources available. A case study is presented of mathematical war work in ballistics (Wolfgang Haack) which illustrates the complexity of the problems involved and the short distance between basic academic research and applied work for the military. The paper is concluded by appendices (tables) giving condensed information on mathematical war work in the seven countries considered.

# **1** Introduction and General Discussion

Wars are primarily waged with technologies – be it for the destruction of material assets and human beings or for the control and corruption of minds. Wars are, however, not waged by directly using the sciences or the humanities, or mathematics for that matter. These serve "only" – however crucially and however in themselves responsible for the development of new war technologies – as intellectual techniques behind the technologies. Thus the two World Wars of the 20th century can not simply be declared as the "chemists" or the "physicists" wars.<sup>1</sup> This is even more true of an alleged "mathematicians' war". According to Mehrtens,

"a 'mathematical war' ... [is] hard to imagine. Even with computer science becoming central to military technology, mathematics still remains in the background, being 'just a tool'. The history of the relation of mathematics to the military and to war has therefore to be analysed as a mediated connection."<sup>2</sup>

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<sup>&</sup>lt;sup>1</sup> According to chemist J. B. Conant WW II was a "physicists' war" (Gray 1943, p. 25). "By early 1942 OSRD had spent four times as many contract dollars in physics as in chemistry." (Kevles (1987), p. 320). On science in WW II in general see Hartcup (2000).

<sup>&</sup>lt;sup>2</sup> Mehrtens (1996), p. 87.

That there has been so much talk of "scientific war" in the public and in the historical literature may have *partly* to do with ideological needs to interpret war as something of a higher moral order and to conceal its dirty reality. In this context, mentality conflicts between the military and engineers may have played a role. Vannevar Bush, the head of the American War Research Organisation *Office of Scientific Research and Development* (OSRD), described it in his recollections of 1970:

Among older military men the engineer was at first regarded as in all probability a thinly disguised salesman, and hence to be kept at arm's length. [(53)...] So all O.S.R.D. personnel promptly became scientists [...]. Even recently when we sent the first astronauts to the moon, the press hailed it as a great scientific achievement. Of course it was nothing of the sort; it was a marvelously skillful engineering job. Now that there is a National Academy of Engineering, perhaps the title of engineer will recover its just recognition.<sup>3</sup>

But this was, of course, only half of the truth, and the other half is going to be at the centre of this paper: Vannevar Bush, the mathematically knowledgeable engineer, author in 1929 of a book on "Operational Circuit Analysis. With an Appendix by N. Wiener" - was certainly aware of the growing importance of basic science in engineering and, in particular, in war technologies. His invention in the early 1930s of the so-called "differential analyzer" - later an important tool in the war - was based on ideas by British "applied mathematicians" Kelvin and Maxwell of the 19th century, even if in itself it was primarily technology.<sup>4</sup> One could go even further and say that the *actual* role of basic science such as mathematics in war-related technologies is concealed rather than exaggerated in the historiography of technology itself because its general nature tends to escape the reader's understanding and - in so far as it is a mere "application" of known knowledge - it is likely to be considered as less "original" than some physical model such as Ludwig Prandtl's boundary layer theory of 1904.<sup>5</sup> Moreover, after the war (!, not during) scientists themselves are often no longer interested in stressing their connection to warfare and they are often rather occupied with an ideological "whitewashing" of their profession.6

So let us come back to the quote by Mehrtens on the "mathematical war": Nota bene: Mehrtens does not talk about a "mathematicians' war" but about a "mathematical war" rather.

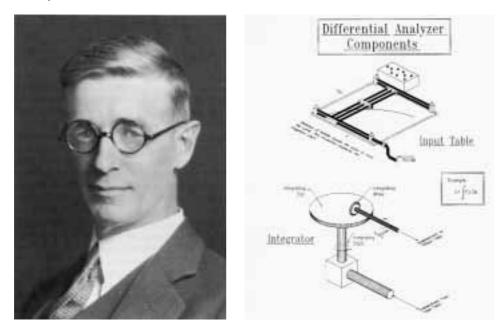
In fact, we have to introduce another distinction: War technologies require – and are promoted by – intellectual techniques. It is rather irrelevant for the

<sup>3</sup> Bush (1970), p. 54.

<sup>4</sup>So argued in Birkhoff (1980), p. 21. See also Bush (1931).

<sup>&</sup>lt;sup>5</sup> I found this confirmed when I tried to evaluate Richard von Mises' importance as an "aerodynamicist". He is often mentioned as such by mathematicians, especially in connection with his theory of the "aerodynamic centre" (1917/20) of an airfoil and his "Mises-transformation" (1927) in boundary layer theory, but particularly less so by engineers, even by mathematically inclined ones such as Theodore von Kármán.

<sup>&</sup>lt;sup>6</sup> As discussed for instance for the British case by Edgerton (1996, p. 20), who also quotes the telling booktitle Beer (1961).



**Figure 1.** *Left:* Vannevar Bush (1890–1974), head of the American War Research Organisation OSRD, leading in the implementation of military funding of pure research [Source: Masani (1990), p. 166] *Right:* A schematic view of his *Differential Analyzer* of 1930, the first analogue computer [Source: Bowles (1996), p. 6]

military who produces and applies these intellectual techniques as long as their existence and application is guaranteed for warfare.

And in fact, we have to ask: Is it really and necessarily the academic mathematicians themselves who are most likely to adapt these intellectual techniques for immediate military use? Are not at least certain social traits required among academic mathematicians or certain social pressure exerted on them before they themselves enter the realm of industrial or military applications? Does a self-serving and self-mobilising<sup>7</sup> "touting" of mathematical theories by mathematicians as "war-relevant" – something which has often occurred in history with or without real basis<sup>8</sup> – already constitute "military work of mathematicians"?

In reality we cannot evade some discussion of the more general type of question such as: "what is mathematics?"; what, in particular, is "applied mathematics?"; who are the people that are doing it?; and what are the historical circumstances necessary for the transfer of that type of intellectual techniques into military technologies?<sup>9</sup>

<sup>7</sup> See the analysis of this notion in Mehrtens (1986/96) and Epple/Remmert (2000).

<sup>&</sup>lt;sup>8</sup> "It is a fact of history that scientists usually have had to take the initiative in making their gifts known to the government." (Gray 1943, p. 41)

<sup>&</sup>lt;sup>9</sup> This discussion somewhat duplicates original historical discussion among mathematicians, physicists, and engineers in various countries, for instance on the topic whether "applied mathematics" is just mathematical physics, or whether it includes engineering mathematics, discussions on the mental divide between mathematicans and engineers such as in Kármán (1940/43) etc.

This is one of the reasons that I decided to gradually extend the topic originally suggested to me, namely "Military Work of mathematicians in World War II", in at least three different directions:

- in the direction of mathematics, not just mathematicians;
- in the direction of considering the whole period which includes the World Wars, because, as it will be seen, the second one is politically, historically, mentally, even scientifically so closely linked to the first, and because the relation between mathematics and the military can only be understood in a broader historical frame;
- in the direction of stressing (if not really discussing) the emerging relations of mathematics to industry exactly in this period between the wars. In particular it would be rather artificial, nearly ridiculous, to separate industrial work of mathematicians during the wars from military work. As one historian put it in connection with his discussion of the militarily and economically highly relevant telephone technologies, which need a lot of mathematics:

In modern societies it makes little sense to distinguish sharply between military and non-military technologies.<sup>10</sup>

Still, some sharp focus shall be given to this paper, and this focus is twofold: it is the role of mathematics in military work and it is an attempt at an international perspective, which has rarely been given in the existing literature.<sup>11</sup>

The article will in particular try to tentatively discuss in the introduction and in several case-studies for the various national environments one central problem: How did the communication channels function between academic mathematics and the application areas, especially the military ones, and to which extent did this depend on the very different social and national preconditions?

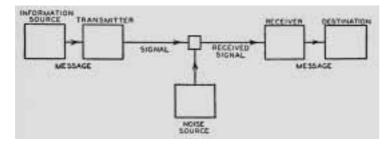
If we first look for a moment at the more "epistemic"<sup>12</sup> side of the problem – and not at the equally important social, political and technological sides – there is no doubt that World War II, in particular, required increasingly – on all sides of the battle line – techniques which resembled those that had been developed within the existing *academic discipline mathematics* at universities world-wide decades or centuries before. It is also clear *which* military technology required the most of mathematics. I only mention catchwords for military technologies such as aeronautics in close connection to ballistics, rockets<sup>13</sup> and the atomic bomb, and – as mathematical techniques – computers, numerical analysis, cryptology, operations

<sup>&</sup>lt;sup>10</sup> Kragh (1996), p. 37. The same point is stressed by mathematician Godement (1978/79), who reserves for mathematics itself only a small part of his study on the American scientific model.

<sup>&</sup>lt;sup>11</sup> Exception is some historical work with international perspective on single cognitive aspects of applied mathematics in war context. In this category belong some publications on the history of computing e.g. Birkhoff (1980), Todd (1990), and Goldstine (1972), as well as on the history of exterior ballistics (McShane et al. (1953)), and aerodynamics (Kármán 1954).

<sup>&</sup>lt;sup>12</sup> In allusion to a recent study (to be discussed below) on Germany, Epple/Remmert (2000), esp. pp. 281ff.

<sup>&</sup>lt;sup>13</sup> For a sketch of the history and mathematics of rockets in various countries (Russia, Germany, U.S.) see Newby (1988).



**Figure 2.** Schematic diagram of a general communication system. [Source: Shannon/Weaver (1949), p. 5]

research, statistical sequential analysis, prediction theory, control theory.<sup>14</sup> It is also clear that some of those military technologies required *new and very modern mathematical tools* which had to be invented even in pure mathematics and were not available in stock, so to speak.<sup>15</sup>

Hodges, the biographer of Alan Turing, stresses one important aspect of these changes:

But rapidly developing, and not only at Bletchley Park and Washington, was a new kind of machinery, a new kind of science, in which it was not the physics and chemistry that mattered, but the logical structure of information, communication, and control.<sup>16</sup>

There were, however, both similar and different "epistemic preconditions" for the military application of mathematics in various countries. Internationally minded mathematicians such as American Norbert Wiener from the MIT in Boston realised this. With respect to his work on anti-aircraft fire-control predictors, strongly inspired by pure mathematical research in statistics and what later would become information and communication theories, Wiener wrote retrospectively in his autobiography of 1956:

At an early stage of my work for the United States military authorities ... the question came up whether anybody abroad was likely to be in possession of ideas similar to mine. I said that they would unquestionably receive no particularly ready reception in Germany; that my own friends Cramer in Sweden, and Lévy, France, might well have been thinking along similar lines, but that if anyone in the world were working on these ideas it would most likely be Kolmogoroff in Russia. This I said because of my knowledge that for twenty or thirty years hardly had either of us ever published a paper on any subject but the other was ready to publish a closely related paper on the same theme.<sup>17</sup>

<sup>&</sup>lt;sup>14</sup> See the contribution by R. V. Gamkrelidze, one of the creators of the theory as an after-war development, in this volume.

<sup>&</sup>lt;sup>15</sup> Rosser's (1982) contention that "except in cryptanalysis, hardly any of the mathematics done for the War effort was of a higher level" (509) seems exaggerated.

<sup>&</sup>lt;sup>16</sup> Hodges (1983), p. 252.

<sup>&</sup>lt;sup>17</sup> Wiener (1956), p. 261/2. On Wiener see Masani (1990).

Wiener refers in his quote to lacking epistemic preconditions in his field in Germany, and to existing ones in the United States, France, Sweden, and the Soviet Union. But what does this mean for the actual mathematical military work done in the countries considered? Not much, because the respective social and political conditions, not discussed in Wiener's quote, were so different:

Failing theoretical preparations and missing indigenous traditions such as in *Germany* would not necessarily have ruled out war work in this field although those conditions have, in fact, some explanatory potential. That indeed work of this kind was apparently *not* done in Germany in the particular field of prediction theory<sup>18</sup> may also be explained by growing problems of international mathematical communication during the war. *France* under German occupation was no likely place for mathematical war research on a large scale, nor was it neutral *Sweden*. Finally: the actual role and full implication of A. N. Kolmogorov's mathematical work in the *Soviet Union's* war effort is not clear until this day.<sup>19</sup> To have a good feeling for applications among pure mathematicians and to have a man with superior physical insight such as Kolmogorov does not guarantee a solution of the precarious problem of transfer from research to development, from mathematical invention to military innovation.<sup>20</sup>

The other side of the coin is this: during the war(s) a lot of at least potentially applicable theoretical work was done in various countries – whether they were involved in the war effort or not – that escaped attention of men such as Norbert Wiener abroad and was likewise not noticed due to the communication blackout during much of the war(s) and even later in the Cold War. Mathematical work or mathematics-related engineering work that was potentially war-important, such as done in France by E. Borel on game theory and émigrés W. Döblin and F. Pollaczek on Markov chains and queuing problems or in Germany by K. Zuse on digital computers, was not, for various historical reasons, actually applied (= transferred into the war effort) and therefore partly or temporarily ignored in the countries that would write the history of the war and set the norms for the scientific enterprise after 1945, especially the United States.<sup>21</sup> Take the following quote by Garrett Birkhoff, who had turned from abstract algebra to aerodynamics during the American war effort:

After the defeat of Hitler, the U.S. (along with Canada) was standing triumphant, virtually unscathed in a world in which most advanced countries were prostrate and in ashes. Its scientific preeminence was taken for granted, and most Americans thought of computing machines as almost a national monopoly. They had largely forgotten the tradition of European scientific superiority that had been generally accepted only a decade earlier. What is now known about ENIGMA and the Zuse machines should help to dispel this illusion.<sup>22</sup>

<sup>&</sup>lt;sup>18</sup> The report by R. Hosemann (1948) on "Verfolgungskurven" (follower curves) is rather pure elementary geometry and does only marginally touch upon probability questions.

<sup>&</sup>lt;sup>19</sup> But see the remarks by A. N. Shiryaev in this volume.

<sup>&</sup>lt;sup>20</sup> This problem was discussed for aeronautics in the Soviet Union in Bailes (1978).

<sup>&</sup>lt;sup>21</sup> See Godement (1978/79) and Lax (1989).

<sup>&</sup>lt;sup>22</sup> Garrett Birkhoff (1980), p. 29.

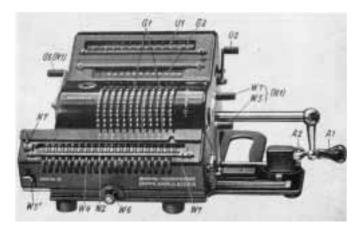


Figure 3. Mechanical computing machine widely used in the pre-computer age, including parts of WW II. [Source: Meyer zur Capellen (1949), p. 60]

As to the conditions that Wiener himself faced with respect to the application of his mathematical ideas in the United States of the 1920s and 1930s, one should not forget the following: the U.S., so rightly considered the leading country in both pure and applied mathematics after World War II, was rather backward with respect to "academic applied mathematics" in the European sense before the immigration of the 1930s, as will be argued in some more detail below.<sup>23</sup> This is what the same Garrett Birkhoff had to say on a period in the 1920s, when his father George David Birkhoff reigned supreme in American mathematics:

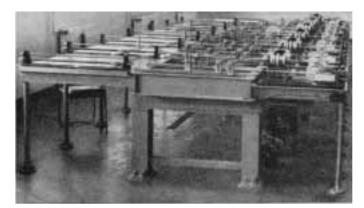
Wiener was also notable as one of the few Americans of his time who was outstanding in both pure mathematics and his (sic!) applications. How much of this can be attributed to his varied and cosmopolitan early background, and how much to his continuing contacts with non-mathematicians such as G. I. Taylor, it is hard to say.<sup>24</sup>

Birkhoff hints at the important role of international communication as a historical precondition of applied mathematics in academic America. It has been mentioned before that, beginning in 1920s, Wiener collaborated closely with the engineer Vannevar Bush at MIT. Bush's analogue computer *Differential Analyzer* was not only central in applied ballistics during WW II,<sup>25</sup> its emulation in Europe<sup>26</sup> promoted applied research elsewhere in the world. Not least, it served to stimulate crucial developments also with digital computers and in information theory (C. Shannon worked with it). Nevertheless, even Norbert Wiener, a pioneer in international communication<sup>27</sup> and in applications, would have his shortcomings

- <sup>25</sup> On the use of the *differential analyzer* in dealing with the many variables in ballistical calculations, for example in order to take into account the Magnus effect see Gray (1943), p. 90. S. also Godement (1979), vol. 204, p. 94.
- <sup>26</sup>Oslo (S. Rosseland), England (D. R. Hartree), partly Germany (W. Cauer, A. Walther).
- <sup>27</sup> As a Guggenheim fellow in Göttingen for example.

<sup>&</sup>lt;sup>23</sup> See Siegmund-Schultze (2003) and some further remarks below.

<sup>&</sup>lt;sup>24</sup> Birkhoff (1977), p. 63.



## Figure 4.

Svein Rosseland's analyzer in Oslo (1937), influenced by V. Bush's machine and envied by the Germans. [Source: Willers (1943), p. 234]







**Figure 5** (above left). Warren Weaver (1894–1978) – head of the American *Applied Mathematics Panel* (APM), founded 1942. [Source: Reid (1976), appendix]

## Figure 6 (above right).

Right: Norbert Wiener (1894–1964) in mathematical war service during World War I. [Source: Masani (1990), p. 69]

## Figure 7 (left).

Chief mathematician Thornton Fry (1892–1991) of Bell Labs. [Source: *American Mathematical Monthly* **89** (1982), p. 80]

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later on as to his potential use in the American war effort. Not only was he ideologically hardly prepared for an undisputed call to arms, given his pacifist views, which even increased after WW II and the atomic bomb, Wiener was also clumsy both socially and manually, traits he obviously shared with Alan Turing in Great Britain, another important contributor to war research. In fact, Warren Weaver of the American *Applied Mathematics Panel* had considerable problems integrating Wiener into his team-work oriented effort.<sup>28</sup>

So "attitudes" of the mathematicians involved and of other "appliers"<sup>29</sup> of mathematics mattered heavily in the process of application of mathematics, especially in times of war, as one would expect, where militaristic or other emotional positions had their motivational part to play. It is no coincidence that many historians and people involved in the processes themselves have stressed that "applied mathematics" is primarily a social distinction, a "social contract"<sup>30</sup> even. Thornton Fry said in 1953 at a conference in New York City devoted to training in applied mathematics, basing much of his judgement on the involvement of *Bell Labs* in the American war effort:

The difference between an applied mathematician and a pure mathematician is not the kind of mathematics he knows, it isn't even whether he can create epoch-making new ideas, or like most of us his ability lies principally in interpreting things that are already known. The distinction resides instead in the nature of his interests; in his attitudes, not in his aptitudes. It is almost a social distinction.<sup>31</sup>

But attitudes did *not* influence the process in a very simple or unequivocal way, as Wiener's example shows: the social and political environment matters just as heavily.

As late as 1941 Thornton Fry, then chief mathematician at the *Bell Laboratories*, stated for American industry a "contrast between the ubiquity of mathematics and the fewness of the mathematicians".<sup>32</sup> This he did in a widely-read report for the U.S. government on "Industrial Mathematics". The fact that in the years before World War II, applications of mathematics had not been as a rule the job of academically trained mathematicians, was, however, not restricted just to the United States. Let me give two additional historical examples:

Michael Fortun and Sylvan S. Schweber discussed the origins of *Operations Research* and *Systems Engineering* in Great Britain and the United States and their military use in World War II. They stressed the importance of mathematical techniques from game theory or statistics being used in *OR*<sup>33</sup> but concluded with respect to the people who were really doing the job:

<sup>&</sup>lt;sup>28</sup> See the discussion in Heims (1980) and Owens (1989).

<sup>&</sup>lt;sup>29</sup> This English word – herewith "created" – might be more appropriate than the traditional attribute "applied" in connection to scientists or mathematicians.

<sup>&</sup>lt;sup>30</sup> Davis (1988).

<sup>&</sup>lt;sup>31</sup> Fry (1953), p. 96.

<sup>&</sup>lt;sup>32</sup> Fry (1941), p. 255.

<sup>&</sup>lt;sup>33</sup> Fortun, M. and S. S. Schweber (1993), p. 338.

Incidentally, mathematicians did not do well as operational analysts during WW II, presumably because they were not used to being parts of 'teams'.<sup>34</sup>

That the problem did not just lie on the subjective side of mathematicians "attitudes" is underlined by Mehrtens, who reports on the job situation for mathematicians coming from universities in the late 1930s in Germany:

Very few mathematicians found jobs in industry, and the few who did were usually hired in lieu of some other sort of specialist. [Mathematician] Wilhelm Magnus, for example, reports that he was employed as a physicist in electrical industry; to employ a mathematician would have appeared absurd to his boss. But with the preparation for war, industrial work intensified and graduated engineers and physicists were rare; thus mathematicians could slip in.<sup>35</sup>

Now if mathematics could frequently be applied without professional mathematicians being present this was not necessarily always the case, especially not in the more modern fields and under the pressing needs of the war, as the British enterprise at Bletchley Park showed:

The enigma machine was the central problem that confronted British Intelligence in 1938. But they believed it was unsolvable [...] In particular this department of classicists, [...] did not include a mathematician.<sup>36</sup>

The exigencies of the war finally showed the need for academically trained mathematicians in the last-mentioned case of cryptology as well as in the United States and other places.

More generally the War seems to have triggered in several countries a new step in the professionalisation of mathematics at large, the systematic training of industrial mathematicians being part of this process. This applies also to Germany where the authorities declared the creation during the war of a new type of mathematician, the "Diplommathematiker", which had been much demanded by the German mathematicians themselves.<sup>37</sup> In the United States, however, there seems to have been *missing one more link* in the chain of epistemic-societal preconditions for the application of mathematics in the war.

In fact, the difference between the United States and Germany was that there did not even exist in America around 1940 a considerable supply of academically trained mathematicians with interest or understanding for the needs of the engineers. By comparison the Germans and other European countries could draw on a long tradition of mathematical training of engineers at Technical Universities

<sup>&</sup>lt;sup>34</sup> Ibid. p. 350. A closer look at ASWORG, the American Antisubmarine Warfare Operations Research Group under the lead of physicist Philip Morse, where half of the collaborators were mathematicians, may somewhat modify this picture. See T. Hoff Kjeldsen (2000) and her contribution to this volume. See also McArthur (1990), Meigs (1990), and Rider (1992).

<sup>&</sup>lt;sup>35</sup> Mehrtens (1996), p. 105.

<sup>&</sup>lt;sup>36</sup> Hodges (1983), p. 148. As to the mathematical nature of cryptology see also Levinson (1970).

<sup>&</sup>lt;sup>37</sup> Mehrtens (1996).

– Theodore von Kármán and Richard von Mises bearing witness to that – and even of institutionalised training in applied mathematics at some universities like Göttingen and Berlin.

The deficiencies of the Americans in this respect even around 1940 were clearly stated in articles by Kármán (1943) and by AMS-president Richardson (1943); the latter saw a close connection between the insufficient state of "Applied Mathematics and the Present Crisis" and promoted his famous summer-schools under the title "Program of Advanced Instruction and Research in Mechanics" which was clearly oriented towards training in applied mathematics and took place beginning in 1941.

The role of the war as a catalyst of the development of *academic* applied mathematics in the United States was stressed after the war by Joachim F. Weyl, son of the famous German émigré Hermann Weyl, who organized a Conference on "Training in Applied Mathematics" at Columbia University, New York, in 1953:

Without the demands, resulting from considerations of national security, applied mathematics in this country might be as dead as a doornail.<sup>39</sup>

Again, what Weyl meant was not just "applications of mathematics in industry" but rather developing germs of institutionalised applied mathematics at American universities – a development which was heavily funded even after the war by state agencies such as the military *Office of Naval Research*. The Second World War, so it seems, had finally led to a transfer of the originally French,<sup>40</sup> later German<sup>41</sup> role model of academically institutionalised applied mathematics to the United States. The pressure of the emergency situation in the U.S. around 1941 was doubtless dominant in this process, but émigrés to the United States such as Richard Courant contributed much to a spread both of this social ideal and of the cognitive mathematical prerequisites<sup>42</sup> for military work of academic mathematicians in WW II.

All examples brought forward so far point to the complicated structure and many-sidedness of epistemic, technical and social prerequisites for the use of

<sup>&</sup>lt;sup>38</sup> See the publication under the same title as no.8 of the Bulletin of Brown University 40 (1943). In 1943 the journal Quarterly of Applied Mathematics was launched in close connection with Richardson's program. The maintenance of the name "mechanics" in the program served obviously to stress the area to which the mathematics was being applied.

<sup>&</sup>lt;sup>39</sup> Weyl (1954), p. 22. Weyl was at that time "Director of Mathematical Sciences Division in the Office of Naval Research". The ONR was the leading state agency in the U.S. after WW II to support both applied and pure mathematics mostly on the grounds that it was defence-related.

<sup>&</sup>lt;sup>40</sup> Ecole Polytechnique of 1794.

<sup>&</sup>lt;sup>41</sup> Göttingen with Felix Klein and Carl Runge.

<sup>&</sup>lt;sup>42</sup> In this latter respect one cannot overestimate the role of mathematical work such as Courant/ Friedrichs/Lewy (1928), Courant/Hilbert (1924/37), and Courant/Friedrichs (1948). A detailed historical analysis of this "epistemic side" of the reception of German mathematical work in the U.S. is given in an unpublished work by this author, of which a summary is Siegmund-Schultze (2003). Some analysis also can be found in Goldstine (1972).

mathematics and mathematicians in the war. What was needed for a "successful application" of mathematics in the war was obviously on the *one hand* a bridging of all the mental divides and differences in attitude of the many people involved in a large scale effort; among them were engineers, military men, scientists, government administrators, coming from very different social groups of society.

On the other hand, and closely related to the more personal side, military application of mathematics required "continuity" also on the "level of things", be it theories or material artifacts and institutions. This was the continuity (or continuum) of mediation between the "epistemic side", the "heights" of theoretical mathematics, stretching over the technical application, the experiments in wind tunnels<sup>43</sup> and the like all along the way until the actual production of weapons. Of course, a country had to be strong enough economically and militarily to secure the continuity of that mediation or otherwise it had to find appropriate allies.<sup>44</sup> This included the above mentioned "transfer problem" of science and technology, which was important both for economic and military applications but had special traits in times of war. It was not necessarily so that the "transfer problem" was easier to solve in times of war, given the "paradox in technics" as described by sociologist Lewis Mumford, namely that "war stimulates invention, but the army resists it.<sup>245</sup> But at least the necessary money for the applications was not the main concern when they gained such a political priority in the war. In order to consider the similarities and dissimilarities in the realisation of the mentioned epistemictechnical, mental and institutional-material continuities in different social surroundings, case studies for different countries are warranted.

# 2 Two Case Studies on Germany and the U.S. and Short Remarks on Mathematical War Work in Five Other Countries

If one looks at the different countries and their organisation of military research between 1914 and 1945 one finds – as one would expect – similarities and differences. What was similar was the feeling of growing competition between the nations, in its sharpest and most articulated form, of course, as military threat, and similar measures for research were taken in different countries at comparable points of time. In 1923, the president of the newly founded *International Educa*-

<sup>&</sup>lt;sup>43</sup> Much on wind tunnel construction in various countries since about 1900 can be found in Anderson (1997), pp. 296–304, and in Lee (1998).

<sup>&</sup>lt;sup>44</sup> This applies to Poland as an ally to Britain in decoding (s. Hodges (1983) and the contribution by E. Rakus-Andersson in this volume), Japan as an ally to Germany in jet propulsion. Britain (unlike the U.S.) was unable to maintain full computer development on the technical scale, with resulting resentments on the part of the theoreticians such as Turing as described in Hodges (1983).

<sup>&</sup>lt;sup>45</sup> Quoted from Gray (1943), p. 41.

*tion Board* of the Rockefeller Philanthropy, Wickliffe Rose, put this very clearly the following way:

The nations that do not cultivate the sciences cannot hope to hold their own; must take an increasingly subordinate place; [...] and must in the end be dominated by the more progressive states even though these states do not seek to dominate.<sup>46</sup>

This general situation of competition had found its sharpest expression in the First World War. Germany had been – according to leftist British scientist John Desmond Bernal – more successful in the beginning of the war because its industries were based on science. Bernal, in his well-known book of 1939 *The Social Function of Science*, continued that

the War, and only the War, could bring home to Governments the critical importance of scientific research in the modern economy. This was recognised in Britain by the formation of the Department of Scientific and Industrial Research.<sup>47</sup>

Not surprisingly I will begin my short case studies on the relation of mathematics and the military with the cases of Germany and the United States, the main opponents in the scientific Second World War. Considerably more uncertainty remains especially about the organisation of scientific war work in the Soviet Union, to be tentatively discussed later. In any case the available primary and secondary (historiographic) literature is most extensive with respect to the German and American environments.

## Germany<sup>48</sup>

Broad in its approach as to the time frame and problems discussed – and therefore very much in the direction of the present paper – is Mehrtens' contribution of 1996 which is partly based on an earlier German publication by the same author of 1986.

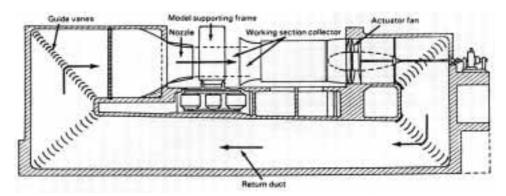
Felix Klein's "small but paradigmatic techno-scientific complex" in Göttingen is discussed, in particular the foundation in 1917 of the *Aerodynamische Versuchs-anstalt* (Aerodynamic Proving Ground) AVA in connection with the scientific *Kaiser-Wilhelm-Gesellschaft* but mainly financed by the military. The AVA, together with the more theoretical Institute for Fluid Mechanics (1925) under the leadership of Ludwig Prandtl, became "the most advanced aerodynamic research installation in the world".<sup>49</sup> Aeronautical Research in Germany was "the main

<sup>&</sup>lt;sup>46</sup>As quoted in Siegmund-Schultze (2001), p. 26.

<sup>&</sup>lt;sup>47</sup> As quoted in Edgerton (1996), p. 12. For science in WW I see also Hartcup (1988).

<sup>&</sup>lt;sup>48</sup> See also appendix 5.1 and the case study on Wolfgang Haack's work in ballistics below. The most relevant secondary literature for the German case is Mehrtens (1986/96) and Epple/ Remmert (2000).

<sup>&</sup>lt;sup>49</sup> Mehrtens (1996), p. 97.



**Figure 8.** Ludwig Prandtl's wind tunnel at Göttingen, a closed-circuit wind tunnel (1916). [Source: Anderson (1997), p. 300]

consumer of applied mathematics, and especially for complex calculations"<sup>50</sup> and stood for the continuity of military mathematical research from World War I to World War II, especially after the great upswing of the Göttingen, Braunschweig, and Berlin facilities in 1933, when the Nazis came to power. The ideological continuity between the World Wars as to the preparedness of German mathematicians and engineers to engage in military research has also been investigated.<sup>51</sup> In Mehrtens and other publications<sup>52</sup> the German tradition of strong mathematical education of engineers at Technical Universities is stressed as a central prerequisite for the application of mathematics in military and industrial contexts. Little has been published, however, on industrial mathematical research,<sup>53</sup> in particular at the various airplane production plants; some information on respective work at the Siemens & Halske laboratories can be found in (Kragh 1996).

For World War II Mehrtens points out the "poorly coordinated mathematical war-work" in Germany, especially deficiencies – in international comparison – in computers and cryptology, but also, that by 1944 only very few mathematicians were left who were not involved in military work.<sup>54</sup> Things seem to have worked more smoothly in more traditional fields of applied mathematics especially in aerodynamics, which was strongly supported by the regime.<sup>55</sup>

<sup>50</sup> Ibid. p. 105.

- <sup>51</sup> For example Siegmund-Schultze (1986) and (1990).
- <sup>52</sup>Ludwig (1974), Hensel/Ihmig/Otte (1989).
- <sup>53</sup> See Gross (1978).
- <sup>54</sup> Mehrtens (1996), pp. 111/12.

<sup>&</sup>lt;sup>55</sup> In a well-known American report after WW II the following is stated: "In contrast to the Army research it must be said that the German Air Force research under Goering was excellently organized and highly successful." (Goudsmit (1947), p. 147) But note that even Goudsmit acknowledged already the effectiveness of rocket research under army command in Peenemünde (ibid., p. 145/46), which relied heavily on computations from A. Walther's "Institute for Practical Mathematics" in Darmstadt.

A recent German study by Epple and Remmert (2000) presents, for the first time, the structures of the organisation of mathematical research in Germany during World War II. It tries to give a definition of "mathematical war research" and finds two criteria crucial:

1. "Mathematical war research" provides means for the realisation of technical projects which are of immediate or mediate term military interest.

2. There exists an organisational framework which confirms the relevance of that work for the war.

The authors declare it irrelevant for a mathematical work to be called "mathematical war research" whether the mathematician himself is aware of its importance for the war or not.<sup>56</sup>

Epple/Remmert then give historical details on four major streams or strings (Stränge) along which "mathematical war research" was organised in Germany during WW II. They locate first governmental-military institutes,<sup>57</sup> second indus-trial-military institutes, thirdly research at universities (including technical ones) commissioned by the first two agencies, and, finally, so-called "self-mobilised" mathematical war research at these universities.

Especially in connection with "self-mobilised" research the study confirms the results already found by Mehrtens (1986/96) that the German mathematicians used the pretext of the war for pursuing plans for the erection of a central German mathematical research institute, partly in competition with American (Fry) and Italian (Picone) examples. When the German institute finally came into existence in Oberwolfach in the Black Forest in the last months of the war – the late date being partly a result of the competence struggles in the Nazi hierarchies – it was no longer able to conduct effective war research and served merely as a survival base for German mathematics, including the very pure brand.

Such a listing of war research institutions as in Epple/Remmert is useful, but even more so the theoretical discussion which follows in the article under the title "Epistemic continuities. The example of conformal mappings".<sup>58</sup> In their discussion of the application of conformal mappings in airfoil design the authors show that the code-word "conformal mapping" was used and propagandistically exploited in many "applied" and "pure" mathematical contexts which reached from very simple graphical methods over traditional "pure" complex function theory (L. Bieberbach, P. Koebe) of recently detected importance for applications to sophisticated analytical investigations of the kind which Oswald Teichmüller<sup>59</sup>

<sup>&</sup>lt;sup>56</sup> Epple/Remmert (2000), p. 266.

<sup>&</sup>lt;sup>57</sup> Among them the most important aerodynamical research institutions in Göttingen, Berlin and, somewhat later Braunschweig, and Alwin Walther's "Institut für Praktische Mathematik" in Darmstadt. Some institutions, such as the Berlin-Adlershof DVL, seem somewhat undervalued in Epple/Remmert.

<sup>&</sup>lt;sup>58</sup> Epple/Remmert (2000), pp. 281ff.

<sup>&</sup>lt;sup>59</sup> The fanatic Nazi Teichmüller could obviously find no fulfillment of his ideological aspirations in his pure mathematical work. He did not see the chain (continuity) till applications, he volunteered for the army and disappeared at the Russian front in 1943. See Schappacher/Scholz (1992).

preferred and performed. This discussion in Epple/Remmert of an example for the "epistemic continuities" is used as another<sup>60</sup> argument for the main thesis which is being brought forward in this study:

During the war a continuous spectrum of mathematical research activities emerged, which reached from technologically immediately applicable and war-relevant work till domains of so-called "pure mathematics", in which the practical importance of the work on the other side of the spectrum was not easily visible.<sup>61</sup>

## USA

On military research in and with mathematics in the US in WW II, there exists a quite extensive literature both of primary and secondary nature<sup>62</sup>, where Goldstine (1972), Rees (1980), and Owens (1989) stand out as to importance. Using a "mathematical approach" to war research in a rather broad and general sense, MIT physicist Philip Morse and his *Antisubmarine Warfare Operational Research Group* (ASWORG) since 1942 were crucial in the American war effort:

In Boston, the operations research scientists showed mathematically that, whatever the tactics employed, the chances of finding a submarine were considerably greater with radar-equipped aircraft than with destroyers.<sup>63</sup>

The cooperation of the U.S. military with scientists in this field of war research has been repeatedly well described.<sup>64</sup> New sophisticated mathematical and statistical methods, such as in linear and non-linear programming, originated over time from this endeavour but were mostly an after-war development.<sup>65</sup>

Aerodynamic research before and during WW II, which had intimate connections to applied mathematics, has been exemplarily documented in publications by the American space agency *NASA*.<sup>66</sup> The lack of academically trained applied mathematicians in the U.S. – unlike Germany at that time – has been mentioned before. Exceptions were Americans N. Wiener, H. Bateman, and F. Murnaghan, the latter two with partly foreign background. Early immigrants, such as Rus-

<sup>&</sup>lt;sup>60</sup> The other one being the continuous spectrum of institutions for mathematical war research as discussed before by the authors.

<sup>&</sup>lt;sup>61</sup> Epple/Remmert (2000), p. 265.

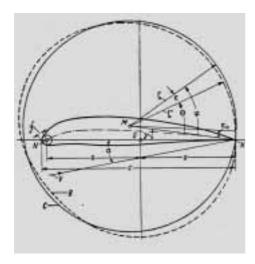
<sup>&</sup>lt;sup>62</sup> See also appendix 5.2, especially for the relevant literature.

<sup>&</sup>lt;sup>63</sup> Kevles (1987), p. 310.

<sup>&</sup>lt;sup>64</sup> McArthur (1990), Meigs (1990), Rider (1992), Fortun/Schweber (1993), and Hoff Kjeldsen (2000).

<sup>&</sup>lt;sup>65</sup> See Hoff Kjeldsen (2000), who shows an impact of the Air Force Programming problem since 1941. See also her contribution to this volume.

<sup>&</sup>lt;sup>66</sup> See Hansen (1987), Roland (1985), and Bilstein (1989), the latter two, together with many original research reports from the 1930s (e.g., by Th. Theodorsen), also being available on the internet.



**Figure 9.** Using complex function theory in airfoil design. [Source: Riegels (1961), p. 83]

sian engineers St. Timozenko and S. Lefschetz,<sup>67</sup> aerodynamicists Th. Kármán (Hungary/Germany) and (as already partly American-educated) Th. Theodorsen (Norway) were particularly notable. But the Nazi-caused immigration after 1933 was even more important. Examples are Abraham Wald (statistical sequential analysis), John von Neumann (computers and non-linear hyperbolic equations), Richard Courant and Kurt Friedrichs (work on shock waves).

As to indigenous American traditions the application and creation of mathematics at the *Bell Laboratories* has to be mentioned particularly, since it had clear war relevance especially in statistical prediction and control theory.<sup>69</sup> Bell Labs was "the world's largest and richest institution for industrial research."<sup>69</sup> Mathematically educated engineers such as G. A. Campbell, J. R. Carson, W. A. Shewhart, and Th. Fry,<sup>70</sup> pioneered at Bell Labs in the application of complex functions, operational calculus, probability, and in work on the foundations of communication. A mathematical research division existed at Bell Labs from 1925, led by Thornton Fry and later H. W. Bode.<sup>71</sup>

Probably the most spectacular development in communications mathematics to take place at Bell Laboratories was the formulation in the 1940s of information theory by C. E. Shannon.<sup>72</sup>

<sup>&</sup>lt;sup>67</sup> Originally an engineer, Lefschetz – after a long excursion to topology – would only return to more applied fields after WW II.

<sup>&</sup>lt;sup>68</sup> Millman (1984), pp. 43ff.

<sup>&</sup>lt;sup>69</sup> Kragh (1996), p. 41.

<sup>&</sup>lt;sup>70</sup> See Fry (1928) and Shewhart (1931). On early statistical control methods being applied in the U.S. see Bayart/Crépel (1994).

<sup>&</sup>lt;sup>71</sup> Bode (1953).

<sup>&</sup>lt;sup>72</sup> Millman (1984), p. 46.



Figure 10. Claude Shannon (1916–2001) – the founder of information theory in the broader context of war work. [Source: Masani (1990), p. 155]

The relevance for the military of this research was obvious; cooperation with MIT's department for electrical engineering was tight.<sup>73</sup>

As to state-funded military work in mathematics, the continuities from World War I to World War II are most palpable in the histories both of the *Aberdeen Proving Ground* in Ballistics and of the NACA-related aeronautical research facilities.

During World War I systematic ballistic research had been done in the US in Washington under astronomer Forest Ray Moulton and at the newly founded *Aberdeen Proving Ground* in Maryland under geometer Oswald Veblen. The introduction of finite difference methods in ballistic computation by Moulton and of higher methods of the calculus of variations by G. A. Bliss in Aberdeen, which lay "on the farther boundary of the explored mathematical domain of today"<sup>74</sup> has been described by Goldstine.<sup>75</sup>

The same author describes the continuation of ballistic research on the eve of World War II, when in 1938 a *Ballistic Research Laboratory* was erected at Aberdeen.<sup>76</sup> This laboratory, to which, once again, Veblen and Bliss were attached, would have a decisive catalytic role to play in the development of scientific computing in the U.S. Since 1935 a copy of Bush's differential analyzer was used to

<sup>&</sup>lt;sup>73</sup> The close contacts between Bell and MIT were personally realized in the collaboration of Wiener and Bush, of Struik and Fry. On the electronic industry in the U.S. and the influence of the War see esp. Godement (1979), vol. 203, pp. 97ff.

<sup>&</sup>lt;sup>74</sup> Bliss (1927), 313, on his use of "line functions" in ballistics war work.

 <sup>&</sup>lt;sup>75</sup> Goldstine (1972), pp. 75ff. See also Dickson (1918), Moulton (1926), and Bliss (1927).
 <sup>76</sup> Goldstine (1972), pp. 123ff.



Figure 11. Howard Aiken (1900-1973), middle, in front of the Harvard Mark I calculator (circa 1944). [Source: Aspray (1990), p. 212]

compute ballistic firing and bombing tables. Even more important was that the needs of computing at Aberdeen triggered digital computing early in the war, even if the full development – similar to linear and non-linear programming – was basically an after-World War II phenomenon.<sup>77</sup> Goldstine writes that around 1940–41 there was

worked out an arrangement which resulted in bringing into the Ballistic Research Laboratory a set of standard punch card machines to do ballistic calculations. Also, in 1944, two special multiplying machines were built by IBM and installed at Aberdeen. [...] This marks a renascence of interest in the digital approach, which had proved so important in the days of Bliss, Gronwall, Moulton and Veblen, but which had been temporarily superseded by the acquisition of the differential analyzer in 1935. This idea of doing calculations digitally was to culminate in the ENIAC, the first electronic digital computer, and eventually in the gigantic computer industry of today.<sup>78</sup>

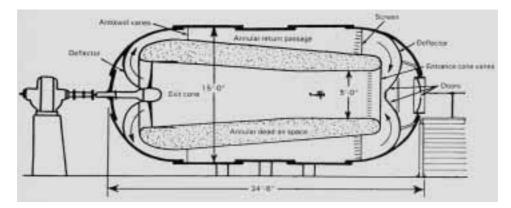
The National Advisory Committee on Aeronautics (NACA), the precursor to the later NASA, was founded in 1915 before the entrance of the United States into the war in 1917.<sup>79</sup> In its experimental and research laboratories, particularly the Langley Laboratory, built 1917–1920, the Americans tried to catch up with European aeronautical research.<sup>80</sup> Hiring Prandtl's student Max Munk in 1920 gave them access to the modern mathematical approach in aerodynamics. Munk had shown with methods of the calculus of variations that the so-called "induced drag" at the wing tips is minimised in case of the so-called elliptic distribution of lift over the

<sup>79</sup> Bilstein (1989).

 $^{80}\,\mathrm{Roland}$  (1985) and Hansen (1987).

<sup>&</sup>lt;sup>77</sup> See Goldstine (1972), esp. his appendix (pp. 349ff.) on the post-war developments in computing in various countries, which were not unaffected by the previous events in the war.

<sup>&</sup>lt;sup>78</sup> Goldstine (1972), pp. 129/130.





## Figure 12.

The NACA Variable Density Tunnel (1922), constructed by German early emigré Max Munk. [Source: Anderson (1997), p. 302]

## Figure 13.

Theodore von Kármán (1881–1963) – the leading American aerodynamicist, an early (1929) immigrant from Europe. [Courtesy: California Institute of Technology Archives]

wing span.<sup>81</sup> Munk introduced a so-called *variable-density wind tunnel* to solve the so-called *Reynold-number scaling problem*. The measurements performed remained partly secret in the 1920s and 1930s and the tunnel would later be seen as an important contribution to the American war effort in WW II.<sup>82</sup> The NACA became a role model for the creation of the OSRD in 1941 and other research committees.<sup>83</sup> But even more important for aerodynamical research in the United States than the NACA facilities proved to be the foundation in 1929 of the *Guggenheim Aeronautics Laboratory at the California Institute of Technology* (GALCIT)

<sup>&</sup>lt;sup>81</sup> Kármán (1954), p. 65.

<sup>82</sup> Munk (1981), p. 1, and Anderson (1997), pp. 301-304.

<sup>&</sup>lt;sup>83</sup> Fortun/Schweber (1993), p. 330, and Kevles (1987), p. 292.

in Pasadena under an early immigrant from Germany (earlier Hungary), Theodore von Kármán. This greater importance came particularly from the "multiplication effect" of Kármán's many students, while Munk at NACA did not have students and got into conflict with the prevailing research philosophy there.<sup>84</sup>

Owens (1989) discusses the policies of the *American Mathematical Society* (AMS) during World War II. He argues that M. Morse and M. H. Stone, as spokesmen for the AMS, were working with all the assumptions of the status quo. They believed that, as leading researchers in pure mathematics, they alone were best suited to organise the utilisation of mathematical skill on behalf of the war. They had only little understanding of or respect for the Washington bureaucracies that were in charge of the general mobilisation of scientific workers.

But when the war came, American mathematicians who sat in a joint committee of the *National Academy of Sciences* (NAS) and the *National Research Council* (NRC) since spring 1942, simply did not know what to do next and Warren Weaver had to step in with his *Applied Mathematics Panel* (APM), founded in November 1942 within the *Office of Research and Development* (OSRD). On the work of the AMP much has been written especially by Rees (1980) and Owens (1989).

The discussion in Owens (1989) shows once again the complexity of the problems of mediation between the various participants in war work to enable mathematics really to be applied in the war. On the more cognitive side problems in mutual understanding between engineers and mathematicians were at the same time discussed by key figures like Kármán (1940/43), who had experiences with both kinds of participants.

## Briefly on mathematical war work in the Soviet Union<sup>85</sup>

Much has been published in recent years on the development of the leading Moscow and Leningrad schools of mathematics before WW II, on ideological intrusion of Stalinism into mathematics and the role of people like E. Kolman, on the political behaviour of great Russian mathematicians such as A. N. Kolmogorov in those years, on the sufferings of mathematicians during the war.<sup>86</sup> Most of the discussion has been about pure research and basic science, with only few passing remarks on applications.<sup>87</sup> Frequently the point is being made that mathematics came out of the various crises relatively undamaged just because it was so pure and apolitical. Not much, however, is known about applied mathematics in general

<sup>&</sup>lt;sup>84</sup> Hanle (1982), p. 90/91. In Munk's case the mentality problems between the theoretician (Munk) and engineers weighed heavily and contributed to his early resignation from NACA in 1926. See Roland (1985), vol. I, pp. 87–98.

<sup>&</sup>lt;sup>85</sup> See also appendix 5.3.

<sup>&</sup>lt;sup>86</sup>Two examples in English are Zdrakovska/Duren (1993) and Lorentz (2002).

<sup>&</sup>lt;sup>87</sup> Lorentz (2002, p. 177) mentions that during WW I pure mathematician and mathematical physicist A. A. Friedmann (Fridman) became interested in plane navigation and construction and served as a pilot for the Russian army. Lorentz reports also on a political trial during the blockade of Leningrad 1941/42 against the "group Rose-Koshlyakov", of which N. V. Rose was dean of the Mathematical-Mechanical Faculty of Leningrad University, counter-admiral

and mathematical war work in the Soviet Union in particular. The work available is mostly in Russian.<sup>88</sup>

Some literature in English<sup>89</sup> on general science policies in the Soviet Union which does not address mathematics complements the picture. It is generally assumed that there existed problems of "transfer" from invention to innovation in some sciences in the SU partly due to a certain divide between theoretical science and engineering.<sup>90</sup> Some Western observers seem to even principally doubt the relevance of Soviet basic science for the war effort.<sup>91</sup> Much of the historical perspective on and interest in Soviet science and mathematics is, of course, inspired by the later successes of this country in the space age. In fact, the foundations of Soviet rocketry and jet propulsion were laid between the two World Wars. Those technologies were clearly relevant for the military; at the same time their smooth development was apparently exceptional in the Soviet research environment, comparable to other exceptions such as results in Soviet synthetic rubber research.<sup>92</sup>

In 1918, Moscow saw the foundation of the *Central aero-hydrodynamic institute* (CAGI or TsAGI) under the famous aerodynamicist N. E. Zukovskij (Joukowski).<sup>93</sup>

Defense-related institutes, such as the Central Aviation Institute (TsAGI) were among the best equipped, and were held up as models.<sup>94</sup>

The Soviet Union could rely on a strong tradition<sup>95</sup> of applied mathematics and theoretical mechanics in Russia (P. L. Cebysev, A. M. Ljapunov, V. A. Steklov).<sup>96</sup> America, partly Germany/France, profited from this tradition and from Russian influence, especially in more applied fields like differential equations and mechan-

- <sup>89</sup> In particular Bailes (1978).
- <sup>90</sup> There was "a strong tradition of pure research in Russian science and (a) high status accorded to such work by comparison with technical innovation." (Bailes 1978, 342). A similar divide existed probably also in other countries such as in the U.K. see Bush (1970), 54, and earlier also in Germany and the US.
- <sup>91</sup> "While the eastern front was turned by sheer brute force, the western powers had space and time for developments in which force was not the only element." (Hodges 1983, p. 251)

<sup>93</sup> Golubev (1955), p. 98. Lenin called Zukovskij the "father of Russian aviation."

<sup>94</sup> Bailes (1978), p. 346.

- <sup>95</sup> So Russians could connect to an uninterrupted tradition of "applied mathematics" in some sense in close neighborhood to academic mathematics from the 19th to the 20th century, unlike the Germans, as is known from Felix Klein's reform around 1900.
- <sup>96</sup> To Lorentz (2002), p. 176, Steklov was more of an organizer than a scholar, but as such important especially due to his influence with Lenin. But this judgement may more reflect on Lorentz as a pure mathematician.

of the Soviet Navy and author of many textbooks. Lorentz says that Rose died in jail while N. S. Koshlyakov "survived in special camp(s) (sharashkas) for working on war-related problems." (Ibid., p. 212)

<sup>&</sup>lt;sup>88</sup> Golubev (1955) and *Moscow University* (1975) are most relevant for mathematical war work. Bogoljubov/Urbanskij (1985), contains material on N. M. Krylov and his Ukranian school in Kiev. Ogibalov/Kudrjasova (1980) and Vogt (1998) give some information on institutions but do not go into the mathematics produced.

<sup>92</sup> Bailes (1978), p. 340.



**Figure 14.** V. V. Golubev (1884–1954) – one of the main organizers of mathematical war work in the SU. [Source: Golubev (1955), p. 28–29]

ics. Among the emigrants from Russia/Soviet Union to America were Timozenko, Tamarkin, Lefschetz, Besicovich, etc.

Golubev (1955) is the most critical of the publications available on mathematical war work, although it was written in the immediate aftermath of war and Stalinism and does not give sources or bibliographic information. Being both a mathematician and a military officer, Vladimir Vasil'evic Golubev (1884–1954) was obviously himself a key figure in the organisation of mathematical war work. He began as a student of "pure" mathematician D. F. Egorov. He started with analytic functions and changed to applied aerodynamics around 1930. In 1932 he became chair for higher mathematics at the Airforce Academy "N. E. Zukovskij" in Moscow and at the same time dean at University faculty math and mechanics. His monograph in Russian of 1948 "Lectures on the theory of wings" gave a systematical outline of Zukovskij's ideas.<sup>97</sup>

There seems to have existed close collaboration of pure and applied mathematicians and common responsibility for the training of students at the *Scientific Research Institute for Mathematics and Mechanics* (NIIMM, founded in Moscow 1922) at least til 1935.<sup>98</sup> Golubev maintains:

<sup>97</sup> Ogibalov/Kudrjasova (1980), p. 11. Golubev (1955), p. 121.

<sup>&</sup>lt;sup>98</sup> There existed for instance practical courses, introduced in 1923, for mathematics and mechanics students who had to work either at TsAGI or in airplane factories (Ogibalov/ Kudrjasova 1980, p. 9). In 1935 the NIIMM was separated, but according to Golubev (1955), p. 110, without hampering the cooperation of pure mathematicians and mechanists.

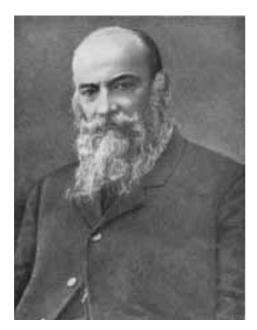


Figure 15. N. E. Zukovskij (1847–1921) – engineer, mathematician and "father of Soviet aviation" (Lenin). [Source: Golubev (1955), p. 82–83]

The period from 1917 till 1930 was a revolution in the development of mechanics at Moscow University. The mutual approachment of University theoretical mechanics and technics ... did not have any support in the ministry before 1917.<sup>99</sup>

Golubev argues that before 1917 mechanics was considered as "applied mathematics" (in the sense of Lagrange), while it now was a "science rather" and was in much closer relation to experimental facilities and applications.<sup>100</sup> Obviously there was a new notion of "applied mathematics" created also in the Soviet Union in the 1920s and 1930s which pointed – similarly to other countries – in the direction of "engineering mathematics." Insofar as the situation in the SU reflected an international process of "modernization" of mathematics, even if Golubev recognizes some "exaggerations such as a 'laboratory brigade method"<sup>101</sup> under the special ideological conditions in the SU which were apparently removed in a new reform 1930–1933.

Venerable traditions and increased cooperation between mechanicists and mathematicians alone could not guarantee successful war research, and details of the "transfer" from invention to innovation in the case of mathematics (for instance in the production of the well-known "Katjusa"-missiles, where apparently

<sup>99</sup> Golubev (1955), p. 107. My translation from Russian.
<sup>100</sup> Golubev (1955), p. 115.
<sup>101</sup> Ibid., p. 108.

mathematical work by A. A. Kosmodemjanskij played a role) under the particular political conditions of the SU have still to be investigated. Little is known, for instance, about the concrete collaboration between mathematicians and the military during the war.<sup>102</sup> For example: To what extent was A. N. Kolmogorov aware of the military applicability of his work on statistical prediction theory or was the latter even commissioned by the military?<sup>103</sup>

The War caused not only the destruction of the aerodynamical laboratory in Moscow by bombardment in February 1942, and with it of Zukovskij's models and wind tunnel of 1904.<sup>104</sup> The evacuation of mathematicians and mechanists from Moscow in the fall 1941, the reduced possibilities for experiments and somewhat lower teaching duties led – according to Golubev – also to some increase in theoretical mathematical reflections among mechanists and to the preparation of after-war publications.<sup>105</sup>

## Briefly on mathematical war work in Great Britain<sup>106</sup>

Britain could, in her mathematical war work, draw on a rather broad and versatile mathematical background ranging from numerical methods (E. T. Whittaker, L. F. Richardson) over substantial aerodynamic research (G. I. Taylor)<sup>107</sup> to applications of pure research such as logic and number theory in cryptology and computer theory (A. Turing). The origins of operational research in Britain in connection with radar development are also well known.<sup>108</sup> Fox (1990) reports on his war work in the Oxford group on "relaxation methods" under R. V. Southwell beginning in 1939 and on his subsequent (1943) engagement in the "new Admiralty Computing Service at Bath."<sup>109</sup>

There existed huge differences as to the institutionalisation and even the notion of "applied mathematics" as compared to Germany and later the U.S.: that notion

<sup>102</sup> More specific questions see in Section 3 below.

<sup>&</sup>lt;sup>103</sup> According to one American observer there was "a very real connection between the success of the superb Russian offensive and the fact that a Stalin prize is given for a paper entitled 'On Dispersion, Probability of Hits and Mathematical Expectations of the Number of Hits'" Gray (1943), p. 149. Shiryaev says (this volume) that the publication was urged by the military, while Kolmogorov saw "primarily methodological interest" in his work. N. Wiener assumes the existence of more detailed work by Kolmogorov, adapted to applications: "I am by no means convinced that Kolmogoroff was not independently aware of the possibility of the applications I made. If that was so, he must have had to keep them out of general publication because of their importance for the military-scientific work." (Wiener 1956, p. 261)

<sup>&</sup>lt;sup>104</sup> Golubev (1955), p. 116/117.

<sup>&</sup>lt;sup>105</sup> Golubev (1955), p. 118.

<sup>&</sup>lt;sup>106</sup> See also appendix 5.4, especially for selected literature.

<sup>&</sup>lt;sup>107</sup> Batchelor (1996). He worked on the statistical theory of turbulence parallel with Kolmogorov.

<sup>&</sup>lt;sup>108</sup> Fortun/Schweber (1993).

<sup>&</sup>lt;sup>109</sup> Fox (1990), p. 292.

did not, as a rule, include engineering mathematics.<sup>110</sup> Some sources report a particularly strong social divide between engineers and scientists in Britain.<sup>111</sup>

There were differences between British and American science policies in the war too, especially with respect to team work and secrecy regulations<sup>112</sup> and with respect to the highly personalised style of directing research, particularly in Britain.<sup>113</sup> Connected to the different social and political environments there remained a certain intellectual superiority complex among some British scientists, especially over Americans, which surfaced for instance in the discussion of ideas underlying computers.<sup>114</sup> However, the lack of economic preconditions commensurate with the Americans was clearly palpable and effective in the war effort, particularly in technical computer development.

There seems to have existed a rather broad tradition in Britain of both leftist and (not necessarily overlapping) pacifist views among some pure mathematicians as revealed in Hardy's book (1940) which triggered some discussion on the morality of war research.<sup>115</sup>

## Briefly on mathematical war work in Italy<sup>116</sup>

Pure mathematical research had a dominant position as to the number of chairs at universities in Italy until the first World War in comparison to other sciences, even to physics.<sup>117</sup> Thus the shift towards applications which was initiated by Italian Fascism under Mussolini, especially after 1927, and which was enhanced due to economic and political autarky policies after 1935, was not just "detrimental" but had some "healthy" aspects for mathematics as well.

However, the concrete forms of change symbolised in mathematics by the substitution of V. Volterra by G. Marconi (later by marshal P. Badoglio) in leading positions (CNR, Academy) had strong political connotations, particularly so when those measures later were supplemented by antisemitic campaigns (racial laws of 1938) as well.<sup>118</sup>

- <sup>111</sup> Bush (1970), p. 54, reflecting on the war years: "It was then true that to the British and they have not quite recovered from it yet the engineer was a kind of second-class citizen compared to the scientist." According to Ashford (1985, p. 117) was the inventor of the relaxation method, R. Southwell, "a rare example of an engineer with outstanding mathematical abilities."
- <sup>112</sup> See Bush (1970), p. 281/2, and Hodges (1983), p. 222, for the British doubts in American secrecy policies because of "lacking the habits of deference, secretiveness and deviousness."
- <sup>113</sup> Think of the well-known Tizard/Lindemann dispute as for instance discussed in Edgerton (1996), p. 21.
- <sup>114</sup> In 1946, Turing spoke of an "American tradition of solving one's difficulties by means of much equipment rather than by thought." (Hodges 1983, p. 352)
- <sup>115</sup> Turing himself said at one point about his work in cryptology that he was "rather doubtful about the morality of such things." (Hodges 1983, 120) See also Levinson (1970). As to L. F. Richardson's pacifist views see Ashford (1985).
- <sup>116</sup> See also appendix 5.5, esp. for selected literature.
- <sup>117</sup> Orlando (1998), p. 149.

<sup>&</sup>lt;sup>110</sup> McVittie (1953), p. 45.

The Volta-Conference<sup>119</sup> of 1935 in Rome on "High Velocities in Aviation", where "the treatment of the foreign guests [...] was almost like that of royalty"<sup>120</sup> revealed a rather developed state of Italian aerodynamic research. One participant, the German applied mathematician A. Busemann (1971) reported on "General Arturo Crocco (the father of Luigi Crocco, who worked in Princeton later), a very able chairman and aeronautical scientist in Italy, in both research and teaching."<sup>121</sup> He also visited "the new Italian aerodynamical research center in Guidonia near Rome, equipped with a variety of high-velocity wind tunnels of Ackeret's [Jacob Ackeret, Swiss] design."<sup>122</sup>

However, the central event as to applications of mathematics in industry and military was the creation in 1927/31 of the "National Institute for the Application of the Calculus" (INAC) of the Italian Research Council under Mauro Picone in Rome.

Picone's biography, on which a collaborator at his institute, L. Amerio (1987), gives detailed information, is interesting because it tells something about the ideological background and motivations for some applied mathematicians to pursue their field.<sup>123</sup> At the same time Picone's biography is indicative of the role of ballistics as a general field of application of mathematics in many different countries (Vahlen, Veblen, Bliss, Moulton, Charbonnier etc.). Trained as a pure mathematician and specialising in partial differential equations around 1910, Picone became involved in ballistic work for the military during World War I. He had to calculate ballistic tables and soon recognised the need for numerical methods.<sup>124</sup> In the 1920s he worked on the famous Ritz method for the application of the calculus of variations in mathematical physics. In June 1923, after Mussolini's "march on Rome" of 1922, Picone wrote a letter to then minister of education Giovanni Gentile, who at that time launched a Fascist educational reform, congratulating him on joining the Fascist party and indicating that he himself was a member.<sup>125</sup> Still in Naples and under continued impression of his experience of WW I, Picone created a privately funded "Institute for Calculus" in 1927, which was later on taken over by the Italian Research Council and renamed in "National Institute for the Application of the Calculus" (INAC) in 1931/32, after Picone had been called to

- <sup>119</sup> In honour of Alessandro Volta (1745–1827).
- <sup>120</sup> Busemann (1971), p. 7. More general on the Volta Conference: Anderson (1997), pp. 400–403.
- <sup>121</sup> Ibid. Crocco constructed a wind tunnel near Rome already in 1903. Anderson (1997), p. 299.

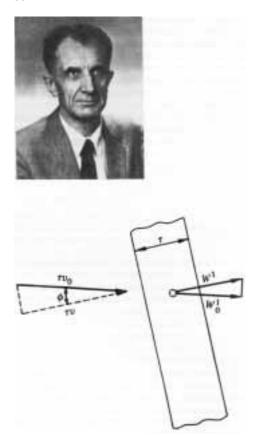
<sup>&</sup>lt;sup>118</sup>On the general political background of Italian mathematics see above all Israel/Nurzia (1989).

<sup>&</sup>lt;sup>122</sup> Ibid., p. 10. See also Kármán (1954, p. 133) reporting on the Volta conference and on Busemann's talk there on aerodynamic properties of sweptback wings.

<sup>&</sup>lt;sup>123</sup> This background he shared with Germans such as the Nazi and ballistician Theodor Vahlen. See Siegmund-Schultze (1984).

<sup>&</sup>lt;sup>124</sup> More details on Picone's ballistic work in WW I are possibly in Picone (1934) which I have not seen.

<sup>&</sup>lt;sup>125</sup> Nastasi (1990), p. 31.



**Figure 16.** Adolf Busemann (1901–1986) and his diagram for a swept wing of 1935. [Source: Anderson (1997), p. 402]

a professorship at the University of Rome. In his report of 1938 on the activities of his institute during the quadriennium years XII til XV (in Roman numerals) of Mussolini's rule, Picone first stressed the theoretical work done by collaborators such as Fabio Conforto and Carlo Miranda on problems of the calculus of variations and eigenvalue problems. In fact, one of the most important pure mathematical after-effects of the INAC was Miranda's book (1955) on elliptical differential equations. But these more general theoretical results notwithstanding, Picone assured industry of its right to an exclusive exploitation of the concrete calculations and algorithms performed and developed for it at the Institute. Focussing more on economic than military applications Picone, nevertheless, concluded his report with the following remarks:

Much of the calculative work done at the Institute was commissioned by the Ministry of National Defence: the stability of aeronautical structures, direction of air bombardment, stabilisation of submarine torpedoes, ballistic tables for long distance artillery, military radio communication have provided material for long and complicated research, which fortunately has been finished in time with the help of the mature mathematicians and numerical procedures which belong to the Institute.

And the Institute for the Application of the Calculus, today under the command of his excellency marshal of Italy Pietro Badoglio [1871–1956, apparently chief of the CNR after Marconi's death in 1937], is decided to maintain and increase its efficacy always ready to respond to any request to secure the power of the Italian army.<sup>126</sup>

Already before World War II there was much collaboration with German applied mathematicians and industry/military; the Germans saw the INAC as somewhat of a role model for their own developments in applied mathematics.<sup>127</sup> There is not much said, in the available reports on the INAC, on the mechanical devices used in the numerical work. Because the reports tell about a great number of "human calculators and computists" one might speculate that the amount of instruments was not very considerable and insufficient even for the time period considered. After WW II (1951) the Picone Institute gained recognition by UNESCO and got the title of a "Centro Internazionale di Calcolo" after a positive review by American Herman Goldstine.<sup>128</sup>

In 1940 the Italian "Royal National Institute of Higher Mathematics" was founded under mathematician Francesco Severi in Rome, and it partly compensated for losses in pure mathematics.

## Briefly on mathematical war work in France<sup>129</sup>

As to the general background of applied mathematics in France in the inter-war years one is often referred in the literature to an overly bureaucratic and Pariscentred science and education system, to a lack of contact between engineering schools and universities quite contrary to the earlier great traditions of the Ecole Polytechnique as a research institution.<sup>130</sup> The leading *Ecole Normale Supérieure* was rather distant to applications;<sup>131</sup> the first modern French applied mathematician became known only after WW II: J.-L. Lions (1928–2001). Misguided developments in French aeronautics seem to have prevented extensive mathematical research in that field and seem to have reversed promising and exemplary developments as the early one at the Eiffel laboratories.<sup>132</sup>

<sup>&</sup>lt;sup>126</sup> Picone (1938), p. 6. Translation from Italian by the author.

<sup>&</sup>lt;sup>127</sup> See documents (letters) reproduced in Picone (1959). On the INAC see also Salvadori (1938).

<sup>&</sup>lt;sup>128</sup> Picone (1959), pp. 160/61, Amerio (1987), p. 23.

<sup>&</sup>lt;sup>129</sup> See also appendix 5.6.

<sup>&</sup>lt;sup>130</sup> On both points mentioned see Germain (1953).

<sup>&</sup>lt;sup>131</sup> Andler (1994).

<sup>&</sup>lt;sup>132</sup> According to Chadeau there was a focus on strong engines instead of aerodynamics: "From 1925 on, when international developments favored innovations with strong scientific components, French products fell behind those of foreign manufacturers. [...] Not until the midfifties, then, did French technology catch up with international standards. In the field of jet propulsion a first step was taken in 1950 [...]" (Chadeau 1988, pp. 28/29)

The poor French reception of international scientific results<sup>133</sup> seems to have improved somewhat in the 1920s, testified to by both developments in ballistics<sup>134</sup> and the founding in 1926 of the *Institut Henri Poincaré* (IHP). However, the developments in stochastics at that institute were theoretical rather than applied.<sup>135</sup> The founding in 1939 of the French CNRS led to the creation of "mathematical laboratories" at IHP.<sup>136</sup> As to their importance for mathematical war research, however, the outcome was doubtful, not least due to the political situation of France under German occupation since June 1940.<sup>137</sup> French deficiencies in computation and communication technologies became clear after WW II.<sup>138</sup> The results in mathematical war research in France given in appendix 5.6 are probably very incomplete and due to the insufficient historical work available. One wonders, for instance, whether the promising developments of M. d'Ocagne's "nomographie" of around 1900 did not have followers in France as well.

## Briefly on mathematical war work in Japan<sup>139</sup>

The main problem of the historiography of mathematical war research in Japan is that almost no primary historical sources are available.<sup>140</sup> The secondary literature on Japan contains almost no information on mathematics in the War. Indirect conclusions from the existing fields of application and general historical works are therefore needed.

According to Sasaki (2002, p. 231) Japanese mathematics began to emancipate itself from the traditional Chinese mathematics called *wasan* before the Meji Restoration which started in 1868. This emancipation was originally "Learning Western Mathematics as Military Science," where navy officers played a leading role. It was followed by "Germanization of the Political System and of Learning, 1881–1945", and at the same time "the intellectual and political mood in Japan became authoritarian and militaristic."<sup>141</sup> On this period Sasaki writes:

At all institutions where higher mathematics was taught prior to the outbreak of World War II – the imperial universities as well as the teacher-training schools – mathematical research was an intellectual activity aimed at

<sup>136</sup> These were the labs for "mathematics" (under É. Cartan), "theoretical physics" (L. de Broglie), statistics (M. Fréchet), ballistics (G. Valiron), astrophysics (M. Mineur), and numerical calculus (L. Couffignal). See Siegmund-Schultze (2001), p. 176.

<sup>141</sup> Sasaki (2002), p. 238.

<sup>&</sup>lt;sup>133</sup> Pyenson (1996).

<sup>&</sup>lt;sup>134</sup> See Charbonnier (1924).

<sup>&</sup>lt;sup>135</sup> Siegmund-Schultze (2001), pp. 156–177.

<sup>&</sup>lt;sup>137</sup> Picard (1990).

<sup>&</sup>lt;sup>138</sup> See the reports by L.Couffignal, from his after-war mission in Germany, ordered by the CNRS (Picard 1990, p. 97). See also Kragh (1996).

<sup>&</sup>lt;sup>139</sup> See also appendix 5.7.

<sup>&</sup>lt;sup>140</sup> According to (Yamazaki 1995, 167) were "the important materials relating to the matter burnt on orders of the military at the end of the War."

turning Japan into a first-class world power. This is one of the reasons why the German cultural ethos was so influential in modern Japan and why so many young Japanese mathematicians studied in Germany. [...] It is therefore not surprising that, with a few exceptions, mathematicians were directly or indirectly mobilized for the war without any serious resistance. After all, some were already engaged in research that had direct military application.<sup>142</sup>

The "German ethos" in mathematics did, however, not necessarily imply a drive for applications and it has been argued that "the distinction between pure and applied mathematics may have been sharper in Japan than in other countries."<sup>143</sup> There existed social and epistemic barriers between mathematicians and engineers, more generally there was a lack of interdisciplinarity in research.<sup>144</sup>

The tradition of Japanese colonialism, militarism<sup>145</sup> and racism was reinforced in 1931 with the Japanese invasion of Manchuria. The period considered between the two World Wars largely coincides with the so-called Showa Period (1926– 1945), another period of modernisation<sup>146</sup> of Japanese science, of increase in the number of graduate students etc.<sup>147</sup> In 1932 the Japanese broke the American code (two-digit encipherment system) but they were apparently unable to read the new one of 1939. Drea (1991) says that Japan's communication intelligence was more successful than commonly assumed. However, no indication of the use of sophisticated mathematical methods in decoding is given in Drea's publications.<sup>148</sup> As to scientific computing, Suekane (1980) argues that the Japanese were unable to produce punched-card machines of their own early in the 20th century and had to import them as late as 1941. Some developments with binary circuits in Japanese industry since about 1938 notwithstanding, the Japanese tradition of the ubiquitous abacus (soroban) contributed to a delay in computer development until well after World War II.

- <sup>143</sup> Sasaki (2002), p. 248. The author adds that "there have been few Japanese mathematicians in the mold of Norbert Wiener." (Ibid., p. 248) "It is certain that modern Japan has produced as many excellent physicists or theoretical engineers as talented mathematicians. Unfortunately, however, as a general rule Japanese mathematicians do not seem to be good at fields connected with other sciences. [...] In general, pure mathematics may have been too autonomous in Japan." (Sasaki 1999, p. 43)
- <sup>144</sup> As stated in Grunden's dissertation of 1998, of which only an abstract was available to this author.
- <sup>145</sup> According to Makino (in this volume), the French mathematician Emile Borel, visiting Japan in the 1920s, found the country "too nationalist", although he himself was used to sharp utterances of nationalism by his French compatriots.
- <sup>146</sup> See Makino (in this volume) on the Japanese mathematician Kinnosuke Ogura (1885–1962), who first protested against Ludwig Bieberbach's racist theories in mathematics (1935), only to support the policies of the Japanese military under the slogan of an "anti-feudalistic modernization".
- <sup>147</sup> Shimao (1989), p. 86.
- <sup>148</sup> Drea (1991), 188. S. Fukutomi's contribution in this volume gives the impression that mathematicians turned to military cryptography rather late in WW II and did not apply very sophisticated methods.

<sup>&</sup>lt;sup>142</sup> Sasaki (2002), p. 246.

The Japanese Technology Board (founded in 1941 with strong focus on aeronautics) was the "true starting point of Japanese science and technology policy."<sup>149</sup>

During WW II Japan had a strong ally, Germany,<sup>150</sup> where several Japanese mathematicians had studied before the war. According to one historian, however, the mobilisation of science in Japan compared to the US was a "downright failure".<sup>151</sup>

## **3** Some Open Historiographical Problems

A listing of some open historiographical problems of different kinds (internal mathematical, sociological, moral etc.), which have been partly touched upon before, may conclude the more general discussion in this article:<sup>152</sup>

- Where exactly is the boundary between industrial and military research in war times, if there exists one?
- How did secrecy regulations during the war influence mathematical research and what connection existed to secrecy regulations in other fields, such as engineering?<sup>153</sup>
- How did mental barriers and ideological prejudices between the different social groups involved (engineers, military, mathematicians, politicians) influence war research in different national surroundings? Can Meigs's observation that "American scientists [...] measured and analyzed, in a way far more unbiased than their counterparts in uniform"<sup>154</sup> be applied to other national contexts?
- Does totalitarian political rule promote or hinder mathematical applications for the military?
- What does the historical material discussed say on the spin-off problem?<sup>155</sup> Is, in particular, Lewis Mumford's "paradox in technics" confirmed, according to which "war stimulates invention, but the army resists it"?<sup>156</sup>

<sup>149</sup> Yamazaki (1995), p. 168.

- <sup>150</sup> For the rather one-sided technology transfer from Germany to Japan see Braun (1987).
- <sup>151</sup> "Nothing was produced by the mobilisation of science comparable with the atomic bomb, radar, rockets, operations research, etc., which were achieved by the mobilization of science in the allied nations. However the Japan programme exerted a great influence upon the postwar order of research." (Low (1990), p. 350) The reference to rockets is somewhat misleading, because Germany led the field here.
- <sup>152</sup> More and more detailed examples suggested by the special national environments are given below in the appendices.
- <sup>153</sup> It seems as though mathematicians were still able to partly communicate mathematical information both nationally and internationally due to the esoteric character of their language. Thus secrecy policies applied to them in quite a different way than to engineers, which made also for mental differences between the two social groups, the engineers and mathematicians. But at the same time there existed national differences in secrecy policies, which were not restricted to the military and not to times of war.
- <sup>154</sup> Meigs (1990), pp. 219/20.
- <sup>155</sup> As discussed for instance in Booss/Høyrup (1984).
- <sup>156</sup> On this one finds much in Meigs (1990), in particular in connection with the discussion of Admiral King's "dogmatic authoritarian personality and an almost bitter determination to protect the Navy from any outside meddling" (Meigs (1990), p. 218).

- Did the War put a halt to much of fundamental (mathematical) research, or has the notion of fundamental research changed rather, for instance in the direction of applied mathematics as an academic discipline?<sup>157</sup>

And more internal:

- What kind of historical connection existed between the development of numerical methods and mathematical instruments: was the correlation necessarily positive or did sometimes a lack of instruments promote the development of more sophisticated numerical algorithms?<sup>158</sup>
- Is it justified to say that geometrical mathematical methods were relatively less important in war research than analytical, as suggested by some authors, and could this be connected to the upswing in numerical analysis and computer development?<sup>159</sup>

# 4 Wolfgang Haack's *Geschoßformen kleinsten Wellenwiderstands* (1941) as a Typical Piece of Mathematical War Research<sup>160</sup>

In cursory form and without going into mathematical details I shall discuss a paper of 1941 by Wolfgang Haack (1902–1994), who started as a "pure" mathematician before the war, but who would later, not least due to his war experiences, become one of the leading applied mathematicians with strong theoretical leanings<sup>161</sup> in the Federal Republic of Germany. Unlike many of his other results,

- <sup>158</sup> As for instance suggested by L. Collatz in Walther (1948–1953), volume 3, p. 34, and in Weissinger (1985), p. 117.
- <sup>159</sup> For industrial research: "It is not easy to say why advanced geometry plays no larger part in industrial research; however, the fact remains that it does not."(Fry (1941), p. 269)
- <sup>160</sup> The title reads in its English translation of 1948 "Projectile Shapes for Smallest Wave Drag" (Haack (1948)). I have not seen this translation which has apparently only limited circulation in American libraries. All translations from German are mine.
- <sup>161</sup> Bernhelm Booss-Bavnbek considers Haack as one of his mathematical ancestors (personal communication). With his joint work with Günter Hellwig in the early 1950s on boundary value problems for elliptic partial differential equations, not satisfying the *Fredholm Alternative*, Haack contributed to what later was to be called the index theory of Atiyah and Singer. Haack reports on this work in his recollections Haack (1989), p. 225ff. See also Haack (1987) and (1998).

<sup>&</sup>lt;sup>157</sup> Note the following remark by Bell Lab president and president of the National Academy of Sciences Frank B. Jewett, which may however have been coloured by his desire to lift restrictions on scientific communication still in place after WW II: "It wasn't long before practically all fundamental research in the physical sciences ceased, as did the advanced training of men." (Jewett 1947/71, p. 405).



Figure 17. Wolfgang Haack (1902–1994) in his retrospection on 85 years. [Source: Haack (1987), p. 23]

Haack's paper, which I am going to discuss, is apparently not, theoretically, of a far reaching importance for "applied mathematics" as a whole or even for "aerodynamics", "ballistics" or "gas dynamics" as scientific fields of their own standing.<sup>162</sup> But lying on the borderlines of the four fields mentioned, the circumstances of the creation of Haack's paper in 1940/41 are revealing with respect to the practices of war research in mathematics at that time and exactly with respect to the "continuity of epistemic, technical, mental and institutional prerequisites for war research," as discussed before. At the same time, work by Busemann of 1942, inspired by Haack's contribution, proved to be of a more lasting, theoretical kind.<sup>163</sup>

In 1939 Haack was professor for mathematics and geometry at the Technical University Karlsruhe. The work on the shape of projectiles was commissioned by the "Aeronautical Proving Ground Hermann Göring" in Braunschweig. Haack did the work as an act of self-mobilisation: he phoned the head of the research department of Göring's aviation ministry, A. Baeumker, immediately after the outbreak of the war in 1939. Haack was a differential geometer and he says in his recollections:

This special field was obviously not very much tuned to war purposes. But nevertheless, it was mathematics, and mathematics is being needed everywhere.<sup>165</sup>

Haack's research, which he did in collaboration with his wife, a theoretical physicist,<sup>166</sup> resulted in a talk in October 1941 in Peenemünde (the well known

<sup>&</sup>lt;sup>162</sup> In a letter to this author, dated 29 February 1992, Haack stressed that he used in his report "but very elementary mathematical methods".

<sup>&</sup>lt;sup>163</sup> Work on conical flow as quoted by Kármán (1954), pp. 117, 141. See also letter by Busemann to Kármán, 6 April 1946 (Kármán Papers 4.27, Caltech) according to which he had meanwhile extended his "criticism of Haack's paper", which he obviously found theoretically insufficient.

<sup>&</sup>lt;sup>164</sup> Haack (1941), p. 15.

<sup>&</sup>lt;sup>165</sup> Haack (1989), p. 2. Later on, in an interview with this author on 26 June 1991, Haack added that he was afraid to be called to the army as an ordinary soldier, because he had been classified as a driver only in examination.

<sup>&</sup>lt;sup>166</sup> Haack interview 26.6.1991, Haack (1941, p. 15): "lively collaboration", also Haack (1989, p. 5).

rocket proving ground, where Wernher von Braun was working) before the *Lilien-thal-Gesellschaft für Luftfahrtforschung*. The talk was published by that society as a secret report with limited circulation (it is therefore still difficult to obtain today)<sup>167</sup> but it was translated after the war in a report by Brown University.<sup>168</sup>

Supersonic speed of projectiles in compressible fluids had been known for long as a phenomenon of ballistics. The new problem of applied mathematics here was to precisely calculate the air resistance to shapes of projectiles (profiles, especially the angles of the noses), which could not be considered as mere pointmasses. Connected to this were phenomena known from fluid dynamics, such as the wellknown boundary layer and its possible separation from the body moving through the flow.<sup>169</sup> Transonic and supersonic flow problems emerged at the same time also in aviation where the resulting consequences for so-called sweptback wings etc. were discussed by applied mathematicians such as Busemann.<sup>170</sup> To interconnect the results of the ballisticians and the aerodynamicists as done in the work by Kármán, Busemann and others was no easy task, not least because of the low level of international cooperation and standardization on the part of the ballisticians, which Kármán would criticise for instance at the Volta Conference in 1935.<sup>171</sup> In the same talk Kármán stated with respect to the experimental material available concerning the drag of different bodies at supersonic speeds that there were "very few wind tunnel experiments and [...] very numerous ballistic tests." (p. 49)

The governing partial differential equations of this type of problems were nonlinear and thus mostly beyond the reach of analytical methods. Simplification of the assumptions concerning the flow, in particular linearisation of the problem, was required, which resulted in a Volterra-type integral equation which still was only solvable after further simplification.

This had already been done by J. Ackeret in 1928 and Kármán and N. Moore in 1932, that means in times of peace, and again by Kármán at the Volta Conference in Rome on "High Velocities in Aviation" in 1935, stressing the analogy "between the wave resistance of slender projectile and the induced drag of a wing."<sup>172</sup> On a more general theoretical level this became later part of a chapter on "Flow in Three Dimensions" of the classic monograph published by the German émigrés to

<sup>172</sup> Sears (1947), p. 361.

<sup>&</sup>lt;sup>167</sup> My copy is from the library of the Deutsche Forschungs- und Versuchsanstalt f
ür Luft- und Raumfahrt e.V. in Göttingen, the successor institution to Ludwig Prandtl's AVA.

<sup>&</sup>lt;sup>168</sup> Haack (1948). See remarks above on this translation.

<sup>&</sup>lt;sup>169</sup> Haack (1941), p. 28. It was Busemann himself, who had helped Haack in the interpretation of his results in the sense of aerodynamical wing theory (Haack (1941), p. 15 and p. 20, fn.). Busemann was head of the institute for gas dynamics at the Braunschweig aeronautical proving ground.

<sup>&</sup>lt;sup>170</sup> See Volta-conference of 1935, and Kármán (1947), esp. 1954, pp. 103–141 "Supersonic Aerodynamics", where Kármán is also shortly alluding to the work of the American task force after the war in 1945 "collecting German papers and documents" (117). Kármán also reported that the Americans were much impressed in 1945 by the German wind tunnel experiments with sweptback wings in Völkenroda near Braunschweig and that this "led to the birth of the present B-47 airplane, the first bomber with sweptback wings in this country." (Kármán 1954, p. 134)

<sup>&</sup>lt;sup>171</sup> Kármán (1936), p. 50, Fn.1.

the U.S. Richard Courant and Kurt Friedrichs in 1948 under the title "Supersonic Flow and Shock Waves".

For the immediate requirements of artillery in the Second World War the results reached by Kármán were, however, not sufficient:

Haack stressed in his publication in the secret reports of the *Lilienthal-Gesell-schaft für Luftfahrtforschung* in 1941 that the practical demands of ballistics (especially anti-aircraft artillery) during warfare required the consideration of calibre and volume (mass) of the projectile in the first place and not, as Kármán had done, calibre and length.

This led Haack to a generalisation of the problem of the optimal cross section (line of profile) of the projectile for the three parameters calibre (K), volume (M), and length (L). He considered the variational problem for the optimal cross section (especially the shape of the nose) in the three different cases where two of the three parameters are fixed. The variational problem and its solution by Haack were closely related to one which the Prandtl student Max Munk had solved in his influential dissertation in wing theory in 1919.<sup>173</sup>

Kármán in 1932 and 1935 had considered the problem of calibre and length fixed ("K-L-projectiles", according to Haack) and had theoretically found a singularity of the solution which hinted at a blunt nose. This, he admitted, was "somewhat contradictory to our assumptions",<sup>174</sup> and only additional assumptions for slender projectiles enabled him to make the optimality (with respect to air resistance) of a *pointed* nose plausible. But Kármán could not derive the pointed nose theoretically because he "ignored of all properties of the solution just a differential-geometric one,"<sup>175</sup> as Haack stated it with some pride in his recollections of 1989, alluding to his field of special mathematical competence. But there were other reasons as well for the fact that Kármán did not reach a definite solution at that time in 1935, reasons which Haack commented on in a letter to the author on February, 29, 1992 with the following words:

Kármán [...] found no solution. The problem was not urgent at that time and remained unsolved ("blieb liegen"). 1940, when supersonic aircraft was built, the interest in the problem had increased. I generalised the problem and succeeded [...] understandable that the aerodynamicists got angry not to have hit on it.

Haack, however, who basically used Ackeret/Kármán's methods of linearisation and subsequent simplification of the occurring Volterra equation, arrived at a different conclusion in the case of fixed calibre and volume (mass):

This solution is the only one [of the three considered] which is in accordance with the method of approximation [the Ackeret/Kármán method], because the projectile is absolutely sharp (acute).<sup>176</sup>

<sup>&</sup>lt;sup>173</sup> Kármán (1954), pp. 65/66.

<sup>&</sup>lt;sup>174</sup> Kármán (1936), p. 45.

<sup>&</sup>lt;sup>175</sup> "Ich hatte beobachtet, dass Herr von Kármán ausgerechnet eine differentialgeometrische Eigenschaft der Lösung übersehen hatte und dadurch zu diesem Widerspruch gekommen war." (Haack 1989, p. 4)

<sup>176</sup> Haack (1941), p. 20.

By "absolutely sharp" Haack meant a cross section with two symmetrical inflection points with respect to the axis (of rotation). So in this case of the K-M-projectiles – unlike Kármán's case of the "K-L-projectiles" – no further theoretical adaption of the solution for slender projectiles seemed necessary. What, however, was necessary was the numerical or graphical calculation of concrete cross sections of K-M-projectiles and an experimental verification of their properties – after all several steps of approximation (and therefore possible sources of error) had been used also in this case – not least due to Ackeret/Kármán's general method of linearization of the original differential equation.

The first confirmation for Haack's theory came soon. As Haack writes in his paper:

Following the Western campaign several French experimental projectiles were captured (erbeutet) which had a curiously sharp shape. They agree almost completely with the optimal projectiles for given volume and calibre.<sup>177</sup>

Further practical and experimental confirmation was needed.

Haack gives in his paper many hints to bolster up the plausibility of his results and to estimate the boundaries of its applicability.<sup>178</sup> He refers to graphical methods by Busemann and numerical methods by G. I. Taylor in the solution of the original differential equations which secure the validity of the linearised theory at least for conic shapes of the projectiles.<sup>179</sup> He reproduces photographic pictures from wind tunnel experiments where the Mach lines (Schlierenaufnahmen, "Machsche Linien") of conic projectiles for supersonic speed are visible.<sup>180</sup>

As to M-L-projectiles, for which Haack also had found a theoretical solution, he had the shapes checked in the Göttingen wind tunnel for Mach-number U/c = 2 – following a proposal and financing by the Braunschweig institute<sup>181</sup> – and found excellent coincidence with the theoretical values for the air resistance (Widerstandsbeiwerte  $c_w$ ).

Finally, in order to get full vindication for his optimal shapes, he let the governing differential equation integrate graphically in two special cases and thanked a "Miss Kistner for the laborious computations".<sup>182</sup> At one point in his research report Haack also hinted at lack of time for further wind tunnel experiments.<sup>183</sup>

<sup>&</sup>lt;sup>177</sup> Haack (1941), p. 14. Photographs of the French projectiles are reproduced in Haack's paper, p. 25. Haack wrote in a letter to this author (20.1.92) that he saw the French projectiles only after he had submitted an earlier version of his manuscript in Braunschweig.

<sup>&</sup>lt;sup>178</sup> "Grenzen für die Brauchbarkeit der folgenden Untersuchungen festzulegen" (Haack 1941, p. 17).

<sup>&</sup>lt;sup>179</sup> Haack (1941), p. 16.

<sup>&</sup>lt;sup>180</sup> Ibid., p. 17.

<sup>&</sup>lt;sup>181</sup> Ibid., p. 15.

<sup>&</sup>lt;sup>182</sup> Ibid., p. 26.

<sup>&</sup>lt;sup>183</sup> Haack (1941), p. 21: "Ist aus Zeitmangel unterblieben."

So there is no doubt that the job he did was urgent and that his results were really used to produce anti-aircraft projectiles. With undisguised pride<sup>184</sup> Haack reported on the "success" of his theory of projectiles which allowed for a considerable extension of reach of German anti-aircraft artillery. Haack, who generally<sup>185</sup> was not given to very critical or self-critical attitudes with respect to the role of mathematics or of himself in the Third Reich, but nevertheless or for that reason was influential in after-war West German mathematics (for example president of the DMV in 1961/62), reports in his recollections of 1989:

In the first months of 1944 mass production [of the projectiles] began in Enzisfeld near [...] Wien. But this could not remain hidden to the enemy (Gegner). He destroyed each large production site of the shells within short time such as their number was just sufficient to protect important military objects with the new ammunition.<sup>186</sup>

Haack says in his recollections of 1989 that Kármán himself acknowledged the value of his war work, when he was in Germany immediately after the war in 1945 to collect German manuscripts:

I am happy to meet you. Yes I did not realise in Rome in 1935 [the Volta Conference mentioned] that these optimal projectiles can be computed with our method, and your paper interested me greatly.<sup>187</sup>

And, as was seen, the baton was passed over to the Americans and they found – in addition to their own results – mathematical "intellectual techniques" in occupied Germany to go on in artillery or in supersonic bomber warfare or wherever else.

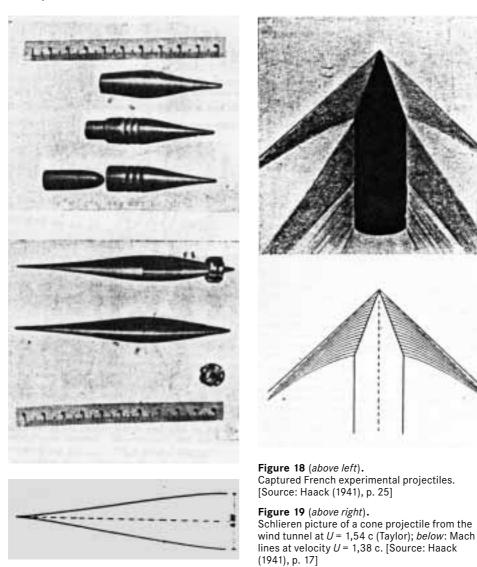
What makes this small piece of mathematical war research so interesting and – I believe – typical, is the close interconnection it reveals, the "continuity" as we called it, of a great many levels of intellectual and practical activities which led to the "success" of the mathematical war work. Particular mathematical activities of very different kinds and theoretical levels (pure analysis, numerical algorithms, graphical methods) were involved. There was a gradual approach of two different engineering disciplines, ballistics and aerodynamics, that were about to recognise what they had in common mathematically. There was a "pure mathematician" (Haack) who made use of his special expertise (differential geometry) at one point, where even the mathematically well-educated engineer Kármán had failed. There were engineering techniques and facilities (wind tunnel experiments, calculation devices) needed. There was a lot of oral communication and collaboration between the pure mathematician Haack and the applied mathematician Busemann, and of a mathematician and a physicist (Haack's wife).

<sup>&</sup>lt;sup>184</sup> Haack would however stress in a letter to this author (20.1.1992) the importance of his work "in an age of civil supersonic flight" trying to somewhat diminish the author's insistence on war research as the topic of Haack's report.

<sup>&</sup>lt;sup>185</sup> See also Haack (1998).

<sup>&</sup>lt;sup>186</sup> Haack (1989), p. 6.

<sup>&</sup>lt;sup>187</sup> Haack (1989), p. 35.



**Figure 20.** K-M-projectile. Optimal projectile (with two inflection points) for given calibre and mass. [Source: Haack (1941), p. 20]

Not just in the background of this story but rather dominant is, of course, the peculiar political situation of the war. One sees the indifference of an allegedly apolitical mathematician<sup>188</sup> to the destructive and deadly aspect of the military use of his science. One notes the abundance of money from the military for math-

<sup>188</sup> The attitude of the mathematician implied to separate his political positions from the scientific part of his work is expressed in Haack's remark, in a letter, dated 20.1.1992, to this

ematical work and wind tunnel experiments. At the same time the urgency of the job becomes clear which had the consequence of stopping the level of logical and mathematical sophistication when the result was needed, while Kármán had broken up his work in 1935 in times of peace for lack of a theoretically satisfying result.

Finally, the case study enables also some general judgement on the epistemic status of fluid dynamics at that point, on which one of its leading representatives said in 1948:

For modern fluid dynamics d'Alembert's saying has been typical in many instances: 'Just go ahead, belief will come afterwards'!<sup>189</sup>

<sup>189</sup>Görtler (1948), p. 11. My translation from German.

author: "The letter to Min.Dir. Bäumker has in my opinion nothing to do with the evaluation of my work and is a pure matter of privacy." Note, however, that Haack himself made this episode public in his recollections of 1989.

## 5 Seven Appendices (Tables) on Mathematical War Work in Different Countries<sup>190</sup>

# 5.1 Military work in mathematics (1914–1945): selected notes on the case of Germany

## 5.1.1 Main institutions and events connected to military application of mathematics

1917:	Aerodynamische Versuchsanstalt (AVA) Göttingen, financed by military
1925:	Institute for Fluid Mechanics Göttingen under Ludwig Prandtl
1928:	Darmstadt: Institut für Praktische Mathematik under Alwin Walther: during WW II calculations for rocket project in Peenemünde
1933:	upswing with rearmament, e.g., 1934 DFL (aviation) in Braunschweig
1937:	upswing of mathematics at DVL (aviation) Berlin-Adlershof: political "oasis" <sup>191</sup>
1942:	"Diplom" in mathematics introduced: "creation" of industrial mathematician
1944/45:	Reichsinstitut in Oberwolfach founded, originally for war-relevant research

### 5.1.2 Some fields of application, methods, mathematicians/appliers involved<sup>192</sup>

field	method	mathematicians/appl.	source
ballistics, super- sonic flight	diff. and integr. equa- tions, calculus of variations	Busemann, Haack	Busemann (1971) Haack (1941)
airfoil design	complex functions	Schmieden, Lagally	Epple/Remmert
rocket-project	calculation of	Darmstadt institute	Walther (1948ff.)
	numerical analysis	Collatz	Walther (1948ff.)

## 5.1.3 Social/epistemic environment for applications of mathematics

- traditions at Technical Universities of strong mathematical education for engineers, and growing understanding of applications among mathematicians (Felix Klein in Göttingen)
- loss of famous applied mathematicians in 1933 (Courant, Mises etc.)

<sup>191</sup> Weissinger (1985), p. 129.

<sup>&</sup>lt;sup>190</sup> In so far as the following tables contain – for the sake of completeness – some overlap in the facts with the previous text, at least the sources will not be indicated again.

<sup>&</sup>lt;sup>192</sup> A particularly important source is Walther (ed., 1948–1953), which contains much German research that had remained secret during the war.

### 5.1.4 Historical primary and secondary literature/sources

Busemann (1971), Epple/Remmert (2000), Görtler (1948), Gross (1978), Haack (1941/ 1989), Ludwig (1974), Mehrtens (1986/96), Petzold (1992), Siegmund-Schultze (1986/89/ 90), Trischler (1992), Walther (1948–53), Weissinger (1985)

### 5.1.5 Some specific problems for historiography

- To what extent did deficiencies in research (statistics, differential equations, celestial mechanics) from before the war influence German war research?
- How did the German material support for mathematics compare to the conditions in the U.S.?

## 5.2 Military work in mathematics (1914–1945): selected notes on the case of United States

## 5.2.1 Main institutions and events connected to military application of mathematics

- 1915: foundation of National Advisory Committee on Aeronautics (NACA)<sup>193</sup>
- 1918: Aberdeen Proving Ground: ballistics under O. Veblen and G. A. Bliss
- 1920: NACA's Langley Laboratory with German immigrant Max Munk
- 1925: mathematical division in Bell Labs
- 1929: immigration of Th. v. Kármán and foundation of GALCIT at Caltech
- 1931: Bush's differential analyzer: since 1935 and during WW II in Aberdeen ballistics
- 1938: Ballistic Research Laboratory at Aberdeen Proving Ground
- 1941: Brown summer school founded (R. G. D. Richardson)
- 1942: Applied Mathematics Panel (APM) under W. Weaver in Bush's OSRD
- 1942: First OR groups founded in US
- 1943: Foundating of Quarterly of Applied Mathematics
- 1943: Los Alamos: Atom bomb project with strong need for calculations
- 1944: Mark I by H.Aiken, Courant/Friedrichs "Shockwaves"

<sup>193</sup> See as an early NACA-report Wilson (1915).

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field	method	mathematicians/appl.	source
air foil design	calculus of variations	Max Munk at NACA	Munk (1981)
ballistics in WW I	finite differences, cal- culus of variations	F. R. Moulton, G. A. Bliss	Moulton (1926), Bliss (1927)
war industry	statistical sequential	A. Wald	Rees (1980)
ballistics, super- sonic flight	diff./int. equations	Kármán	Kármán (1936)
computers	logic, electrotechni- ques	von Neumann, Bush	Goldstine (1972)
gas dynamics, atom bomb	nonlinear hyperbolic equations	von Neumann, Courant	Goldstine (1972), Courant/Friedrichs (1948)
operations	game theory, statistics	von Neumann, Ph. Morse	Fortun/Schweber (1993)
anti-aircraft	statistical prediction	N. Wiener	Wiener (1956)
information	stochastics, computing	Cl. Shannon	Millman (1984)

## 5.2.2 Some fields of application, methods, mathematicians/appliers involved

### 5.2.3 Social/epistemic environment for applications of mathematics

- lack of academic applied mathematics with exception of MIT (Boston) and Caltech (Pasadena)
- strong traditions in industrial research at Bell labs (statistics and electrical engineering)
- important influence of British statistics (estimation tests, J. Neyman) in the 1930s
- transfer of German/European ideals to America with immigrations after 1933
- superior material conditions of American mathematics compared with Europe (instruments)

### 5.2.4 Historical primary and secondary literature/sources

Birkhoff (1977/80), Bilstein (1989), Bliss (1927), Bode (1953), Bush (1929, 1931, 1970), Dickson (1918), Fortun/Schweber (1993), Fry (1928, 1941, 1953), Goldstine (1972), Gray (1943), Hanle (1982), Hansen (1987), Heims (1980), Jewett (1947/71), Kármán (1936ff.), Masani (1990), McShane et al (1953), Meigs (1990), Millman (1984), Morawetz (1992), Morse/Hart (1941), Moulton (1926), Munk (1981), Owens (1989), Rees (1980), Richardson (1943), Roland (1985), Rosser (1982), Rothrock (1919), Shewhart (1931), Todd (1990), Vivian (1919), Wallis (1980), Weyl (1953), Wilson (1915), Zant (1943/44).

#### 5.2.5 Some specific problems for historiography

- How differently would applied mathematics look in the U.S. today without WW II, and what would have been the consequence for pure mathematics, which seems to have profited greatly from military support even after the war?

## 5.3 Military work in mathematics (1914–1945): selected notes on the case of the Soviet Union

## 5.3.1 Main institutions and events connected to military application of mathematics

- 1891: wind tunnel in Moscow of 2 feet diameter built by Zukovskij (Joukowski)<sup>194</sup>
   WW I mathematical physicist A. A. Friedmann (Fridman) serves as a pilot. Training of pilots for the war at Moscow University under Zukovskij<sup>195</sup>
- 1918: Central aero-hydrodynamic institute (CAGI or TsAGI) in Moscow, director Zukovskij, afterwards S. A. Caplygin, later M. V. Keldys, who became Academy president 1961–1975
- 1921: Academy institute in Petrograd/Leningrad under Steklov, named after him and shifted to Moscow following his death in 1926
- 1922: Scientific Research Institute for Mathematics and Mechanics (NIIMM), founded at Moscow University, led for example by A. Ja. Khinchin since 1932 until split in 1935
- 1928: new wind tunnel at CAGI<sup>196</sup>
- 1929: journal "Vestnik prikladnoj matematiki i mechaniki" ("Applied mathematics and Mechanics")
- 1932: V. V. Golubev has chair for higher mathematics at the Airforce Academy "N. E. Zukovski", at the same time dean of University faculty math and mechanics
- 1941: (June) mathematicians of University of Moscow gradually evacuated to Aschabad, Turkmenistan, later to Sverdlovsk (Ukraine); in September 1943 returned to Moscow<sup>198</sup>

#### 5.3.2 Some fields of application, methods, mathematicians/appliers involved<sup>199</sup>

field	method	mathematicians/appl.	source
aerodynamics: flutter	part. diff. equations	M. V. Keldys	MU, p. 81
naval anti- aircraft	nomography	N. A. Glagolev	MU, p. 78
airplane design	geometry, gas dynamics	S. A. Christianovic	MU, p. 80, Gol., p. 120
Katjusa rockets	gas dynamics, diff. equations	A. A. Kosmodemjanskij	MU, p. 83
stability of pro- jectile	Ljapunov functions	N. G. Cetaev	MU, p. 82
shrapnel	plasticity	A. A. Iljusin (not S.V.)	MU, p. 85, Gol., p. 117
anti-aircraft	statistical prediction	A. N. Kolmogorov	Shiryaev (this vol.)

<sup>194</sup> According to Kármán (1954) p. 14. Zukovskij's wind tunnel is also mentioned by Lee (1998), p. 11. Lee and Anderson (1997, p. 299) refer to another one built by Zukovskij with support by the wealthy patron D. P. Rjabucinskij at Koutchino near Moscow in 1904. There was obviously another one in Moscow, mentioned by Golubev (1955), p. 117.

## 5.3.3 Social/epistemic environment for applications of mathematics

- strong tradition of applied mathematics and theoretical mechanics in Russia (N. G. Cebysev, A. M. Ljapunov, V. A. Steklov)
- particularly strong tradition in mathematical foundations of aerodynamics around 1900:
   N. E. Zukovskij, S. A. Caplygin
- influence of Russian/Soviet emigrants above all in America
- language problems and political problems of mathematical communication with the West, reinforced 1933
- secret German-Soviet collaboration in aeronautics until 1933
- close collaboration of pure and applied mathematicians at Moscow University
- foundations of Soviet rocketry and jet propulsion laid between the Two World Wars
- mathematician Keldys "theoretician of Soviet space program" after World War II
- possibly problems of "transfer" from invention to innovation in SU
- propagandistic role of aeronautics between the World Wars, also intentionally with respect to defence,<sup>200</sup> involved mathematician and geophysicist O. Yu. Shmidt (1891– 1956)
- superiority of German military aviation over the Soviets became clear already during the Spanish Civil War 1937
- speed of war planes belatedly (1939) declared as the first priority over distance (bombers)  $^{201}$

#### 5.3.4 Historical primary and secondary literature/sources

Bailes (1978), Golubev (1955), Moscow University (MU, 1975), Lorentz (2002), Ogibalov, P.M. and L.V.Kudrjasova (1980), Palmer (2000), Tobies (1985), Vogt (1998)

## 5.3.5 Some specific open problems for historiography

- Was there a specific influence of Stalinism, in particular of the Moscow trials (1936/37), on mathematics, possibly in favour of applied as opposed to "pure" mathematics?<sup>202</sup>

<sup>195</sup> Golubev (1955), p. 97.

- <sup>196</sup> This apparently even influenced Kármán's tunnel in Aachen (Tobies 1985, p. 73).
- <sup>197</sup> Vogt (1998), p. 181.
- <sup>198</sup> Ogibalov/Kudrjasova (1980), p. 14.
- <sup>199</sup> Most of the details from Golubev (1955) [Gol] or *Moscow University* [MU]... (1975). All mathematicians mentioned are from Moscow.
- <sup>200</sup> Bailes (1978), Palmer (2000), Vogt (1998). However according to Bailes (p. 394) "the achievement of world aviation records cost [the Soviet Union] dearly in military unpreparedness."

<sup>202</sup> Golubev (1955), p. 108. In spite of the well-known attacks against pure mathematicians such as N. N. Lusin, which had some utilitaristic overtones (see Lorentz (2002), p. 206, Lusin subsequently did some more applied work), there was obviously no automatism in this respect. Erickson (1972), p. 256, argues that the military purge in June 1937 damaged the "radio-technical programme" severely.

<sup>&</sup>lt;sup>201</sup> Bailes (1978), pp. 394/95.

- Which foreign support as to scientific information received the Soviets during the War, for instance under the American Lend-Lease Act?<sup>203</sup>
- Which were the meeting places and channels of communication between Soviet mathematicians and the military?
- How were secrecy regulations enforced or did mathematicians follow self-imposed rules?
- Which developments in mathematics during the war proved crucial in the later Soviet space program?

## 5.4 Military work in mathematics (1914-1945): selected notes on the case of Great Britain

- 5.4.1 Main institutions and events connected to military application of mathematics
- 1909: Advisory Committee on Aeronautics (later Aeronautical Research Committee or Council)
- 1913: Mathematical Laboratory at University of Edinburgh<sup>204</sup> Royal Aircraft Establishment, one of the principal research establishments in inter-war<sup>205</sup> 1939:
- Bletchley Park project started
- 1939: Operational Analysis starts at Royal Air Force Command
- 1935-1945: Results on relaxation methods at Imperial College London during war only published in "confidential version of the Royal Society Proceedings"206
- Admiralty Computing Service at Bath<sup>207</sup> 1943:

## 5.4.2 Some fields of application, methods, mathematicians/appliers involved

field	method	mathematicians/appl.	source
cryptology	logic, coding theory, statistics	A. Turing	Hodges (1983), Hinsley/Stripp (1993)
meteorology	finite difference methods	L. F. Richardson	Ashford (1985)
elasticity	relaxation method	R. Southwell	Zienkiewicz (2000), Fox (1990)
aeronautics	statistic turbulence	G. I. Taylor	Kármán (1954)
aeronautics	complex functions	H. Glauert, S. Goldstein	Goldstein (1969)
operations research	various, including sta- tistics	physicists, mathematici- ans at Bomber Command	Edgerton, Fortun/ Schweber (1993)

<sup>203</sup> Erickson (1972, p. 243) argues that the scientific support for the Soviet Union due to Lend-Lease has been overestimated and domestic development undervalued at least with respect to radar.

<sup>204</sup> Whittaker/Robinson (1924).

### 5.4.3 Social/epistemic environment for applications of mathematics

- particularly strong social divide between engineers and scientists?
- different notion of "applied mathematics" as compared to Germany and later U.S.: not including engineering mathematics
- Kelvin/Maxwell as mathematical background for Bush's "differential analyzer"
- Air Ministry in inter-war years largest R&D spending institution in Britain<sup>208</sup>
- certain superiority complex over Americans maintained, in connection with ideas underlying computers, but lack of economic preconditions commensurate with the Americans
- technical-economic superiority of U.K. over the Poles in connection with cryptology<sup>209</sup>
- differences between British and American science policies in the war, especially with respect to team work and secrecy regulations (Hodges, Bush)
- highly personalised style of leading research: Tizard/Lindemann dispute

### 5.4.4 Historical primary and secondary literature/sources

Ashford (1985), Batchelor (1996), Birkhoff (1980), Bush (1970), Edgerton (1996), Fortun/ Schweber (1993), Fox (1990), Hardy (1940), Hinsley/Stripp (1993), Hodges (1983), Levinson (1970), Whittaker/Robinson (1924), Zienkiewicz (2000)

## 5.4.5 Specific problems for historiography

- Why did OR not fare well in England after the war (Fortun/Schweber (1993), p. 343), although the origins were in Britain?

## 5.5 Military work in mathematics (1914–1945): selected notes on the case of Italy

## 5.5.1 Main institutions and events connected to military application of mathematics

- 1903:first wind tunnel constructed near Rome by Arturo CroccoWW I ballistic work by Mauro Picone
- 1923: Consiglio Nazionale delle Ricerche (CNR), restructured 1927
- 1931: "National Institute for the Application of the Calculus" (INAC) under CNR and Mauro Picone in Rome
- 1935: International Volta-Conference in Rome on "High Velocities in Aviation"

about 1935: Italian aerodynamic research centre in Guidonia near Rome

<sup>205</sup> Edgerton (1996), p. 4.

<sup>206</sup>Zienkiewicz (2000), p. 9.

<sup>207</sup> Fox (1990), p. 292.

<sup>&</sup>lt;sup>208</sup> Edgerton (1996), p. 3.

<sup>&</sup>lt;sup>209</sup> British cooperated with the Poles who made early use (1932) of mathematicians in cryptology (Hodges 1983, p. 170). The Polish had gradually "to turn to the technically superior West" (Hodges 1983, p. 235), clearly so, of course after 1939. See also Rakus-Andersson, this volume.

### 5.5.2 Some fields of application, methods, mathematicians/appliers involved

field	method	mathematicians/appl.	source
ballistics	numerical methods	Picone	Amerio (1987)
aerodynamics		Arturo Crocco	Anderson (1994)
aerodynamics: stability, flutter	eigenvalue problems, calculus of variations, numerical methods	Picone-Institute INAC	Picone (1938), (1959)
pure spin-off	elliptic differential equations	C. Miranda at INAC	Picone (1938)

#### 5.5.3 Social/epistemic environment for applications of mathematics

- pure mathematics had a dominant position in Italy until WWI in comparison to other sciences: thus partly healthy reorientation towards applications under Mussolini, especially after 1927
- change symbolized in mathematics by substitution of Volterra by Marconi (later by marshal P. Badoglio) in leading positions (CNR, Academy) with strong political connotation
- CNR's committee for mathematics later suppressed in favor of a sub-committee for applied mathematics of the committee of physics<sup>210</sup>
- 1923 Picone congratulates minister of education Gentile on membership in Fascist Party
- 1935: Ethiopian War, followed by boycott of Italy, growing autarky policies and increased demands for Picone's institute
- missing steel industry created problems for airplane production especially in isolation<sup>211</sup>
- 1938 racial laws in Italy: expulsion of Jewish mathematicians (Orlando)
- 1940 Italian "Royal National Institute of Higher Mathematics" under Severi in Rome: partly compensated for losses in pure mathematics
- collaboration with German applied mathematicians and industry/military: Italians as a role model for Germans (Picone 1959)
- after WW II (1951) recognition of Picone Institute by UNESCO under the title "Centro Internazionale di Calcolo" after a positive review by American Herman Goldstine

#### 5.5.4 Historical primary and secondary literature/sources

Amerio (1987), Busemann (1971), Guerragio (1987), Israel/Nurzia (1989), Nastasi (1990), Ogburn (1949), Orlando (1998), Picone (1934-1959), Russo (1986), Salvadori (1939), Todd (1990)

### 5.5.5 Specific problems for historiography

- Unclear level of computing facilities at INAC: large number of "human calculators and computists" mentioned by Salvadori and others indicates rather low level

<sup>210</sup> See Russo (1986), p. 307. <sup>211</sup> Ogburn (1949), p. 90.

## 5.6 Military work in mathematics (1914–1945): selected notes on the case of France

## 5.6.1 Main institutions and events connected to military application of mathematics

- 1909: Ecole Nationale Supérieure d'Aeronautique<sup>212</sup>G. Eiffel's laboratories with aerodynamic experiments<sup>213</sup>
- 1922: Le Mémorial de l'Artillerie Française: internationally oriented
- 1926: Institut Henri Poincaré (IHP): theoretical rather than applied stochastics

1939: CNRS with laboratories at IHP: doubtful outcome

## 5.6.2 Some fields of application, mathematical methods, mathematicians/ appliers involved

field	method	mathematicians/appl.	source
ballistics in WW I	analytic functions	A. Denjoy, G. Valiron, P. Lévy	Charbonnier (1924)

### 5.6.3 Social/epistemic environment for applications of mathematics

- overly bureaucratic and Paris centred science and education system (Germain)
- lack of contact between engineering schools and universities (Germain)
- indirect shot since WW I urged new mathematics in French ext. ballistics (Charbonnier)
- poor French reception of international results (Pyenson), seems to improve in 1920s
- historic influence of metric system in French artillery on British and American artillery  $^{214}\,$
- leading Ecole Normale Supérieure distant to applications
- criticism of the state of applied mathematics and mathematical physics in France in the context of the foundation of the IHP 1926<sup>215</sup>
- misguided developments in French aeronautics: focus on engines instead of aerodynamics
- 1938 disbandment of French Strategic Air Force
- no defence research under German occupation (1940) possible without collaboration

<sup>213</sup> Hashimoto (1994), Both Prandtl and von Mises referred to the experiments there. However as a private institution the Eiffel laboratories were rather isolated in France itself (Chadeau 1988, p. 40). For the influence on Japanese aerodynamics see Braun (1987, p. 13).

<sup>215</sup> Siegmund-Schultze (2001).

<sup>&</sup>lt;sup>212</sup> This school was nationalized after the creation of the French Air Ministry in September 1928, but mainly produced bureaucrats who as students originally had often done brilliant theoretical work (Chadeau 1988, p. 39).

<sup>&</sup>lt;sup>214</sup> Gray (1943), p. 77.

- attempts to involve French prisoners of war in German war research<sup>216</sup>
- French deficiencies in computation and communication technologies clear after WW II

### 5.6.4 Historical primary and secondary literature/sources

Charbonnier (1924), Chadeau (1988), Germain (1953), Hashimoto (1994), Pestre (1994), Picard (1990), Pyenson (1996), Andler (1994), Orlando (1998), Kragh (1996), Siegmund-Schultze (1986/2001)

## 5.6.5 Some specific problems for historiography

- According to Charbonnier (1924): superior French mathematical methods in WW I ballistics
- How did the inner logic of an applied science such as ballistics stimulate the interest of mathematicians in new applications, including applications in wars?<sup>217</sup>
- "Phony war" (drôle de guerre) also in French science 1939, i.e., defence research not taken seriously?

## 5.7 Military work in mathematics (1914–1945): selected notes on the case of Japan

- 5.7.1 Main institutions and events connected to military application of mathematics
- 1931: Japanese invasion of Manchuria, new scientific institutions, partly in occupied territories
- 1932: Japanese broke American code (two-digit encipherment system)<sup>218</sup> but unable to read the new one of 1939
- 1933: Ultra-nationalists oppose introduction of metrical system which is delayed till  $1958^{219}$

Wind tunnel erected by W. Margoulis, former director of the Eiffel laboratories in Paris.<sup>220</sup> "Extensive wind tunnel tests" for the *Aeronautical Research Institution of Tokyo Imperial University*<sup>221</sup>

1937: Japanese-Chinese War, Japan becomes the first country outside the United States to build and successfully operate a cyclotron<sup>222</sup>

<sup>217</sup> Charbonnier (1924), p. 577, is talking about tendencies in ballistics to create a coherent logical structure of the field. According to him there were no applications in the moment: but hopefully, as he maintained, in the future.

<sup>&</sup>lt;sup>216</sup> Siegmund-Schultze (1986).

<sup>&</sup>lt;sup>218</sup> Drea (1991), p. 188.

<sup>&</sup>lt;sup>219</sup> Shimao (1989), p. 58.

<sup>&</sup>lt;sup>220</sup> Braun (1987), p. 13. See also Hashimoto (1994) for the general importance for aerodynamics of the Eiffel experiments and tables, not just in Japan.

<sup>&</sup>lt;sup>221</sup> Yamazaki (1995), p. 171.

<sup>&</sup>lt;sup>222</sup> Low (1990), p. 350.

1941: Technology Board (focus on aeronautics): "true starting point of Japanese science and technology policy"<sup>223</sup>

Pearl Harbor: "with few exceptions mathematicians were directly or indirectly mobilised for the war"  $^{\rm 224}$ 

- 1943: S. Tomonaga (mathematical physicist)<sup>225</sup> driven into applied electrical engineering (Naval Institute of Technology).
- 1944: Institute of Mathematical Statistics established in 1944<sup>226</sup>
- 1946: late foundation of "Japan Mathematico-Physical Society": sign of belated modernisation<sup>227</sup>

### 5.7.2 Some fields of application, methods, mathematicians/appliers involved

field	method	mathematicians/appl.	source
electrical engineering		S. Tomonaga	Low (1990)
circuit design	lattice theory, logic	Nakajima at NEC	Suekane (1980)
war production	statististics	statistical Institute	Itagaki (2002)
airplane design		aeronautical institute Tokyo	Braun (1997)

### 5.7.3 Social/epistemic environment for applications of mathematics

- tradition of Japanese colonialism, militarism and racism since Chinese-Japanese War of 1895, reinforced 1931
- backwardness in scientific computation, partly due to strong abacus tradition (Suekane (1980))
- 1926-1945 Showa Period: period of modernisation of Japanese science
- pioneering results in theoretical physics: H. Yukawa: nobel prize theoretical physics 1949 for prediction of meson
- Japan had a mathematically strong ally, Germany, where several Japanese had studied
- social and epistemic barrier between mathematicians and engineers felt: pure mathematics too autonomous, more generally, lack of inter-disciplinarity in research

- <sup>225</sup> S. Tomonaga had worked on the mathematical foundations of meson theory before, esp. on "renormalization theory, which made quantum electrodynamics consistent with the special theory of relativity." (Low (1990), p. 356)
- <sup>226</sup> According to a personal communication by Prof. Itagaki Ryoichi (2002), based on "The hundred-years history of mathematics in Japan" of 1983/84, this institute was erected by the Ministry of Education on the ground that statistical research will help to increase the production of goods in wartime.
- <sup>227</sup> Interpreted this way by Sasaki (1999).

<sup>&</sup>lt;sup>223</sup> Yamazaki (1995), p. 168.

<sup>&</sup>lt;sup>224</sup> Sasaki (1999), p. 37.

- 1940–42 Japanese military showed interest in atomic weapons, but found prospects discouraging during the war, therefore concentration on radar<sup>228</sup>
- mobilisation of science in Japan compared to the US a "downright failure"<sup>229</sup>

## 5.7.4 Historical primary and secondary literature/sources

Braun (1987), Drea (1991/95), Hashimoto (1994), Low (1990), Sasaki (1999), Shimao (1989), Suekane (1980), Yamazaki (1995), Itagaki Ryoichi (personal inf. 2002), *The hundred-years history of mathematics in Japan* (Japanese), 2 vols. Tokyo: Iwanami-Shoten 1983–1984

## 5.7.5 Specific open problems for historiography

- Controversial statements in the literature with respect to dependence on Germany<sup>230</sup>
- Unclear whether Japan's communication intelligence used sophisticated mathematical methods

<sup>&</sup>lt;sup>228</sup> Low (1990), p. 350.

<sup>&</sup>lt;sup>229</sup> Low (1990), p. 350.

<sup>&</sup>lt;sup>230</sup> According to Braun (1987), p. 15, the Japanese were largely independent in jet technology, while Shimao (1989), p. 171, maintains that they did import it from Germany.

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