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On the Defense Work of A. N. Kolmogorov during World War II

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In the paper we give a short description of the defense work of Andrei N. Kolmogorov on the Theory of Firing during World War II.

Similarly to his great predecessors P. L. Tschebyshev and A. M. Lyapunov, the academician A. N. Kolmogorov (25.04.1903–20.10.1987) was not only a universal mathematician, proposing and developing fruitful ideas, he also readily answered challenges presented to him in his capacity as an applied mathematician. He had an amazing ability to get to the root of a problem, to determine fundamental issues, and to clarify debatable situations.

A typical example of such work by Kolmogorov are his investigations on **firing dispersion theory** that he performed during World War II.

His paper *Estimation of the center and spread of dispersion for a bounded sample*, [Kolmogorov 1942], was dated September 15th, 1941, i.e., three months after the war began on the Soviet territory. It was published in response to a request from the appropriate military agencies to "give his conclusion on the subject of discrepancies among the existing methods to estimate precision from experimental data." Kolmogorov remarked that this paper presents primarily methodological interest due to critical comparison of distinct approaches. Nevertheless, Kolmogorov and his colleagues from the Mathematical Institute of the Academy of Sciences, Department of Mechanics and Mathematics at Moscow State University, as well as the immediate practitioners from the Institute of Marine Artillery Research, developed largescale *theoretical and computational efforts* toward measuring the efficiency of firing strategies. Simultaneously, he taught a course at the university on firing dispersion theory. He made it *compulsory* for students majoring in probability.

This work concluded in the publication of a separate volume of "Proceedings of the Steklov Mathematical Institute" (1945), nicknamed by Kolmogorov himself *Collection of Articles on the Theory of Firing*, [Kolmogorov 1945]. It included two of his articles, *On the number of hits among several shots and general principles of characterizing efficiency of firing strategies and Artificial dispersion in case of single shot kill and onedimensional dispersion*, as well as two articles by his

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Translated from the Russian by Galina Shubina.

ИЗВЕСТИЯ АКАДЕМИИ НАУК СССР
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А. Н. КОЛМОГОРОВ

ОПРЕДЕЛЕНИЕ ЦЕНТРА РАССЕИВАНИЯ И МЕРЫ ТОЧНОСТИ ПО ОГРАНИЧЕННОМУ ЧИСЛУ НАБЛЮДЕНИЙ

Настоящая статья обязана своим появлением двум обстоятельствам:

1° Предполагая в нескольких дальнейших статьях опубликовать изложение своих точек зрения и исследований по вопросу теоретико-вероятностного обоснования математической статистики, автор считает целесообразным предпослать этим статьям детальный критический разбор существующих методов, проведенный на материале какой-либо достаточно простой классической задачи математической статистики. Для этого было вполне естественно остановиться на задаче оценки параметров Гауссовского закона распределения по данным n независимых наблюдений.

2° К автору обратились с просьбой дать свое заключение по поводу разногласий, имеющихся среди артиллеристов, относительно приемов оценки меры точности по опытным данным (см., например, [10], [11], [12]). В связи с рассмотрением этих разногласий автору стала ясной желательность познакомить артиллеристов с результатами Student'а и Фишера, относящимися к малым выборкам. Этими запросами и определился окончательно конкретный материал настоящей статьи.

Из сказанного ясно, что статья претендует по преимуществу лишь на методологический интерес. Новыми с фактической стороны автору представляются в ней определение исчерпывающей статистики и исчерпывающей системы статистик в § 2 и уточнения остаточных членов в предельных теоремах в § 4.

Необходимость критического сопоставления различных подходов к разбираемым задачам привела неизбежно к тому, что статья получилась довольно объемистой по сравнению с элементарностью разбираемых в ней вопросов.

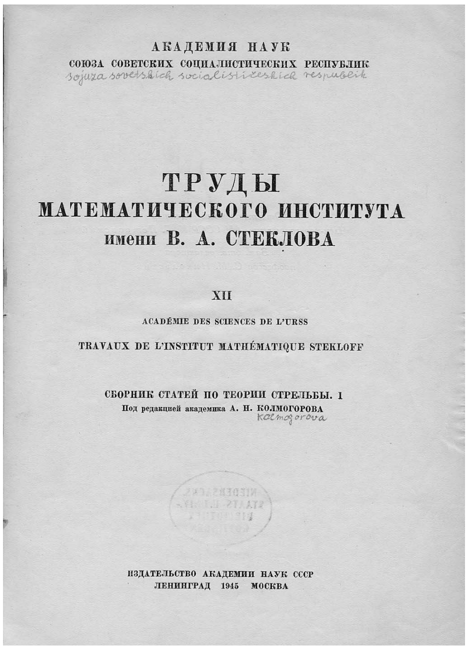
Введение

Допустим, что случайные величины

$$x_1, x_2, \ldots, x_n$$

независимы и подчинены каждая Гауссовскому закону распределения с общим им всем центром рассеивания a и мерой точности h. Как известно, в этом случае n-мерный закон распределения величин x_i определяется следующей плотностью вероятности:

$$f(x_1, x_2, \ldots, x_n \mid a, h) = \frac{h^n}{\pi^{\frac{n}{2}}} \exp(-h^2 S^2), \tag{1}$$



АКАДЕМИЯ НАУК
СОЮЗА СОВЕТСКИХ СОЦИАЛИСТИЧЕСКИХ РЕСПУБЛИК
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ТРУДЫ
МАТЕМАТИЧЕСКОГО ИНСТИТУТА
имени В. А. СТЕКЛОВА

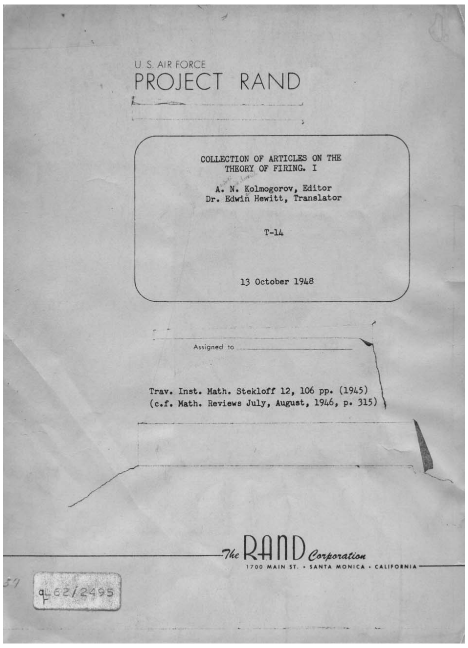
XII

ACADÉMIE DES SCIENCES DE L'URSS

TRAVAUX DE L'INSTITUT MATHÉMATIQUE STEKLOFF

СБОРНИК СТАТЕЙ ПО ТЕОРИИ СТРЕЛЬБЫ. I
Под редакцией академика А. Н. КОЛМОГОРОВА
Kolmogorova

ИЗДАТЕЛЬСТВО АКАДЕМИИ НАУК СССР
ЛЕНИНГРАД 1945 МОСКВА



U. S. AIR FORCE
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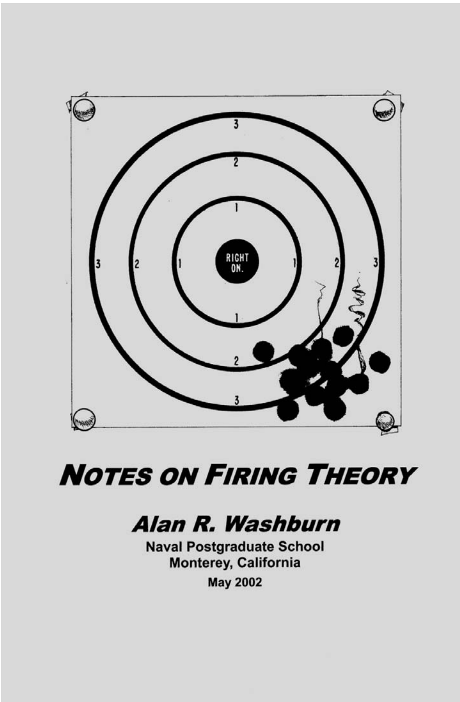

NOTES ON FIRING THEORY

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May 2002

Figure 1.
Kolmogorov's *Theory of Firing*, four front pages
1942 (*opposite page*), 1945 (*above left*),
1948 (*above right*), 2002 (*left*).

The work by Kolmogorov and collaborators on Firing Theory was interesting enough to be translated by the RAND Corporation in 1948 and is still regarded as fundamental in quite recent US military education. It is one of the apparent paradoxes of the relation between mathematics and warfare that an earlier paper by Kolmogorov on the same topic was published in 1942 for everybody to read (including the enemy). Could it be that this early paper (in contrast to the 1945 book) was too mathematical and too general to inform military practice directly?
Kolmogorov himself modestly remarked that the article does not claim anything more than a certain methodological value. On the other hand: was the 1945 work after all too closely linked to a specific problem to inspire further mathematical development? [Editor's note]

collaborators, A. A. Sveshnikov (*Optimal artificial dispersion of firing for some cases*) and I. A. Gubler (*Solution of the firing problem with artificial dis persion for some cases*). Kolmogorov was both its editor and the author of the preface.

The paper *On the number of hits among several shots and general principles of characterizing efficiency of firing strategies* considers the number of hits μ in a group of n shots ($\mu = 0, 1, \dots, n$). Let $P_k = P(\mu = k)$ and E_μ be the expectation of the number of hits. Kolmogorov defines a new term

"efficiency criterion of a firing strategy."

He notes that the "thoughts accepted in the literature on comparative pros and cons of 'expectation criterion' and 'probability criterion' do not provide suffcient clarity." He formulates a problem: can the probabilities $P_0, P_1, \dots, P_n$ that characterize the firing strategy by the hit probability distribution, "be replaced by a single quantity $W = f(P_0, P_1, \dots, P_n)$ that depends on them all, and which could then be declared the efficiency criterion of a firing strategy." Following discussion of this question, A. N. Kolmogorov derives several exact formulas for probabilities P_k as well as their practical approximations.

The next batch of problems this paper touched upon concerns two issues. The first one is the classification of factors that affect firing in order to determine a practical firing strategy. The second is the problem of artificial dispersion (the term itself is discussed in detail also), and is defined as follows.

Let $p_i = p_i(\alpha_i, \beta_i)$ be the probability that the i-th shot is a hit, depending on *azimuth* α_i and *elevation* β_i. Let $(\bar{\alpha}_i, \bar{\beta}_i)$ be a pair (usually unique) of values of α_i and β_i that maximizes the hit probability:

$$\max p_i(\alpha_i, \beta_i) = p_i(\bar{\alpha}_i, \bar{\beta}_i)$$

and

$$\bar{\alpha} = (\bar{\alpha}_1, \dots, \bar{\alpha}_n),\ \bar{\beta} = (\bar{\beta}_1, \dots, \bar{\beta}_n).$$

The question under consideration is whether $\max W = W(\bar{\alpha}, \bar{\beta})$, i.e., is it suffcient to maximize the hit probability of each individual shot in order to maximize our overall firing efficiency?

It is noted in the paper that the property $\max W = W(\bar{\alpha}, \bar{\beta})$ holds in two special cases. The first is when $W = E_\mu$ (where μ is the number of hits out of n shots). In the second case, $W = c_1 P_1 + \cdots + c_n P_n$ ($c_i \geq 0$) and the events of hitting the target are independent. Thus in those particular cases, the most practical firing strategy is to *maximize the hit probability for each individual shot*.

However this is not true for some other firing efficiency criteria. In other words, in order to maximize a given criterion, one sometimes has to deliberately choose settings (azimuth and elevation) on a per-shot basis that deviate from the values maximizing the individual hit probability. This is called firing with artificial dispersion. It is practical, for example, in the case when it is "imperative to achieve at least a few hits, the number of hits being possibly much smaller than the total number of shots n."

In the second paper *Artificial dispersion in the case of a single shot kill and onedimensional dispersion* from the *Collection of Articles on the Theory of Firing*, Kolmogorov considers the question of *artificial dispersion* when firing at “small” targets. In an earlier draft of this paper, he sets out the following visual justification for dispersion: “Occasionally, when firing repeatedly at the same target, it is beneficial to artificially increase the dispersion of projectiles in order to maximize the hit probability; similarly a hunter prefers to use pellets with sufficiently wide dispersion cone when aiming at a flying bird. The most dramatic example of this is when it is sufficient to hit once in order to destroy a target.”

In considering this very case, Kolmogorov simplifies the problem further by allowing dispersion only in one dimension. A typical example of such a situation would be: bombardment of a narrow stretch (say, a bridge), located perpendicularly to the plane of bombardment (in this case, errors in azimuth are immaterial, and the hit probability depends only on the elevation) or mine shelling at sea (where only the choice of azimuth is essential). Kolmogorov remarks that the formulas derived in this case, “belonging to such a ‘onedimensional’ case allow for direct *practical* applications.”

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Kolmogorov, A. N. (1945) (ed.): *Collection of papers on the theory of artillery fire*. I. (Russian). Trav. Inst. Math. Stekloff 12, Inst. Math. Stekloff, Moscow, (1945); MR **7** (1946), 315h (Reviewer: J. L. Doob). American translation by Edwin Hewitt, RAND Corporation, Santa Monica, California, 1948.

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