

# Chapter 2

## The Physical Layer

### 2.1 Introduction

This chapter covers some of the most important aspects of the Physical Layer of the protocol stack related to the study of WSNs. The chapter is not meant to cover available communication technologies, such as modulation techniques, noise and interference issues, and the like. Rather the chapter provides the reader with basic understanding of some fundamental concepts that are useful for later sections and chapters, such as wireless channel propagation models, energy consumption models, and sensing and error models in WSNs.

### 2.2 Wireless Propagation Models

This section provides an introduction to wireless propagation models. In wireless communications, signals travel from sender to receiver through the radio channel. These signals, which are sent at a particular power by the transmitter, suffer attenuation in the radio channel. The receiver, at the other end, is only able to receive the sender's transmission if the signal is received with a power level greater than the sensitivity of its transceiver. The attenuation, commonly known as the *path loss* of the channel, directly depends on the distance between sender and receiver, the frequency of operation and other factors. Path loss models exist to predict if there is a radio channel between two nodes. This section describes the three most commonly known path loss models available in the literature: the free space model, the two-ray ground model, and the log-distance path model [101].

### 2.2.1 The Free Space Propagation Model

The Friis' free-space propagation model applies when there is a direct and unobstructed path between sender and receiver, i.e., there is line-of-sight. The received power at a distance  $d \geq d_0$  meters between sender and receiver is given by Friis' path loss equation 2.1.

$$P_{rx}(d) = \frac{P_{tx} \times G_{tx} \times G_{rx} \times \lambda^2}{(4\pi)^2 \times d^2 \times L} = C_f \times \frac{P_{tx}}{d^2} \quad (2.1)$$

where  $P_{rx}(d)$  is the power received at the receiver over distance  $d$ ,  $P_{tx}$  is the power at which the signal was transmitted,  $G_{tx}$  and  $G_{rx}$  are the gains of the antennae of the transmitter and receiver, respectively,  $L \geq 1$  includes the losses in the transmitter and receiver circuits,  $\lambda$  is the wavelength in meters, and  $C_f$  is a constant that depends on the transceivers.

As it can be seen from Equation 2.1, the signal attenuates proportional to the square of the distance  $d$  that it has to travel. From the transmitter's point of view, Equation 2.1 says that a disk of radius  $r = \sqrt{C_f \times P_{tx}}$  is created and centered at the node equivalent to its area of coverage.

### 2.2.2 The Two-Ray Ground Model

The two-ray ground model assumes a more realistic scenario in which the receiver receives signals that travel directly from sender to receiver and other signals that reach the receiver, as shown in Figure 2.1. This model is definitively more accurate than the Friis' free space model described before. The power received at the receiver over distance  $d$  is now given by Equation 2.2.

$$P_{rx}(d) = \frac{P_{tx} \times G_{tx} \times G_{rx} \times h_{tx}^2 \times h_{rx}^2}{d^4} = C_t \times \frac{P_{tx}}{d^4} \quad (2.2)$$

where  $h_{tx}$  is the height of the antenna of the transmitter,  $h_{rx}$  is the height of the antenna of the receiver,  $G_{tx}$  and  $G_{rx}$  are the gains of the antennae of the transmitter and receiver, respectively,  $P_{tx}$  is the power at which the signal is transmitted, and  $C_t$  is a constant that depends on the transceivers. As it can be seen from Equation 2.2, with the two-ray ground model, the signal now attenuates proportional to the fourth power of the distance  $d$ . Therefore, the transmitter has now a disk of radius  $r = \sqrt[4]{C_t \times P_{tx}}$  equivalent to its area of coverage.

### 2.2.3 The Log-Distance Path Model

The log-distance path model was derived based on field measurements and curve fitting the collected data. This is a commonly used approach to derive path models in uncommon environments, such as caves or other places with abundant obstacles or reflecting materials. The log-distance model is given by Equation 2.3.

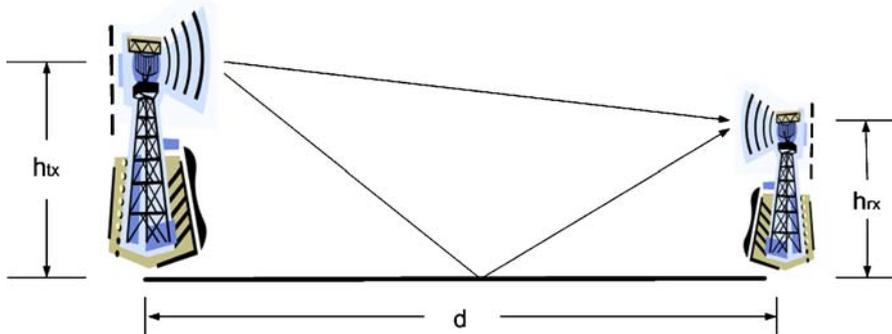


Fig. 2.1 The two-ray propagation model.

$$P_{r,x}(d) \propto \frac{P_{t,x}}{d^\alpha} \quad (2.3)$$

which says that the path loss is proportional to the transmission power  $P_{t,x}$  and to the distance  $d$  between sender and receiver raised to the path loss exponent  $\alpha$  that depends on the environment. In this case, the transmitter is able to have a disk of radius  $r = \sqrt[\alpha]{P_{t,x}}$  equivalent to its area of coverage.

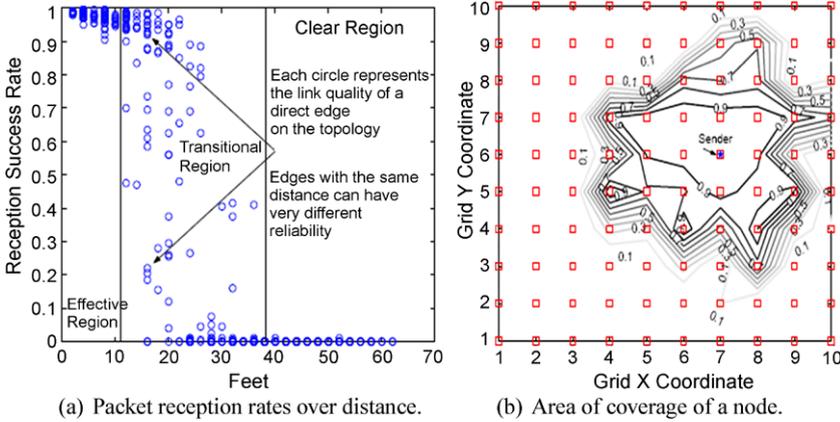
Equation 2.3 can be expressed in decibels as:

$$PL_{r,dB}(d) = PL_{dB}(d) + 10\alpha \log\left(\frac{d}{d_0}\right) + X_{\sigma,dB} \quad (2.4)$$

where  $PL_{r,dB}(d)$  is the received power in dB,  $PL_{dB}(d)$  is the path loss in dB from sender to receiver over a distance  $d$ ,  $d_0$  is a reference distance,  $\alpha$  is the path loss exponent, and  $X_{\sigma,dB}$  is a zero-mean Gaussian random variable in dB with standard deviation  $\sigma$ .

Two interesting conclusions can be derived directly from Equation 2.4. First, the received power decreases with the frequency of operation. Second, the received power depends on the distance  $d$  according to a power law. A node at a distance  $\alpha d$  to some receiver must spend  $\alpha^2$  times the energy of a node at distance  $d$  from the same receiver with the same received power  $P_{r,x}$ .

Although analytical models and equations are good to predict the received power, empirical studies are important to validate the analytical results and better understand the specifics of a particular environment. One typical example is the assumption of a perfect disk area of coverage, as given by the equations above. An interesting empirical study of connectivity with a platform of MICA motes was carried out in [129] to show that connectivity is not binary but instead a probability of successful communication modified by fading, interferences, and other sources of loss. Figure 2.2(a) shows how nodes that are similar distances apart obtain very different packet reception rates, especially within the transitional region where individual pairs exhibit high variation. A link quality model with respect to distance was de-



**Fig. 2.2** Empirical packet reception rates and area of coverage using an empirical path loss model [129] © 2003 ACM, Inc. Included here by permission.

rived from the observations in Figure 2.2(a) and later used to build a node's area of coverage, which definitively doesn't show a perfect circle (Figure 2.2(b)).

### 2.3 Energy Dissipation Model

Energy dissipation models are very important in WSNs, as they can be utilized to compare the performance of different communication protocols from the energy point of view. A very simple and commonly used energy dissipation model is the first order radio model introduced in [47]. The model, shown in Figure 2.3, considers that to transmit a  $k$ -bit packet from sender to receiver over a distance  $d$ , the system spends:

$$E_{Tx}(k, d) = E_{elec-Tx}(k) + E_{amp-Tx}(k, d) \quad (2.5)$$

$$E_{Tx}(k, d) = E_{elec} \times k + E_{amp} \times k \times d^2 \quad (2.6)$$

where  $E_{Tx}(k, d)$  is the energy consumed by the transmitter to send a  $k$ -bit long packet over distance  $d$ ,  $E_{elec-Tx}(k)$  is the energy used by the electronics of the transmitter, and  $E_{amp-Tx}(k, d)$  is the energy expended by the amplifier. Similarly, at the receiving node, to receive the message the transceiver spends:

$$E_{Rx}(k) = E_{elec-Rx}(k) \quad (2.7)$$

$$E_{Rx}(k) = E_{elec} \times k \quad (2.8)$$

where  $E_{Rx}(k)$  is the energy consumed by the receiver in receiving a  $k$ -bit long packet, which is given by the energy used by the electronics of the receiver  $E_{elec-Rx}(k)$ .

In the same paper, the authors consider  $E_{elec} = 50$  nJ/bit as the energy consumed by both the transmitter and receiver circuitry,  $E_{amp} = 100$  pJ/bit/m<sup>2</sup>, as the

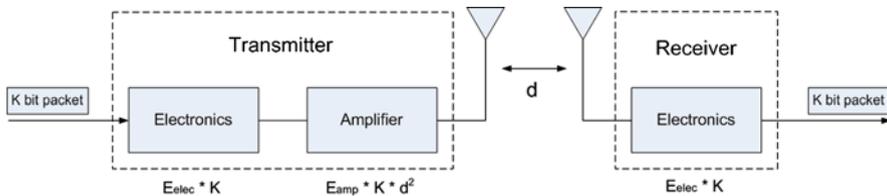


Fig. 2.3 The first order energy dissipation model introduced in [47].

energy consumed by the amplifier, and a path loss exponent  $\alpha = 2$ . For these numbers, which are typical values, it can be concluded that receiving packets is not a low cost operation; therefore, care must be taken in energy-efficient routing because paths with many short links may consume more energy than paths with fewer but longer links.

## 2.4 Error Models

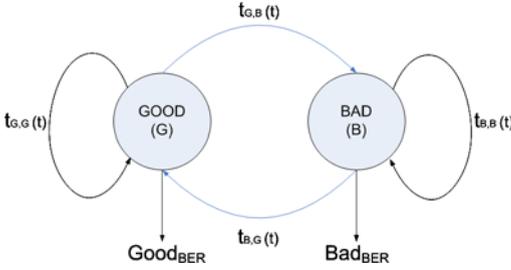
Errors occur in different patterns and quantity according to the physical media and some other phenomena such as node speed, obstructions, multi-path, fading, etc. For example, in fiber optic-based channels, errors are considered to occur randomly and with very low rates, in the order of  $10^{-9}$  or better. However, wireless channels are usually in the opposite side of the spectrum with bit error rates in the order of  $10^{-6}$  or worse and errors occurring in “bursts”. The manner errors occur and the quantity of errors have direct and important implications in the design and analysis of communication protocols and mechanisms, as it will be shown in later chapters. Two error models commonly used to model these two different scenarios and analyze the performance of communication protocols and mechanisms are the **independent error model** and the **two-state Markov error model**, which are described next.

### 2.4.1 The Independent Error Model

The first model, called the independent error model, assumes that errors occur at random and, therefore, they are independent. This model is “memoryless” as there is no temporal correlation between the symbols, i.e., the probability that one symbols is in error is not affected by what happened to any other symbol. This model is widely used because of its simplicity for mathematical analysis. Under the independent model, the probability that a frame of size  $L$  bits is received in error,  $P_e$ , is given by Equation 2.9 as:

$$P_e = 1 - (1 - p)^L \quad (2.9)$$

where  $p$  is the Bit Error Rate (BER) of the channel. This model is easy to understand and simple to use but it does not reflect the real behavior of several channels, and wireless channels in particular. In wireless channels, errors occur in “bursts” and, therefore, they are correlated.



**Fig. 2.4** Two-state Markov error model for wireless channels.

#### 2.4.2 The Two-State Markov Error Model

A more appropriate model for wireless channels is the two-state Markov error model described in [122]. This model, illustrated in Figure 2.4, is implemented using a Discrete Time Markov Chain (DTMC) that models channel conditions at the bit level. During the “Good” state the channel is assumed to be in good condition with certain  $Good_{BER}$ . On the other hand, the “Bad” state indicates that the channel is experiencing a degradation in its quality providing a worse BER,  $Bad_{BER}$ , that produces a larger number of errors.

The final effect of the two-state Markov model is shown in Figure 2.5. Normally, the model stays more time in the Good state during which the channel exhibits a good quality. On the other hand, the chain stays in the Bad state for a rather short duration of time during which a higher BER is introduced. These long and short durations produce a “bursty” error model that reproduces common wireless effects, such as short fading, multi-path cancellations, etc., very accurately.

The two-state Markov model is characterized by four transition probabilities, the initial state probability distribution, and the error probability matrix. The transition probabilities indicate the probability of the chain of being in state  $S = \{G(Good), B(Bad)\}$  at time  $t + 1$  given that it was in state  $S$  at time  $t$ . Mathematically, these probabilities can be written as:

$$\begin{aligned}
 t_{G,G}(t) &= P\{S_{t+1} = G | S_t = G\} \\
 t_{G,B}(t) &= P\{S_{t+1} = G | S_t = B\} \\
 t_{B,G}(t) &= P\{S_{t+1} = B | S_t = G\} \\
 t_{B,B}(t) &= P\{S_{t+1} = B | S_t = B\}
 \end{aligned} \tag{2.10}$$

which can be expressed in matrix form as:

$$T(t) = \begin{bmatrix} t_{G,G}(t) & t_{G,B}(t) \\ t_{B,G}(t) & t_{B,B}(t) \end{bmatrix} \tag{2.11}$$

which is called the *state transition matrix*.

It is well known that the state distribution  $\Pi_{t+k}$  at time  $t + k$  can be easily found in an iterative manner using the state transition matrix as:

$$\Pi_{t+k} = \Pi_t \times T^k \tag{2.12}$$

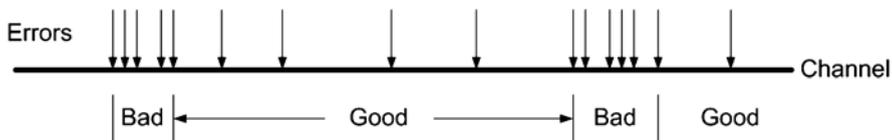


Fig. 2.5 Example of error pattern as produced by a two-state Markov error model.

where  $\Pi_t$  for  $t = 0$  is the initial state probability distribution  $\Pi_0$ , which can be set to any value. The steady state distribution is then equal to:

$$\Pi_{ss} = [\pi_G \pi_B] \quad (2.13)$$

which is the same  $\Pi_{t+k}$  for a sufficiently large  $t$  and arbitrary value of  $k$ .

Finally, the *error probability matrix*  $E$  is defined as:

$$E = \begin{bmatrix} P\{C|G\} & P\{C|B\} \\ P\{M|G\} & P\{M|B\} \end{bmatrix} \quad (2.14)$$

where  $P\{C|G\}$  is the probability that a Correct decision was made given that the chain was in the Good state, or the probability of having a good bit given that the channel was in the Good state;  $P\{M|G\}$  is the probability of making a Mistake, or the probability of having a bad bit given that the channel was in the Good state;  $P\{C|B\}$  is the probability of having a bad bit given that the channel was in the Bad state; and  $P\{M|B\}$  is the probability of having a good bit given that the channel was in a Bad state.

By simple matrix multiplication, the probabilities of making a Correct decision or making a Mistake are calculated a follows:

$$[P_C \ P_M] = \Pi_{SS} \times E^T \quad (2.15)$$

where  $E^T$  is the transpose of matrix  $E$ .

Note that if the system is error free during the Good state, the well-known two-state Gilbert model is obtained, which was used by Gilbert to calculate the capacity of a channel with bursty errors [41].

An important question is, how can we derive the state transition matrix  $T$  and the error probability matrix  $E$  from either channel simulation results or real channel measurements, so that the two-state Markov chain accurately models the channel that originated those results or measurements? The answer to this questions is found in the well-known iterative procedure given by the Baum–Welch algorithm [8], which given a sequence of observations  $O = \{O_1, O_2, \dots, O_t, \dots, O_T\}$ , finds the maximum likelihood estimator  $\Gamma = \{T, E\}$  that maximizes the probability that the sequence of observations were produced by the estimated parameters of the model  $T$  and  $E$ ,  $P\{O|\Gamma\}$ . For the interested reader, all the necessary steps and software code needed to calculate and implement the Baum–Welch algorithm and, therefore, calculate the channel model  $\Gamma$  are very well detailed in [122].

Another way to calculate the probabilities of the two-state Markov model is given in [68, 101]. Under this method, the  $P(\text{Good})$ ,  $P(\text{Bad})$  and the transition probabilities of the DTMC  $t_{G,B}$  and  $t_{B,G}$  are calculated assuming a Rayleigh fading channel. The method calculates the average number of positive crossings of the signal per second as:

$$N = \sqrt{2\pi} \times f_m \times \rho \times e^{-\rho^2} \quad (2.16)$$

where  $f_m$  is the maximum Doppler frequency and  $\rho$  is the Rayleigh fading envelope normalized to the local rms level. The average time  $T$  during which the signal is below the threshold  $R$  is:

$$T = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} \times f_m \times \rho} \quad (2.17)$$

With these equations,  $P(\text{Good})$  and  $P(\text{Bad})$  are calculated as follows:

$$P(\text{Good}) = \frac{1/N - T}{1/N} = e^{-\rho^2} \quad (2.18)$$

$$P(\text{Bad}) = \frac{T}{1/N} = 1 - e^{-\rho^2} \quad (2.19)$$

And the transition probabilities are then given by:

$$t_{G,B} = \frac{N}{R_t \times P(\text{Good})} \quad (2.20)$$

$$t_{B,G} = \frac{N}{R_t \times P(\text{Bad})} \quad (2.21)$$

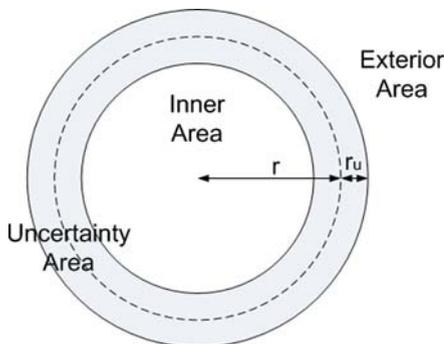
where  $R_t$  is the transmission rate in symbols per second of the communication system under consideration.

Using these results, the final average BER can be calculated as the summation of the probability of being in either state times the BER of each state as:

$$BER = \text{Good}_{BER} \times P(\text{Good}) + \text{Bad}_{BER} \times P(\text{Bad}) \quad (2.22)$$

## 2.5 Sensing Models

Similar to the communication coverage of a node, which is determined by the transmission power of the transceiver and the propagation models discussed before, sensors have a their own *sensing coverage*. An important aspect to emphasize here is that the sensing coverage does not necessarily match the communication coverage, therefore ensuring connectivity and communication coverage does not mean that the network also ensures sensing coverage. There are two widely known sensing models utilized in the literature to model the sensing coverage: the **binary sensing model** and the **probabilistic sensing model**.



**Fig. 2.6** The probabilistic sensing model.

### 2.5.1 The Binary Sensing Model

The binary sensing model assumes that the sensing coverage of a sensor  $s$ ,  $C(s)$ , is given by a disk with a fixed radius  $r$ . Any event that occurs at a point  $p$  in the 2-dimensional plane,  $(x_p, y_p)$ , will be detected by sensor  $s$ , if the Euclidean distance between  $s$  and  $p$ ,  $d(s, p)$ , is within the sensing range of  $s$ , as follows [147]:

$$C(s) = \begin{cases} 1, & \text{if } d(s, p) \leq r \\ 0, & \text{otherwise} \end{cases} \quad (2.23)$$

According to Equation 2.23, the sensor will detect event  $e$  with probability 1, if  $d(s, p)$  is less than or equal to  $r$ , and will not detect it otherwise. This deterministic model is not very realistic given the imprecise sensor detections. In reality, the sensor coverage is not a perfect circle and the coverage is modeled using a probabilistic model.

### 2.5.2 The Probabilistic Sensing Model

Under the probabilistic sensing model, sensors have three very well defined areas, as shown in Figure 2.6. The inner area is given by the distance  $r - r_u$ , which ensures that the sensor will detect event  $e$  with probability 1, where  $r_u$  is known as the uncertainty distance. The exterior area is just the opposite. This is the part where the probability of sensing event  $e$  is zero. In other words,  $d(s, p)$  is greater than  $r + r_u$ . Finally, there is the uncertainty area (gray area in the figure) in which the sensor will detect event  $e$  with certain probability that decays exponentially with the distance. Mathematically, sensor  $s$  sensing coverage can be expressed as [147]:

$$C(s) = \begin{cases} 1, & \text{if } r - r_u \geq d(s, p) \\ e^{-\lambda \alpha^\beta}, & \text{if } r - r_u < d(s, p) \leq r + r_u \\ 0, & \text{if } r + r_u < d(s, p) \end{cases} \quad (2.24)$$

where  $\alpha = d(s, p) - (r - r_u)$  and  $\beta$  and  $\lambda$  are parameters that yield different detection probabilities that can be used to model different types of physical sensors, in particular, range sensing devices, such as infrared and ultrasound sensors.