1 Basic electromagnetism

1.1 Introduction

Two thousand years ago, the Chinese invented the compass, a special metallic needle with one end always pointing to the North Pole. That was the first recorded human application of magnetism. Important understandings and developments were achieved in the mid 19th century and continue into the present day. Indeed, today, magnetic devices are ubiquitous. For example, to name just two: energy conversion devices provide electricity to our homes and magnetic recording devices store data in our computers. This chapter provides an introduction to basic magnetism. Starting from the simple attractive (or repelling) force between magnets, we define magnetic field, dipole moment, torque, energy and its equivalence to current. Then we will state the Maxwell equations, which describe electromagnetism, or the relationship between electricity and magnetism.

A great tutorial is provided by Kittel [1], which may be used to support students studying Chapters 1–4.

1.2 Magnetic forces, poles and fields

In the early days, magnetic phenomena were described as analogous to electrical phenomena: like an electric charge, a magnetic pole was considered to be the source of magnetic field and force. The magnetic field was defined through the concept of force exerted on one pole by another. In cgs units, the force is proportional to the strength of the magnetic poles, defined as

$$F = \frac{p_1 p_2}{r^2}, \tag{1.1}$$

where r is the distance between two poles (in units of centimeters) and the unit of force F is the dyne. There is no unit for the pole, p. This equation defines the pole strength as one unit, when the force is 1 dyne and the distance between the poles is 1 cm. Analogous to Coulomb's Law of electric charge, one may consider a magnetic pole, say p_1, which generates a magnetic field H, and H exerts a force on the other pole, p_2. Thus,

$$F = \left(\frac{p_1}{r^2}\right) p_2 = H\, p_2, \tag{1.2}$$

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where H is given by

$$H = \frac{p_1}{r^2}. \tag{1.3}$$

Thus, a magnetic field H of unit strength exerts a force of 1 dyne onto one unit of magnetic pole. The unit of the magnetic field in cgs units is the oersted (Oe). To get a feel for the strength of the magnetic field, at the end of a magnetic bar on a classroom white board the magnetic field can be as high as 5000 Oe, whereas the earth's magnetic field is smaller than 0.5 Oe.

1.3 Magnetic dipoles

Although a magnetic pole is the counterpart of an electric charge, there is a difference. Magnetic poles always come in pairs: a north pole and a south pole. A monopole has never been found. This pair of positive and negative poles occurs at the same time and forms a dipole. For example, a bar magnet always has a north pole at one end and a south pole at the other. Magnetic field lines emit from one pole, diverge into the surroundings and then converge and return into the other pole of the magnet. Figure 1.1 shows the field lines around a magnet.

If a bar magnet with a north pole and a south pole is dissected into two bars, will these two bars becomes two magnets? The answer is yes, since poles are always in pairs.

In 1820, H. C. Oersted discovered that a compass needle could be deflected when electric current passes through a wire positioned near to the compass. This was the first time electricity was linked to magnetic phenomena. Subsequent work by André-Marie Ampère established the basis of modern electromagnetism. He established the relationship between a magnetic dipole and a circulating current in a conductor loop around an axis. The direction of the dipole is along the axis of the loop, which is orthogonal to the loop plane. Figure 1.2 illustrates

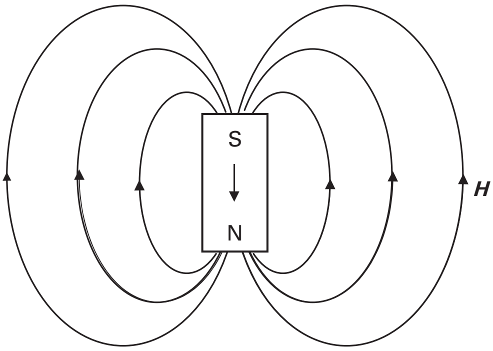

Figure 1.1. Magnetic field lines from a magnet.

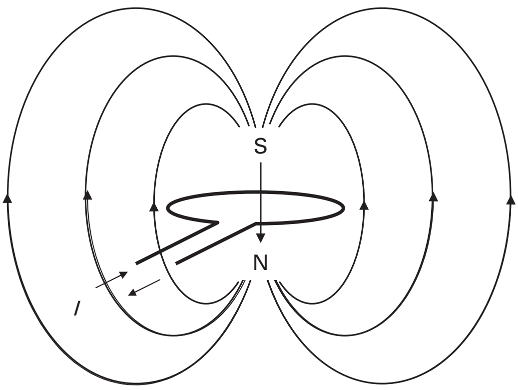

Figure 1.2. Magnetic field from a circular current loop.

the relationship between the dipole and the current loop. The polarity is dictated by the direction of the current, I. Reversing the direction of the electric current changes the polarity of the dipole. Thus, the magnetic dipole is another form of electric current, or moving electric charge.

Although both the electric field and the magnetic field are originated from electric charges, the difference is that the magnetic field must come from moving electric charges or electric current, rather than a stationary electric charge. The moving electric charge concept adequately explained the origin of magnetic poles at the time. The observation was later proven incorrect, however, when electron spin was taken into consideration. We will discuss this topic in a later part of this book.

1.4 Ampère's circuital law

Ampère further established that the relationship between the magnetic field H and the current I is given by

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = 4\pi \cdot 10^{-4} I \tag{1.4a}$$

where, in cgs units, $\boldsymbol{H}$ is in oersted, $d\boldsymbol{l}$ is in centimeters and I is in milliamperes, and later one finds that it is more convenient to calculate H in thin films using

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = 4\pi I \tag{1.4b}$$

($\boldsymbol{H}$ is in oersted, $d\boldsymbol{l}$ is in micrometers and I is in milliamperes), where $d\boldsymbol{l}$ is the segment length of an arbitrary closed loop where the integration is performed and I is the current within the closed loop. This law is simple in concept and is

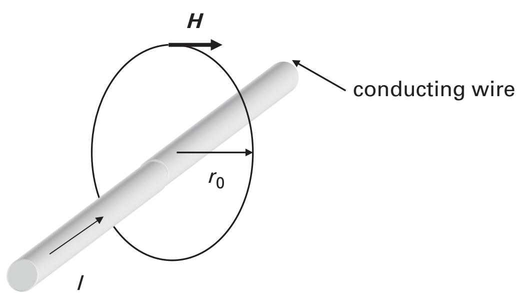

Figure 1.3. Magnetic field around a conducting wire carrying a current I. The magnetic field at a distance r_0 from the wire is $H = I/2\pi r_0$.

particularly useful in computing the field generated by the current in a long conductor and conducting thin film.

Here, we would like to discuss the units. Historically, there have been two complementary ways of developing the theory and definitions of magnetism. As a result, there are two sets of units for magnetic field, and thus for a magnetic pole. The definitions are similar, but not entirely identical. The major difference lies in how the magnetic field is defined inside the material. Centimeter-gram-second (cgs) units are used for studying physics, such as the origin of the magnetic pole and the magnetic properties in a material. The Systéme International d'Unités (SI units) are frequently used for obtaining magnetic field strength from circulating currents. Engineers working on electromagnetic waves, electric motors, etc. like to use SI units. This book will use both sets of units, depending on whichever makes more sense and in line with journal publications.

In SI units, Ampère's law is given as

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I. \tag{1.5}$$

(In SI units, $\boldsymbol{H}$ is given in amperes/meter, $d\boldsymbol{l}$ is in meters and I is in amperes.) From these two equations, one finds that a magnetic field of 1 (Oe) = 1000/4π (A/m) ~ 80 (A/m).

Example 1.1: The magnetic field lines go around a current-carrying wire in closed circles, as illustrated in Fig. 1.3. At a distance r_0 from the conductor, the magnitude of the field $\boldsymbol{H}$ is constant. This makes the line integral of Ampère's law straightforward. It is simply given by

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = 2\pi r_0 \boldsymbol{H} = I,$$

and so the field $\boldsymbol{H}$ is given by

$$H = \frac{I}{2\pi r_0}.$$

1.5 Biot–Savart Law

An equivalent statement to Ampère's circuital law (which is sometimes easier to use for certain systems) is given by the Biot–Savart Law. The Biot–Savart Law states that the fraction of a field, $\delta\boldsymbol{H}$, is contributed by a current I flowing in an elemental length, $\delta\boldsymbol{l}$, of a conductor:

$$\delta\boldsymbol{H} = \frac{1}{4\pi r^2} I \; \delta\boldsymbol{l} \times \boldsymbol{n}, \tag{1.6}$$

(in SI units), where r is the radial distance from the current element and $\boldsymbol{n}$ is a unit vector along the radial direction from the current element to the point where the magnetic field is measured. Note that the direction of the vector $\delta\boldsymbol{H}$ is orthogonal to the plane formed by $I\,\boldsymbol{dl}$ and $\boldsymbol{n}$, as a result of the vector operation "$\times$" of two vectors $I\,\boldsymbol{dl}$ and $\boldsymbol{n}$, and the amplitude of $|I\,\boldsymbol{dl}|\sin\theta$, where θ is the angle between vectors $\boldsymbol{dl}$ and $\boldsymbol{n}$.

Example 1.2: Field from a current in a loop wire The magnetic field at the center of the loop plane as shown in Fig. 1.2 is calculated by the Biot–Savart Law as follows.

The radius of the loop is r_0, and H can be in the positive or negative z-direction, depending on the current direction, and only in the z-direction. The vector sum is simplified into a scalar sum. On the loop plane, $z = 0$. So, $|\boldsymbol{H}| = H_0$, where H_0 is the integral of the field contributed by each segment $\boldsymbol{dl}$ of the loop, and

$$H_0 = 2\pi r_0 \left[\frac{1}{4\pi r_0^2} I \right] = \frac{1}{2r_0}, \tag{1.7}$$

(in SI units) and

$$\boldsymbol{H} = H_0 \boldsymbol{n}_z,$$

where $\boldsymbol{n}_z$ is the unit vector in the z-direction.

1.6 Magnetic moments

Next we need to introduce the concept of magnetic moment, which is an angular moment exerted on either a bar magnet or a current loop when it is in a magnetic field. The angular moment causes the dipole to rotate.

For a bar magnet positioned at an angle ϕ to a uniform magnetic field, $\boldsymbol{H}$, as shown in Fig. 1.4, the forces on the pair of poles are given by $\boldsymbol{F}_+ = +p\boldsymbol{H}$ and $\boldsymbol{F}_- = -p\boldsymbol{H}$. The two forces are equal but have opposite direction. So, the moment acting on the magnet, which is just the force times the perpendicular distance from the center of the mass, is

$$pH\sin\phi(l/2) + pH\sin\phi(l/2) = pH\;l\sin\phi = mH\sin\phi, \tag{1.8}$$

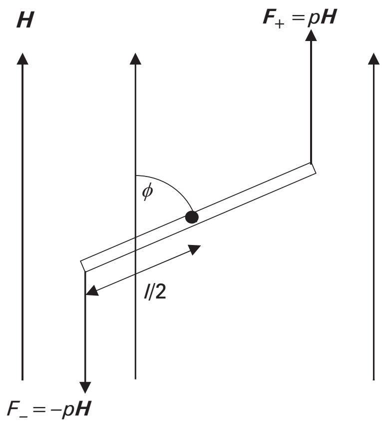

Figure 1.4. Dipole moment of a bar magnet in a uniform magnetic field.

where $m = pl$, the product of the pole strength and the length of the magnet, is the amplitude of the magnetic moment. The magnetic moment is a vector, pointing to a direction normal to the plane formed by the magnet and the magnetic field. One cgs unit of magnetic moment is the angular moment exerted on a magnet when it is perpendicular to a uniform field of 1 Oe. The cgs unit of magnetic moment is the emu (electromagnetic unit).

Since a magnetic dipole is equivalent to a current loop, it can be quantified by loop area A and a current I in the loop, and its magnetic moment is defined as

$$\boldsymbol{m} = IA\boldsymbol{n}, \tag{1.9}$$

where $\boldsymbol{n}$ is a vector normal to the plane of the current loop. In SI units, magnetic moment is measured in amperes times squared meters (A m^2).

1.7 Magnetic dipole energy

A magnetic dipole can be defined in two ways. First, it is the magnetic moment, $\boldsymbol{m}$, of a bar magnet at the limit of very short but finite length. Second, it is the magnetic moment, $\boldsymbol{m}$, of a current loop at the limit of a very small but finite loop area. Either way, there is a finite magnetic moment.

The energy of a magnetic dipole is defined to be zero when the dipole is perpendicular to a magnetic field. So the work done in turning through an angle ϕ against the field is given by

$$\delta E = 2(pH \sin \phi)(l/2)\ d\phi = mH \sin \phi\ d\phi,$$

and the energy of a dipole at an angle ϕ to a magnetic field is given by

$$E = \int_{\pi/2}^{\phi} mH\ \sin\ \phi\ d\phi = -mH \cos \phi = -m \cdot H \tag{1.10}$$

This expression for the energy of a magnetic dipole in a magnetic field is in cgs units. In Eq. (1.10), E is in erg, $\boldsymbol{m}$ is in emu and $\boldsymbol{H}$ is in Oe. Equation (1.10) is also known as the formula for magnetostatic energy. In SI units the energy is $E = -\mu_0 \boldsymbol{m} \cdot \boldsymbol{H}$. When the dipole moment, $\boldsymbol{m}$, is in the same direction as $\boldsymbol{H}$, the magnetostatic energy takes its lowest value.

The torque exerted on a dipole moment is the gradient of the dipole energy with respect to the angle ϕ, or

$$\Gamma = dE/d\phi = mH \sin \phi. \tag{1.11}$$

The torque is exerted in the direction that lowers the dipole energy and the unit is expressed in erg/radian. When $\boldsymbol{m}$ and $\boldsymbol{H}$ are parallel, or $\phi = 0$, the energy is at a minimum, and the torque is zero. The torque is maximum when $\phi = \pi/2$. We will be using the concept of magnetic dipoles, and this expression for its energy in a magnetic field is used extensively throughout this book.

1.8 Magnetic flux

Here, we introduce another parameter: the flux Φ. Flux is defined as the integrated strength of a normal component of magnetic field lines crossing an area, or

$$\Phi = \int (\boldsymbol{H} \cdot \boldsymbol{n}) dA, \tag{1.12}$$

where $\boldsymbol{n}$ is the unit vector normal to the plane of the cross-sectional area, A. In cgs units, the flux is expressed in oersted times squared centimeters (Oe cm^2).

Magnetic flux is an important parameter in electric motor and generator design. The time-varying flux induces an electric current in any conductor which it intersects. Electromotive force ε is equal to the rate of change of the flux linked with the conductor:

$$\varepsilon = -\frac{d\Phi}{dt}. \tag{1.13}$$

This equation is Faraday's Law of electromagnetic induction. The electromotive force provides the potential difference that drives the electric current in a conductor. The minus sign indicates that the induced current sets up a time-varying magnetic field that acts against the change in the magnetic flux. This is known as Lenz's Law. The units in Eq. (1.13) as expressed in SI are: flux in webers (Wb), time in seconds and an electromotive force in volts.

1.9 Magnetic induction

When a magnetic field, $\boldsymbol{H}$, is applied to a material, the response of the material to $\boldsymbol{H}$ is called magnetic induction, $\boldsymbol{B}$. The relationship between $\boldsymbol{B}$ and $\boldsymbol{H}$ is a property of the material. In some materials (and in free space) $\boldsymbol{B}$ is a linear function of $\boldsymbol{H}$. But in general $\boldsymbol{B}$ saturates at high $\boldsymbol{H}$ field and sometimes $\boldsymbol{B}$ is history-dependent and multiple-valued for each value of $\boldsymbol{H}$. The equation relating $\boldsymbol{B}$ and $\boldsymbol{H}$ is given by

$$\boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}, \tag{1.14}$$

where $\boldsymbol{M}$ is the magnetization of the medium and $\boldsymbol{B}$ and $\boldsymbol{H}$ are given in cgs units gauss and oersted, respectively. The magnetization is defined to be the magnetic moment per unit volume:

$$\boldsymbol{M} = \boldsymbol{m}/V \tag{1.15}$$

in units of emu/cm^3. Note that $\boldsymbol{M}$ is a property of the material that depends on both the individual magnetic moments of the constituent ions, atoms and molecules, and on how these dipole moments interact with each other. Note that, although $\boldsymbol{M}$ is expressed in emu/cm^3, the unit of $4\pi\boldsymbol{M}$ in Eq. (1.14) is not emu/cm^3, but gauss. See [2, 3] for good reference articles. In a vacuum, the magnetic induction $\boldsymbol{B}$ equals the magnetic field $\boldsymbol{H}$ since $\boldsymbol{M} = 0$. Thus, 1 Oe field induces 1 gauss induction in a vacuum.

In SI units, the relation between $\boldsymbol{B}$, $\boldsymbol{H}$ and $\boldsymbol{M}$ is given by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}), \tag{1.16}$$

where μ_0 is the permeability of free space. The units of $\boldsymbol{M}$ are obviously the same as those of $\boldsymbol{H}$ (A/m), and those of μ_0 are Wb/(A m), also known as henry/m. So the units of $\boldsymbol{B}$ are Wb/m^2, or tesla (T), and 1 gauss = 10^{-4} tesla.

The magnetic induction, $\boldsymbol{B}$, is the same thing as the density of flux, $\boldsymbol{\Phi}$, inside the medium. So, $\boldsymbol{B} = \boldsymbol{\Phi}/A$ in a material, by analogy with $\boldsymbol{H} = \boldsymbol{\Phi}/A$ in free space. In general, the flux density inside a material is different from that outside. In fact, magnetic materials can be classified based on the difference between their internal and external flux.

1.10 Classical Maxwell equations of electromagnetism

The magnetic dipole is the product of a circulating charged particle around an axis. The circulating charged particle around an axis forms a circulating electric current. Thus, the magnetic dipole is another form of electric current. Therefore, magnetic and electric phenomena have the same origin. Maxwell studied their relationship and elegantly described them in the four classical Maxwell equations of electromagnetics in SI units as follows [4]:

$$\nabla \cdot \boldsymbol{D} = \rho, \tag{1.17}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.18}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_{\mathbf{c}} + d\boldsymbol{D}/dt, \tag{1.19}$$

$$\nabla \times \boldsymbol{E} = -d\boldsymbol{B}/dt. \tag{1.20}$$

They are frequently brought together with the current charge relation:

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}_{\mathbf{c}} = -d\rho/dt. \tag{1.21}$$

The mathematical operator $\boldsymbol{\nabla}$ is called *del*; $\boldsymbol{\nabla}\cdot$ means *divergence*. When operated on a vector $\boldsymbol{D}$, it means the *divergence of vector* $\boldsymbol{D}$. The result is a scalar, not a vector. In an (x, y, z)-coordinate system, it can be expressed as

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}. \tag{1.22}$$

The notation $\boldsymbol{\nabla}\times$ means *curl*. When operated on a vector $\boldsymbol{H}$, it means the *curl of vector* $\boldsymbol{H}$. The result is also a vector. In an (x, y, z)-coordinate system, it can be expressed as

$$\begin{aligned}\boldsymbol{\nabla} \times \boldsymbol{H} = \begin{vmatrix} \boldsymbol{n_x} & \boldsymbol{n_y} & \boldsymbol{n_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} &= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\boldsymbol{n_x} - \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\boldsymbol{n_y} \\ &+ \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\boldsymbol{n_z},\end{aligned} \tag{1.23}$$

where $\boldsymbol{n_x}$, $\boldsymbol{n_y}$, $\boldsymbol{n_z}$ are unit vectors in the x-, y- and z-directions, respectively.

The first equation, Eq. (1.17), shows that electrical flux $\boldsymbol{D}$ diverges out from an electric charge ρ. The second equation, Eq. (1.18), tells us that magnetic induction $\boldsymbol{B}$ does not diverge. Thus, $\boldsymbol{B}$ is continuous, forming a closed loop. There is no isolated magnetic charge, or pole, like there is in electric charge. Rather, the source of $\boldsymbol{H}$ is a current element $\boldsymbol{J}_{\mathbf{c}}$ or a time-varying electrical flux $\boldsymbol{D}$, as described in Eq. (1.19)! The current element $\boldsymbol{J}_{\mathbf{c}}$ acts like a magnetic dipole. The curl of $\boldsymbol{H}$ indicates that the direction of the magnetic field $\boldsymbol{H}$ is orthogonal to the direction of $\boldsymbol{J}_{\mathbf{c}}$. For example, if $\boldsymbol{J}_{\mathbf{c}}$ is in the z-direction, $\boldsymbol{\nabla} \times \boldsymbol{H}$ has no component in the z-direction, or $\boldsymbol{H}$ is in the x-y plane. This is another way to describe Ampère's Law.

Equation (1.20) demonstrates that the electric field $\boldsymbol{E}$ is induced by a time-varying magnetic induction $\boldsymbol{B}$. This is the principle of electric generators and electric motors. Although Eq. (1.21) is not one of the Maxwell equations of electromagnetism, it indicates that current $\boldsymbol{J}_{\mathbf{c}}$ can be viewed as the moving electric charge ρ. Together, these equations are the basis of electromagnetic wave propagation and energy conversation.

Since the Maxwell equations do not explicitly link material parameters and the behavior of magnetism, they are not commonly utilized in analyzing the behavior of magnetic materials and thin films.

1.11 Inductance

A time-varying current I in a conducting wire will generate a time-varying magnetic induction B around the wire. According to the Maxwell equation (1.20), the time-varying B induces an electric field in the wire. Thus, a voltage appears at the

Table 1.1 Cgs to SI parameter table

	cgs	SI
Force between poles	$F = \frac{p_1 p_2}{r^2}$ (dyne)	$F = \frac{1}{4\pi\mu_0}\frac{p_1 p_2}{r^2}$ (N)
Field of a pole	$H = \frac{p}{r^2}$ (Oe)	$H = \frac{1}{4\pi\mu_0}\frac{p}{r^2}$ (A/m)
Magnetic moment	$m = p(length)$ (emu)	$m = A\,I$ ($\mathrm{A \cdot m^2}$)
Magnetization	$M = m/(volume)$ ($\mathrm{emu/cm^3}$)	M (tesla)
Magnetic induction	$\boldsymbol{B} = \boldsymbol{H} + 4\pi\,\boldsymbol{M}$ (gauss)	$\boldsymbol{B} = \mu_0\,\boldsymbol{H} + \boldsymbol{M}$ (tesla)
Energy of a dipole	$E = -\boldsymbol{m} \cdot \boldsymbol{H}$ (erg)	$E = -\mu_0\,\boldsymbol{m} \cdot \boldsymbol{H}$ (J)
Magnetic susceptibility	$\chi_m = \frac{M}{H}$ ($\mathrm{emu/cm^3/Oe}$)	$\chi_m = \frac{M}{H}$ (dimensionless)
Permeability	$\mu = \frac{B}{H} = (1 + 4\pi\chi_m)$ (gauss/Oe)	$\mu = \frac{B}{H} = \mu_0(1 + \chi_m)$ (henry/m)

two ends of the wire. In electronic circuitry, the relationship between the induced voltage V and the time-varying current I in a conducting wire is given by

$$V = L\frac{dI}{dt}, \tag{1.24}$$

where L is called the *inductance*. The unit of L is the (volt · second)/ampere or the *henry* (H); L is also called the *self-inductance*, since the time-varying current in the wire itself is the source of the voltage across the two ends of the wire through magnetic induction.

Equation (1.20) does not restrict the source of time-varying B; this equation also allows a time-varying I_1 in the first wire to produce a time-varying B, which extends to a neighboring second wire and induces a voltage V_2 in the neighboring second wire. The relation between the current and induced voltage is given by

$$V_2 = L_{21}\frac{dI_1}{dt}, \tag{1.25}$$

where L_{21} is called *mutual inductance*.

Inductance is an electronic circuit element, like resistance and capacitance. It stores and releases magnetic energy.

1.12 Equation tables

While cgs units are used in physics and in the study of magnetic materials, SI units are utilized when engineers investigate energy conversion in electric motors/generators, as well as in electromagnetic wave propagation. Parameters are listed in Table 1.1. Note that two parameters, *magnetic susceptibility* and *permeability* of