Alla memoria di
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Preface

The birth of minimal surfaces dates back to 1762, with the paper *Essai d'une nouvelle méthode pour determiner les maxima et les minima des formules intégrales indéfinies*, by Lagrange. Some years later Meusnier supplied a geometric interpretation of the Lagrange equation as the vanishing of the mean curvature. Moreover, he discovered the helicoid, which, after the catenoid (constructed by Euler), was the first example of a non-parametric minimal surface.

During the nineteenth century minimal surfaces became widely known as a result of the work of the Belgian physicist Plateau. He realized minimal surfaces as soap films. The theory itself had a rapid development too. The major achievement was perhaps the discovery of the deep connection between minimal surface theory and Complex Analysis (Enneper-Weierstrass representation). The power of these methods is showed by the beautiful constructions of many minimal surfaces in the space (Enneper, Riemann, Schwartz...).

In the first half of the past century minimal surfaces were studied from the point of view of the solution either of the Plateu problem or, by Bernstein, of differential equations. The influence and interaction of minimal surfaces with the calculus of variations and the theory of partial differential equation is one of the main features of the theory.

In the second half of the twentieth century the global theory of minimal surface in flat space had a unexpected and rapid blossoming. Some of the classical problems were solved and new classes of minimal surfaces found; it is enough to mention the work of Osserman on the relationship between the Gauss curvature of a minimal surfaces and the topology and conformal structure of the associated Riemann surfaces. Subsequently, a new impetus was given by the discovery of new complete embedded examples having bounded curvature. The first nontrivial examples, other than the plane and the catenoid, were in fact constructed just ten years ago by Costa, Hoffman and Meeks. New crucial progress in the general theory culminated with the solution of such outstanding problems as the Xavier-Fujmoto-Osserman result on the Gauss map and the work of Pascal Collin on the Nitsche conjecture.

Minimal surfaces are studied from several different viewpoints: methods and techniques from Analysis (Real and Complex), Topology and Geometry are used. Some of the methods and recent developments were presented at the CIME course and are reported in the present Lecture Notes.

In his lectures , William Meeks develops the analytic and geometric tools as well as the basic techniques, like the maximum principle and the Geometric Dehn lemmas, that are instrumental in understanding the topology and the asymptotic geometry of properly embedded minimal surfaces. Properly embedded minimal surfaces in flat spaces are perhaps the most natural and important subclass of minimal surfaces. Periodic surfaces are studied in some detail. Many basic results and deeply fascinating conjectures are discussed.

In their contribution, Antonio Ros and Joaquín Pérez consider, among the properly embedded minimal surfaces in Euclidean 3-space, those with finite total curvature. These surfaces have finitely many ends which are all parallel and asymptotic to planes or half-catenoids. The structure of the moduli space $\mathcal{M}$ of surfaces of this type that have a fixed topology is studied and very important results are explained, among them the Schoen and López-Ros characterizations of the catenoid and the compactness properties of $\mathcal{M}$. Pérez and Ros also study embedded minimal surfaces with vertical forces and prove that, under suitable global restrictions, any surface of this type must be a topological annuli. In particular they prove, in the symmetric case, Meeks's conjecture about compact minimal surfaces bounded by a pair of convex curves in parallel planes.

Harold Rosenberg considers a quite interesting new subject, i.e. surfaces of constant curvature one in the hyperbolic 3-space: the Bryant surfaces. They are related to certain special minimal surfaces in flat space, the cousin surfaces. So, this new class of examples sheds also some new light on the classical theory of minimal surfaces. The cousin's constructions and the Bryant parametrization by meromorphic data are carefully studied. Then a recent theorem (of Collin, Hauswirth and Rosenberg) about properly embedded Bryant surfaces of finite topology is proved; it states that they have finite total curvature and that the Gauss map extends meromorphically to the conformal compactification.

The lectures were a great success with the participants to the CIME session on minimal surfaces. I can only hope that this volume preserve the crisp clarity of those Lectures. I would like to thank Margherita Solci, without whose help I could not have edited this volume.

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