3

Satellite TV Link Analysis

3.1 Radio Transmission Equation Overview

Figure 3.1 shows the thermal noise model of a point-to-point radio transmission system, which can be used in a satellite channel that is mainly affected by attenuation and AWGN.

The *carrier-to-noise ratio* (C/N), defined as the quotient between the modulated carrier power at *intermediate frequency* (IF) and the corresponding noise power in the noise bandwidth of the IF amplifier (B), can be written as

$$\mathrm{C/N} = \frac{P_T G_T G_R}{kTL_b L_a B} \tag{3.1}$$

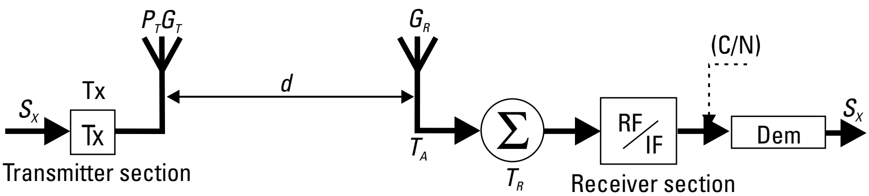

Figure 3.1 Thermal noise model of a point-to-point radio transmission system.

where:

P_T: Transmitter radiated power at carrier frequency, in watts (W);

G_T: Transmitting antenna gain;

T: System's noise temperature, in Kelvin (K);

L_b: Free-space loss.

L_a: Additional loss (relative to free-space loss);

k: Boltzmann's constant ($1.38 \cdot 10^{-23}$ dB.J/K).

The system's noise temperature T is defined as

$$T = T_A + T_R \tag{3.2}$$

where T_A is antenna noise temperature (external noise) and T_R is the receiver noise temperature (internal noise), both in Kelvin. (External and internal noise powers can be added because they are uncorrelated noise sources.)

The free-space loss is defined as

$$L_b = \left(\frac{4\pi d}{\lambda} \right)^2 \tag{3.3}$$

where d is the distance (in meters) between the transmitting antenna and the receiving antenna, and λ is the wavelength (in meters) in the vacuum. The relationship between the carrier frequency f (in gigahertz) and wavelength λ (in meters) is

$$\lambda(\mathrm{m}) = \frac{0.3}{f(\mathrm{GHz})} \tag{3.4}$$

For satellite system applications, it is possible to introduce the following parameters:

- In the transmitting side:

$$\mathrm{EIRP} = P_T G_T \text{, W} \tag{3.5}$$

- In the receiving side:

$$G/T = \frac{G_R}{T_A + T_R} \text{;K}^{-1} \tag{3.6}$$

Now, using (3.5) and (3.6), it is possible to write (3.1) as

$$C / N = \frac{\text{EIRP} \cdot G / T}{k L_b L_a B} \tag{3.7}$$

Using decibels,

$$\begin{aligned} C / N(\text{dB}) &= \text{EIRP}(\text{dBW}) + G / T(\text{dB/K}) \\ &- L_b(\text{dB}) - L_a(\text{dB}) - B(\text{dB.Hz}) + 228.60(\text{dBW / K.Hz}) \end{aligned} \tag{3.8}$$

Example 3.1

A transmitter feeds a power of 120W into an antenna that has a gain of 32 dBi. Calculate the EIRP in watts and decibels relative to 1W.

Solution. Using (3.5), obtain

$$\text{EIRP} = 120 \cdot 10^{32/10} = 190.2 \text{ kW}$$

and

$$\text{EIRP} = 10 \log(120) + 32 = 52.8 \text{ dBW}$$

Example 3.2

A receiving system employs a 36-dBi parabolic antenna operating at 12 GHz. The antenna noise temperature is 50K, and the receiver front-end noise temperature is 110K. Calculate G/T in decibels per Kelvin.

Solution. Using (3.6), obtain

$$G / T = 36 - 10 \log(50 + 110) = 13.96 \text{ dB/K}$$

3.2 Satellite Link Geometry

3.2.1 Slant Range

Figure 3.2 shows the basic geometric model of the satellite-ground terminal path.

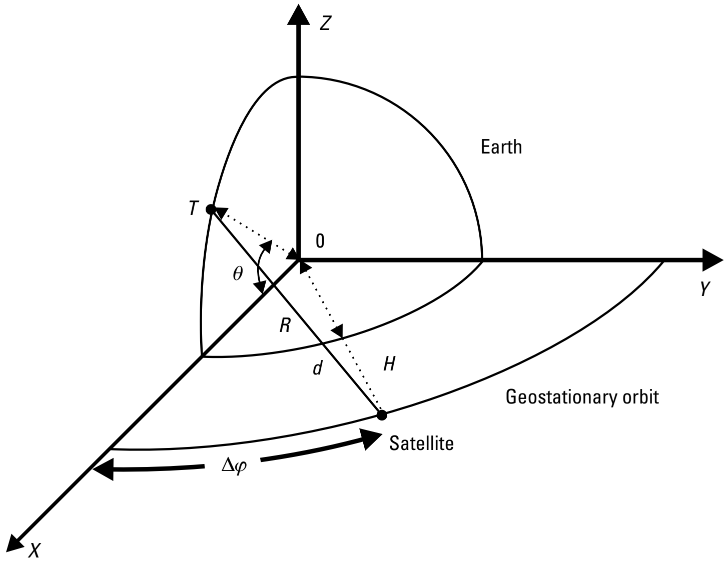

Figure 3.2 Satellite link geometry.

Let θ be the ground terminal (T) latitude, φ_T be the ground terminal longitude, and φ_S be the satellite's orbital position. North latitudes and east longitudes will be considered positive-signed; south latitudes and west longitudes are negative-signed. Define $\Delta\varphi$ as

$$\Delta\varphi = \varphi_S - \varphi_T \tag{3.9}$$

The rectangular coordinates of the ground terminal (point T on the ZX plane) are

$$R\cos\theta;\, 0;\, R\sin\theta$$

and of the satellite (point S on the XY plane) coordinates are

$$\left[(R+H)\cos\Delta\varphi;\, (R+H)\sin\Delta\varphi;\, 0\right]$$

The slant range d can be calculated using the well-known formula of distance between two points in the space knowing the coordinates of T and S. The end result is

$$d = \sqrt{(R+H)^2 + R^2 - 2R(R+H)\cos\Delta\varphi\cos\theta} \quad (3.10)$$

Substituting the numerical values of R and H in (3.10), obtain

$$d = 4.264 \cdot 10^4 \sqrt{1 - 0.296\cos\Delta\varphi\cos\theta}, \text{km} \quad (3.11)$$

Using (3.3), (3.4), and (3.11) and decibels, it is possible to obtain

$$L_b(\text{dB}) = 185 + 20\log f(\text{GHz}) + 10\log(1 - 0.296\cos\Delta\varphi\cos\theta) \quad (3.12)$$

Example 3.3

A ground terminal located at 22° N, 80° W is receiving signals from geostationary satellite Galaxy V (125° W).

Determine the free-space loss at 4 GHz (C band).

Solution. Using (3.12),

$$L_b = 185 + 20\log(4) + 10\log[1 - 0.296\cos(-45°)\cos(22°)] = 196.1\,\text{dB}$$

Example 3.4

Repeat Example 3.3, but now the ground terminal receives a signal from EchoStar I (119° W) operating at 12.5 GHz (Ku band).

Solution. Again, using (3.12), obtain the following result:

$$L_b = 185 + 20\log(12.5) + 10\log[1 - 0.296\cos(-39°)\cos(22°)] = 205.9\,\text{dB}$$

3.2.2 Elevation and Azimuth Angles

The elevation (EL°) and azimuth (AZ°) angles allow an antenna of a ground terminal to be accurately pointed to a satellite in the geostationary orbit (Figure 3.3).

It can be shown (Appendix A) that the elevation angle is expressed as

$$\text{EL}^\circ = \arctan\left(\frac{\cos\Delta\varphi\cos\theta - 0.1513}{\sqrt{1 - \cos^2\Delta\varphi\cos^2\theta}}\right) \quad (3.13)$$

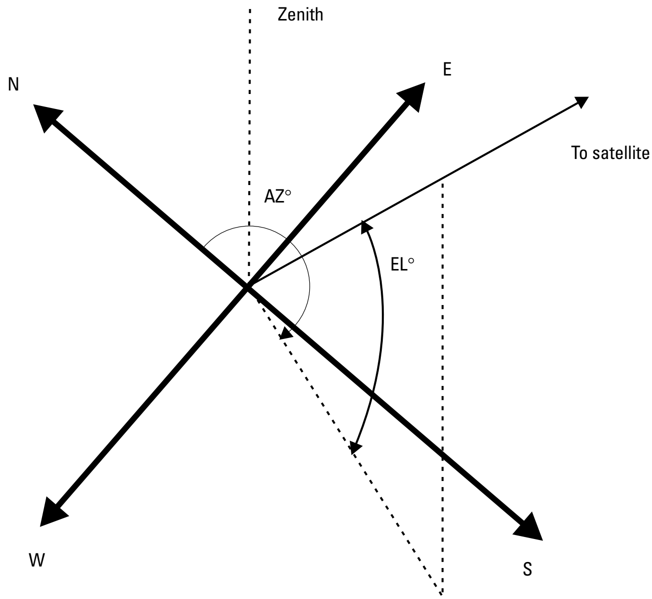

Figure 3.3 Elevation (EL°) and azimuth (AZ°) angles.

and the azimuth angle as

$$AZ° = 180° - \arctan\left(\frac{\tan \Delta\varphi}{\sin \theta}\right) \tag{3.14}$$

Example 3.5

Calculate the azimuth and elevation angles for a ground terminal located at 22° N, 80° W receiving digital TV from EchoStar 1 (119° W).

Solution.

$$EL° = \arctan\left(\frac{\cos(-39°)\cos(22°) - 0.1513}{\sqrt{1-\cos^2(-39°)\cos^2(22°)}}\right) = 39.4°$$

$$AZ° = 180° - \arctan\left(\frac{\tan(-39°)}{\sin(22°)}\right) = 245.2°$$

Example 3.6

Allowing a 5° elevation angle at ground terminals, verify that the geostationary arc of 55–136° W covers CONUS.

Solution. Let us consider two locations in the United States: Washington, D.C., (39° N, 77° W) on the East Coast, and very near to Seattle (48° N, 125° W) on the West Coast.

Let

$$x = \cos \Delta\varphi \cdot \cos \theta$$

in the elevation angle formula. The variables latitude (θ) and relative longitude ($\Delta\phi$) are constrained by

$$0 \le |\theta| < 90°$$
$$0 \le |\Delta\varphi| < 90°$$

and then $0 < x \le 1$. For EL° = 5° and using the elevation angle formula, obtain

$$x^2 - 0.3x - 0.0047 = 0$$

with the only possible solution $x = 0.315$. The other one is neglected ($x = -0.015$) because of the physical constraints. Then,

$$\cos \Delta\varphi = \frac{0.315}{\cos \theta}$$

and, using the definition of $\Delta\phi$, it is possible to set

$$\varphi_S = \varphi_T \pm \arccos\left(\frac{0.315}{\cos \theta}\right)$$

From the location with coordinates 39° N, 77° W, the visible geostationary arc is

$$11° \text{ W} \le \varphi_S \le 143° \text{ W}$$

and it contains 55–136° W.

From the location with coordinates 48° N, 125° W, the visible geostationary arc is

$$50° \text{ W} \le \varphi_S \le 185.66° \text{ W}$$

and it also contains 55–136° W.

3.3 Pointing an Antenna Dish Using a Compass as a Reference

Once the azimuth angle has been calculated, if one wishes to point a dish using a magnetic compass, it is necessary to know the right direction of true north/south by using a compass and correcting for magnetic variation of the receive site. Points of equal magnetic variation on the Earth are represented with isogonal lines. When magnetic variation is zero, the contour is called the *agonic line*. Magnetic variation is added to the azimuth angle if the direction of magnetic north lies to the east of the true north and subtracted if it lies to the west. For example, if the azimuth angle is 153° and the magnetic variation of the receive site is 6° W, the compass bearing is 159°.

3.4 Overall RF Satellite Link Performance

The overall performance of the total satellite link depends on the uplink, downlink, transponder, and interference. Figure 3.4 shows a schematic model of a nonregenerative transponder including uplink and downlink. Only the effects of (thermal) noise will be considered.

The C/N for the uplink is defined as

$$(\text{C}/\text{N})_U = \frac{P_U}{N_U} \tag{3.15}$$

In the same way, for the downlink

$$(\text{C}/\text{N})_D = \frac{P_D}{N_D} \tag{3.16}$$

The overall noise power arriving to the demodulator input of the receiving ground terminal is

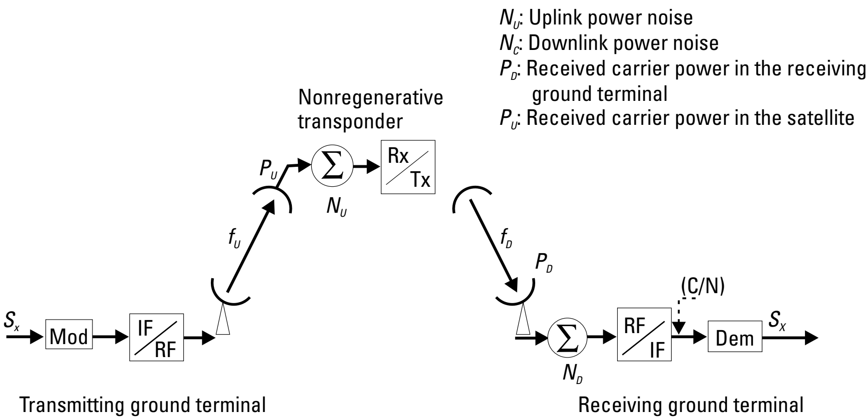

Figure 3.4 Thermal noise model of an overall satellite link using a nonregenerative transponder.

$$N = N_D + \frac{N_U \cdot G_{TR}}{L_D} \tag{3.17}$$

where G_{TR} represents the transponder gain and L_D represents the downlink transmission loss (including antenna gain).

The overall C/N (measured in the demodulator's input of the receiving ground terminal) is

$$\mathrm{C/N} = \frac{P_D}{N} \tag{3.18}$$

Taking the reciprocal of C/N in (3.18) and substituting (3.17), obtain

$$(\mathrm{C/N})^{-1} = \frac{N_D}{P_D} + \frac{N_U}{\left(\dfrac{P_D \cdot L_D}{G_{TR}}\right)} \tag{3.19}$$

The term in parentheses in (3.19) is P_U; then,

$$(\mathrm{C/N})^{-1} = \frac{N_D}{P_D} + \frac{N_U}{P_U} \tag{3.20}$$

Substituting (3.15) and (3.16) in (3.20), obtain

$$(\mathrm{C/N})^{-1} = (\mathrm{C/N})_U^{-1} + (\mathrm{C/N})_D^{-1} \tag{3.21}$$

Note that this has exactly the same form as the equation for resistance of resistors in parallel.

It is possible to rearrange (3.21) in the following way:

$$(\mathrm{C/N})^{-1} = (\mathrm{C/N})_D^{-1}\left(1+\frac{(\mathrm{C/N})_D}{(\mathrm{C/N})_U}\right) \tag{3.22}$$

and then

$$(\mathrm{C/N})_D = \mathrm{C/N}\left(1+\frac{(\mathrm{C/N})_D}{(\mathrm{C/N})_U}\right) \tag{3.23}$$

The term in brackets physically means the uplink noise contribution to overall noise in the satellite link. If this uplink noise contribution is denoted by ΔN_U, then

$$\Delta N_U = \left(1+\frac{(\mathrm{C/N})_D}{(\mathrm{C/N})_U}\right) \tag{3.24}$$

Substituting (3.24) in (3.23), finally obtain

$$(\mathrm{C/N})_D = \mathrm{C/N}\cdot\Delta N_U \tag{3.25}$$

Applying decibels in expression (3.25), it is possible to write

$$(\mathrm{C/N})_D(\mathrm{dB}) = \mathrm{C/N}(\mathrm{dB}) + \Delta N_U(\mathrm{dB}) \tag{3.26}$$

Example 3.7

The C/N values for a satellite link are 30 dB for the uplink and 14 dB for the downlink. Calculate the ΔN_U value.

Solution. Using (3.24), obtain

$$\Delta N_U = 10 \log\left(1 + \frac{10^{14/10}}{10^{30/10}}\right) = 0.1077 \text{ dB}$$

3.5 Uplink Budget Analysis

Figure 3.5 shows the uplink thermal noise model.

Using (3.7) it is possible to write, for the uplink case

$$(\mathrm{C/N})_U = \frac{(\mathrm{EIRP})_T (\mathrm{G/T})_S}{kL_a L_b B_T} \tag{3.27}$$

where $(\mathrm{EIRP})_T$ represents the EIRP of the transmitting ground terminal, $(\mathrm{G/T})_S$ represents the G/T of the satellite's receiving section, and B_T is the transponder bandwidth.

The value of $(\mathrm{EIRP})_T$ must warrant that the transponder operates in saturation, which is usual in broadcast applications. Therefore, the *power flux density* (PFD) at the input of the transponder must be equal to the SFD.

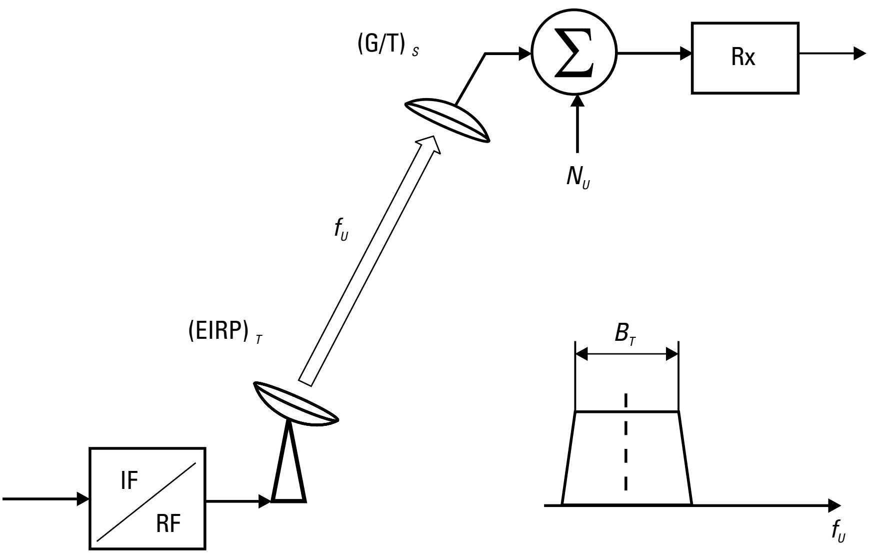

Figure 3.5 Uplink thermal noise model.

Then,

$$\mathrm{SFD} = \frac{(\mathrm{EIRP})_T}{4\pi d^2}, \mathrm{W/m}^2 \tag{3.28}$$

For the uplink case

$$L_b = \left(\frac{4\pi d}{\lambda_U}\right)^2 \tag{3.29}$$

and

$$\lambda_U(\mathrm{m}) = \frac{0.3}{f_U(\mathrm{GHz})} \tag{3.30}$$

Substituting (3.28)–(3.30) into (3.27) and using decibels, it is possible to obtain

$$\begin{aligned}(\mathrm{C/N})_U(\mathrm{dB}) &= \mathrm{SFD}(\mathrm{dBW/m}^2) + (\mathrm{G/T})_S(\mathrm{dB/K}) \\ &- 20\log f_U(\mathrm{GHz}) - B_T(\mathrm{dB.Hz}) - L_a(\mathrm{dB}) + 207.15\end{aligned} \tag{3.31}$$

Example 3.8

Determine the uplink C/N for a transmitting ground terminal sending a modulated carrier to Transponder 23, Galaxy V (129° W), (6.385 GHz). The ground terminal is located at 22° N, 80° W. The additional loss is 0.5 dB. The transponder operates in saturation and has a 36-MHz bandwidth. The SFD is –83.5 dBW/m^2 and $(G/T)_S$ is –1.6 dB/K.

Solution. Using (3.31),

$$\begin{aligned}(\mathrm{C/N})_U &= -83.5 - 1.6 - 20\log(6.385) \\ &- 10\log(36\cdot 10^6) - 0.5 + 207.15 = 30\ \mathrm{dB}\end{aligned}$$

3.6 Downlink Budget Analysis

Figure 3.6 shows the downlink thermal noise model. In the analysis that follows, it is assumed that the TWTA (and the associated transponder) is

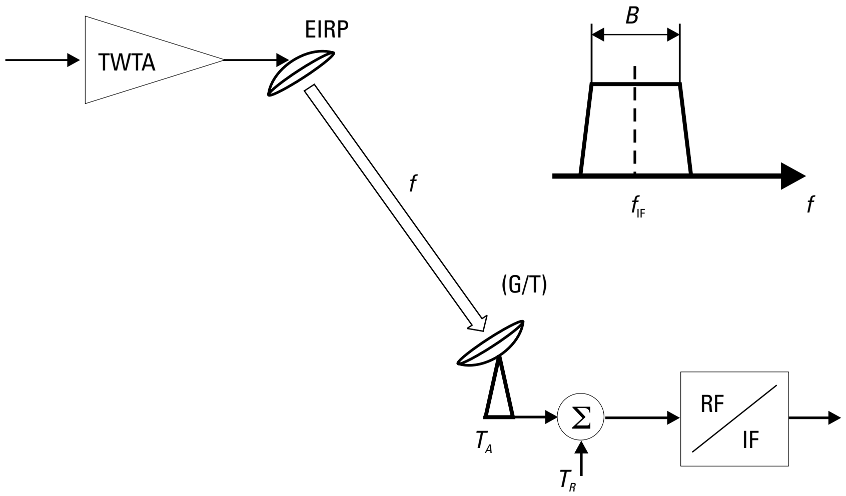

Figure 3.6 Downlink thermal noise model.

operating at saturation and in a single-carrier mode, which is typical in broadcast applications. Also it is assumed that there is not any interference at the input of the ground reception system. The frequency represents the downlink frequency band. For C-band systems this value can be considered to be 4 GHz, and for BSS Ku-band systems the typical value is approximately 12 GHz.

Again, using (3.7), the C/N for the downlink case can be expressed as

$$(\mathrm{C/N})_D = \frac{\mathrm{EIRP} \cdot \mathrm{G/T}}{kL_b L_a B} \tag{3.32}$$

where EIRP corresponds to satellite power output, G/T corresponds to the receiving ground terminal, and *B* is the noise bandwidth of the IF amplifiers of the receiving ground terminal (also called the tuner). Because of the high selectivity of the IF amplifiers, the noise bandwidth and the signal bandwidth have almost the same numerical value. It is a common practice to call it the receiver or IRD bandwidth.

Substituting (3.25) in (3.32) and using decibels

$$\begin{aligned}(\mathrm{C/N})(\mathrm{dB}) &= \mathrm{EIRP}(\mathrm{dBW}) + (\mathrm{G/T})(\mathrm{dB/K}) - L_b(\mathrm{dB}) \\ &- L_a(\mathrm{dB}) - B(\mathrm{dB.Hz}) - \Delta N_U(\mathrm{dB}) + 228.6(\mathrm{dBW/K.Hz})\end{aligned} \tag{3.33}$$

Example 3.9

Calculate the overall C/N for a satellite link with the following parameters:

- Satellite's EIRP: 45 dBW;
- Downlink free-space loss: 205.4 dB (12 GHz);
- Uplink noise contribution: 0.5 dB;
- Additional losses: 0.5 dB;
- Receiver ground terminal bandwidth: 20 MHz;
- Receiver ground terminal (G/T): 20 dB/K.

Solution. The receiver ground terminal bandwidth, expressed in decibels, is

$$B(\text{dB.Hz}) = 10\log\left(20 \cdot 10^{6}\right) = 73 \text{ dB.Hz}$$

Using (3.33),

$$\text{C / N} = 45 + 20 - 205.4 - 0.5 - 73 - 0.5 + 228.6 = 14.2 \text{ dB}$$

3.7 Downlink Performance Analysis: C Band Versus Ku Band

Figure 3.7 shows the main characteristics of a satellite downlink for typical TV broadcast applications, for which the satellite transmitter antenna gain is fixed by the beamwidth implications of the coverage area requirement, and the receiver antenna size is as large as possible, considering convenience and cost. If A is the coverage area, the solid angle Ω is

$$\Omega = \frac{A}{d^{2}}, \text{ steradians} \tag{3.34}$$

If it is assumed that the energy is concentrated in the main beam, the antenna gain G_T of the onboard transmitting antenna is inversely proportional to the beam's solid angle,

$$G_T = \frac{K_p}{\Omega} = \frac{K_p d^{2}}{A} \tag{3.35}$$

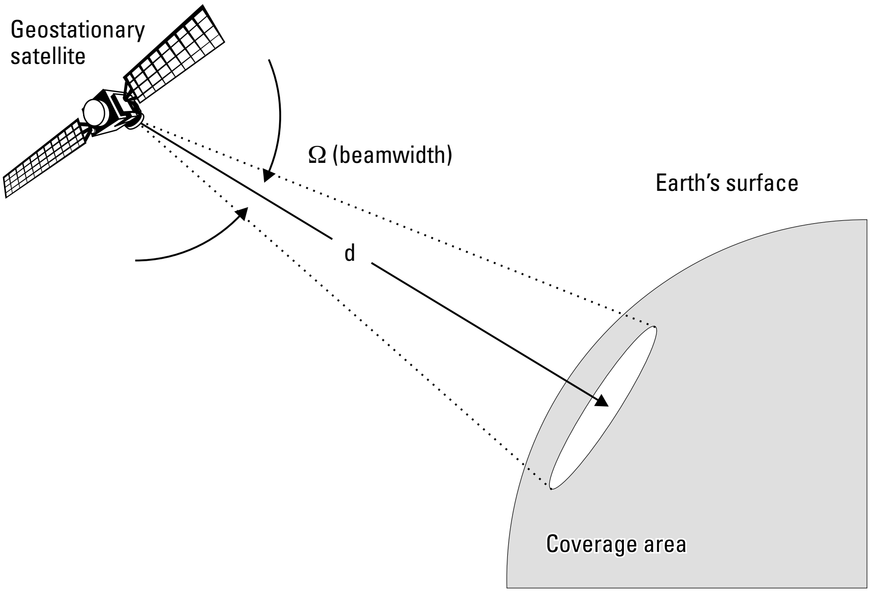

Figure 3.7 Satellite downlink for broadcast applications: fixed antenna gain at transmitter end and fixed antenna size at receiver end.

where K_p is the constant of proportionality. If the receiving antenna on the ground terminal has a fixed size, then the antenna gain G_R is

$$G_R = \eta\left(\frac{\pi D}{\lambda}\right)^2 \tag{3.36}$$

where η is the antenna's efficiency. Substituting (3.3), (3.35), and (3.36) into (3.1),

$$\mathrm{C/N} = \frac{P_T\left(\frac{K_P d^2}{A}\right)\cdot\left[\eta\left(\frac{\pi D}{\lambda}\right)^2\right]}{k\left(\frac{4\pi d}{\lambda}\right)^2\cdot L_a\cdot B} \tag{3.37}$$

Simplifying (3.37) obtains

$$C / N = \left[\frac{\eta K_p}{16 A k}\right] \cdot \frac{P_T D^2}{L_a B T} \tag{3.38}$$

where the factor in brackets has a constant value.

Now, it is possible to derive the following conclusions from (3.38):

- The C/N, as a performance measure of the satellite downlink, is no longer dependent on the carrier frequency. The reduced size of the antenna diameter on Ku-band ground-receiving terminals is because of the larger satellite EIRP used in these systems, compared with the lower-EIRP power-constrained C-band systems—not because the higher frequency carrier allows a higher value in receiving antenna gain. In both cases the satellite EIRPs are limited by regulatory PFD limits on the Earth's surface.
- The downlink performance is more sensitive to antenna diameter than the system's noise temperature.
- The additional loss L_a is higher in Ku-band systems (rain attenuation) than in C-band systems.
- The downlink performance is no longer dependent on distance. Nonetheless, the use of geostationary satellites involves a more viable technology and has a beneficial and economic impact on ground receiving terminals compared to any other kind of satellite (LEO and HEO).

Example 3.10

Determine the relationship between the antenna diameters of two ground receiving terminals, one operating in the C band (4 GHz) and the other in the Ku band (12 GHz). The ground terminal has the following characteristics:

- C-band and Ku-band transponders have the same EIRP value and bandwidth.
- The additional loss is 0 dB in the C band and 4 dB in the Ku band (rain attenuation).
- The system noise temperature in the C band is 55K, while in the Ku band it is 110K.

Solution. Let D_C and D_{Ku} represent the antenna diameters for the C band and the Ku band, respectively. Using (3.38) it is possible to write

$$\frac{(C/N)_C}{(C/N)_{Ku}} = \left(\frac{D_C}{D_{Ku}}\right)^2 \cdot 10^{0.4} \cdot \frac{110}{55} = 1$$

and

$$D_C = 0.45\, D_{Ku}$$

Then the antenna diameter could be lower in C-band systems than in Ku-band systems if both satellites had the same power output. However, note that in this scenario the C-band antenna is bigger by f^2 to yield the same gain pattern as that of the Ku-band spacecraft.

3.8 Radio Propagation Impairments in Satellite TV Links

The transmission of a satellite signal occurs almost in free-space conditions (more than 97% of the slant path). However, the troposphere (less than 100 km above the Earth's surface) and ionosphere (extending from 90 to 1,000 km above the Earth's surface) introduce significant impairments, whose importance depends on carrier frequency, elevation angle, atmosphere and ionosphere status, and solar activity. Rain influence is perhaps the most important single phenomenon over 10 GHz.

3.8.1 Gaseous Atmospheric Absorption

Absorption due to oxygen, water vapor, and other atmospheric gases, as distinguished from rain and other "hydrometeors," is basic and unavoidable. The attenuation is negligible at frequencies less than 10 GHz. There are specific frequency bands where absorption is high. The first band, caused by water vapor, is about 22 GHz, while the second band, caused by oxygen, is about 60 GHz. Absorption increases as the elevation angle is reduced. A variation given by cosec (EL°) can be applied to transform the attenuation over a zenith path. In the frequency range of 1 to 20 GHz, the zenith one-way absorption is approximately in the range 0.03 to 0.2 dB and, for 10° of elevation, the gaseous absorption is in the range 0.17 to 1.15 dB.

3.8.2 Ionospheric Scintillation

Scintillation is a rapid fluctuation of signal amplitude, phase, polarization, or angle of arrival. In the ionosphere, scintillation occurs because of multipath due to small variations of the refractive index caused by local concentrations of ions. Ionospheric scintillation decreases as $1/f^2$ and has a significant effect below 4 GHz. Tropospheric scintillation can also occur, again with multipath as a cause, and in this instance increases with frequency. Generally this is small or negligible, amounting to tenths of decibels at the Ku band.

3.8.3 Faraday Rotation

The electrons in the ionosphere along with the Earth's magnetic field cause a rotation in the plane polarization known as the Faraday rotation. The rotation diminishes inversely with the square of the frequency and sometimes has a maximum value of 150° at 1 GHz. At 4 GHz, the Faraday rotation is 9°, and it can be neglected above 10 GHz.

3.8.4 Rain Influence

The main effect of rain above 10 GHz is to attenuate the signal and, because of its behavior as a lossy attenuator, to increase the antenna noise temperature. In addition to these effects, rain has a depolarizing effect, which creates a cross-polarized component with linear polarization and a loss of circular polarization. The attenuation is caused by the scattering and absorption of radioelectrical waves by drops of liquid water. The attenuation increases as the wavelength approaches the size of a typical raindrop, which is about 1.5 mm.

3.8.4.1 Rain Attenuation

The calculation of rain attenuation can be divided into two main steps. The first one is to estimate the rain rate R in mm/hr as a function of the cumulative probability of occurrence probability ($r \leq R$), where r is the random variable rain rate and R is a specific value. This probability helps to determine the grade of service to be provided and thus the values of margin required. The second step is to calculate the attenuation resulting from those rain rates, given the elevation angles, the ground terminal latitude, and the carrier frequency.

There are two authoritative rain models that are widely used: Crane Global [1] and ITU-R [2]. The Crane Global model is an empirically based model that uses data from geographical regions to develop a relationship between the path average rain and the point rain rate. A revision of this model that accounts for both the dense center and fringe area of a rain cell is the so-called two-components model. The ITU-R model is the empirically based model recommended by the ITU. The model calculates the attenuation due to a rain rate that occurs 0.01% of the time. Then the model uses a reduction factor and an interpolation procedure to determine the rest of the distribution. The model is based on point rainfall statistics.

Both models use the following basic expressions to predict rain attenuation:

$$\gamma_R = kR^a \tag{3.39}$$

$$A_R = \gamma_R \cdot L_e \tag{3.40}$$

where γ_R is the specific attenuation (in decibels per kilometer) and R is the point rain rate (in millimeters per hour) for a specific outage. The k and α values are calculated from theoretical formulae and L_e represents the effective path length (in kilometers) through rain. The rain rate is a measure of the average size of the raindrops. When the rain rate increases (it rains harder), the raindrops are larger and thus there is more attenuation. Reference [3] reports a method that combines rain attenuation and other propagation impairments along Earth satellite paths. For system planning, the method mentioned in Appendix B is good enough for engineering practice. At the C band, the rain attenuation has practically a negligible effect. At the Ku band, the attenuation ranges between 2 and 10 dB and, although it is a large value, is manageable in the link budget. However, at the downlink frequency of 20 GHz, the attenuation for equivalent link availability would be higher than 10 dB.

3.8.4.2 Antenna Noise Temperature Increase Due to Rain

In addition to causing attenuation, rain increases the downlink system noise temperature. The physical reason is that rain acts like an attenuator and that any warm attenuator produces additional thermal noise. The thermal noise rain model is illustrated in Figure 3.8.

The antenna noise temperature under rain conditions, T_A', is

$$T_A' = 10^{-A_R/10} \cdot T_{\text{sky}} + \left(1 + 10^{-A_R/10}\right) \cdot T_0, \text{K} \tag{3.41}$$

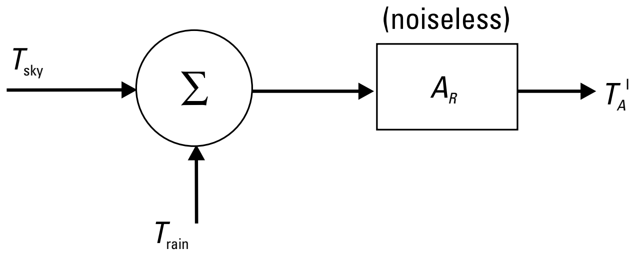

Figure 3.8 Thermal noise rain model. T_{sky} is the sky noise temperature, T_{rain} is the rain noise temperature, and T_A' is antenna noise temperature under rain conditions. A_R is the attenuation in decibels.

where T_0 is the thermodynamic rain temperature. The antenna noise temperature increment ΔT_A can be calculated as

$$\Delta T_A = 10^{-A_R/10} \cdot T_{sky} + \left(1 - 10^{-A_R/10}\right) \cdot T_0 - T_{sky}, \text{K} \qquad (3.42)$$

Rearranging (3.42),

$$\Delta T_A = \left(1 - 10^{-A_R/10}\right) \cdot \left(T_0 - T_{sky}\right) \qquad (3.43)$$

The difference ($T_0 - T_{sky}$) can be approximately substituted by 240K [4] in the 12-GHz band (downlink). Then,

$$\Delta T_A = 240 \cdot \left(1 - 10^{-A_R/10}\right), \text{K} \qquad (3.44)$$

3.8.4.3 Depolarization

Rain also changes the polarization of the electromagnetic wave. Due to resistance of the air, a falling raindrop assumes the shape of an oblate spheroid. Consequently, the transmission path length through the raindrop is different for different carrier polarizations, and the polarization of the received carrier is altered. For a satellite communication system with dual linear polarizations, the change in polarization has two effects. First, there is a loss in the carrier strength because of misalignment of the antenna relative to the clear sky orientation. Typical values are less than 0.5 dB. Second (and usually more significant), there is additional interference due to the admission of a component of any cofrequency carrier in the opposite polarization.

3.8.5 Statistical Analysis of Rain Fading

In the design of any engineering system, it is impossible to guarantee the performance under each possible condition. One sets reasonable limits based on the conditions that are likely to occur at a given level of probability. Also, in the design of a satellite link, a power margin is included to compensate the effects of rain at a given level of service interruption (outage). The rain fading can be analyzed using a flat fading model, because a well-designed, fixed ground terminal is not affected with shadowing, blockage, or delayed multipath rays, and rain scattering is almost negligible [5]. Then, the analysis can be mathematically treated on the basis of the received carrier power in terms of C/N at the demodulator input, as is shown in Figure 3.9.

The margin M_0 relative to the required value $(C/N)_0$ to achieve the desired quality performance is defined as [6]

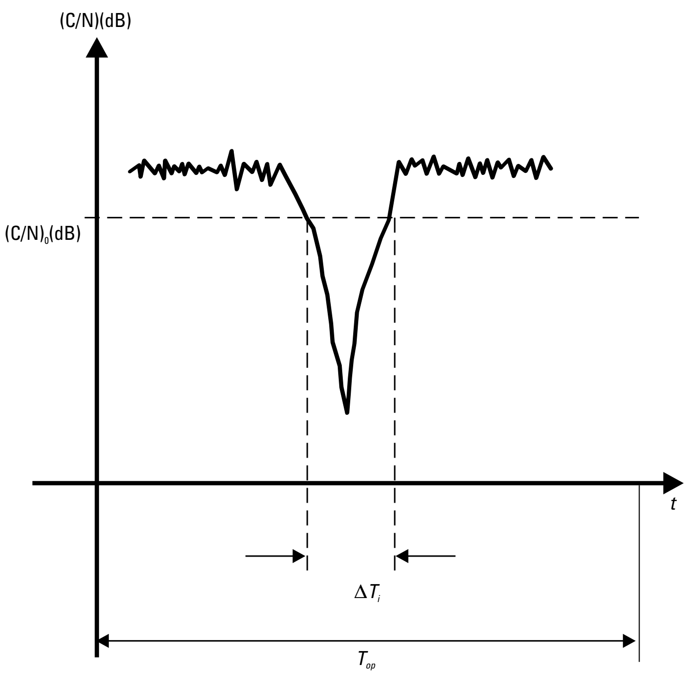

Figure 3.9 C/N temporal variations due to rain fading; T_{op} is a reference time.

$$M_0 = \frac{C/N}{(C/N)_0} \geq 1 \tag{3.45}$$

Using decibels,

$$M_0(\text{dB}) = C/N(\text{dB}) - (C/N)_0(\text{dB}) \geq 0 \tag{3.46}$$

The probability of outage is the percentage of time at which the system performance is unacceptable and it is represented as p (%). It can be defined as

$$p(\%) = \text{Prob}\left(M_0 < 0 \text{ dB}\right) \cdot 100 = \frac{\sum_{\forall i} \Delta t_i}{T_{op}} \cdot 100 \tag{3.47}$$

Example 3.11

The system's availability is the complementary event of the probability of outage. If the system's availability is 99%, determine the time outage in a year (average).

Solution. The time outage in a year (average) is

$$\frac{1}{100} \cdot 365 = 3.65 = 3 \text{ days, } 15 \text{ hours, } 36 \text{ minutes}$$

During this time the system cannot fulfill the expected quality performance, averaged in a year.

Let us consider now the additional loss L_a. It may be split as

$$L_a = L_R + \Sigma L, \text{dB} \tag{3.48}$$

where L_R represents, in decibels, the rain loss as a stationary random process and ΣL (also in decibels) represents other time-invariant losses.

The probability of outage can also be written as

$$p(\%) = \text{Prob}\left(L_R \geq A_R, \text{dB}\right) \cdot 100 \tag{3.49}$$

where A_R is a specific numerical value of the random variable L_R. Since one knows the cumulative probability distribution law in (3.49), it is possible to solve it for A_R. The value thus obtained is represented by $A_R(p\%)$ to specify the selected value of probability outage. Substituting (3.33) in (3.46) and using the developed concepts through (3.48) and (3.49), it is possible to write

$$\begin{aligned} M_0 + A_R &= \mathrm{EIRP} + \mathrm{G/T} - L_b - \Sigma L - B \\ &-\Delta N_U - (\mathrm{C/N})_0 - 228.6,\ \mathrm{dB} \end{aligned} \tag{3.50}$$

Let (G/T)′ be the receiving ground terminal figure of merit in rain conditions and G/T be the same in clear-sky conditions. Taking into account the definition of G/T (3.6), then

$$\frac{(\mathrm{G/T})'}{\mathrm{G/T}} = \frac{G_R / (T_A + \Delta T_A + T_R)}{G_R / (T_A + T_R)} = \frac{1}{1 + \dfrac{\Delta T_A}{T_A + T_R}} \tag{3.51}$$

where T_A is the antenna noise temperature in clear-sky conditions and T_R is the receiver noise temperature. If one defines a factor ΔT (not to be confused with a real temperature increase), in decibels, as

$$\Delta T(\mathrm{dB}) = 10 \log\left(1 + \frac{\Delta T_A}{T_A + T_R}\right) \tag{3.52}$$

then

$$(\mathrm{G/T})'(\mathrm{dB}) = \mathrm{G/T}(\mathrm{dB}) - \Delta T(\mathrm{dB}) \tag{3.53}$$

Substituting (3.44) into (3.52) and using a maximum value of 50K in T_A (typical in Ku-band offset antennas),

$$\Delta T = 10 \log\left[1 + \frac{240\left(1 - 10^{-A_R/10}\right)}{50 + 290\left(10^{F_R/10} - 1\right)}\right] \tag{3.54}$$

where F_R is the receiver noise figure in decibels and is another way to characterize the receiver noise, as is the noise temperature T_R. The relationship between F_R (adimensional) and T_R is the well-known formula

$$F_R = 1 + \frac{T_R}{290} \tag{3.55}$$

where T_R is in Kelvin. Note that in (3.55), F_R is linear.

Taking into consideration the antenna noise increment due to rain, (3.50) can be rewritten as

$$\begin{aligned} M_0 + A_R &= \mathrm{EIRP} + (\mathrm{G/T})' - L_b - \Sigma L \\ &- B - \Delta N_U - (\mathrm{C/N})_0 + 228.6 \end{aligned} \tag{3.56}$$

Substituting (3.53) into (3.56) and rearranging some terms,

$$\begin{aligned} M_R(\mathrm{dB}) &= \mathrm{EIRP}(\mathrm{dBW}) + \mathrm{G/T}(\mathrm{dB/K}) - L_b(\mathrm{dB}) \\ &- \Sigma L(\mathrm{dB}) - B(\mathrm{dB.Hz}) - \Delta N_U(\mathrm{dB}) - (\mathrm{C/N})_0(\mathrm{dB}) + 228.6 \end{aligned} \tag{3.57}$$

where

$$M_R(\mathrm{dB}) \geq A_R(p\%) + \Delta T \tag{3.58}$$

3.9 Application to Satellite TV System Design: Downlink Margin Equation

Equations (3.57) and (3.58) constitute the formal basis to the downlink budget analysis and dimensioning of satellite TV systems. They can be used in a straightforward way for Ku-band analog systems. For Ku-band digital systems, they can be further developed, remembering that [7]:

$$\begin{aligned} (\mathrm{C/N})_0(\mathrm{dB}) &= (E_b/N_0)_0(\mathrm{dB}) \\ &+ R_b(\mathrm{dB.bps}) - B(\mathrm{dB.Hz}) - G_C(\mathrm{dB}) \end{aligned} \tag{3.59}$$

where $(E_b/N_0)_0$ is the average bit energy-to-noise spectral density ratio for a prescribed BER; and a specific kind of modulation, R_b, denotes the MPEG-2 transport stream information bit rate (see Chapter 4); and G_C is the coding gain of the channel encoder-decoder (see Appendix C). Substituting (3.59) into (3.57), one obtains

$$M_R(\text{dB}) = \text{EIRP}(\text{dBW}) + \text{G / T}(\text{dB / K}) - L_b(\text{dB}) - \Sigma L(\text{dB})$$
$$-R_b(\text{dB.bps}) - \Delta N_U(\text{dB}) + G_C(\text{dB}) - (E_b / N_0)_0(\text{dB}) + 228.6 \quad (3.60)$$

which can be applied with (3.58) in associated digital cases.

Example 3.12

Determine the rain margin for the downlink design in a satellite TV system operating in the Ku band (12 GHz) with H polarization. The ground terminal is located in 22° N, 80° W and uses a receiver with a 0.8-dB noise figure; in addition, its height above mean sea level is 200m, and the elevation angle pointed to the satellite is 30°. Consider 99% of availability (average year).

Solution. According to the map of Figure B.1 (Appendix B), the ground terminal is in zone N, and $R_{0.01}$ is 95 mm/hr (Table B.1). The corresponding values of k_H, k_V, α_H, and α_V are 0.0188, 0.00168, 1.217, and 1.2, respectively (Table B.2). The value of τ = 0°, and k and α values are

$$k = \frac{\left[0.0188 + 0.0168 + (0.0188 - 0.0168) \cdot \cos^2 30°\right]}{2} = 0.01855$$

$$\alpha = \frac{\left[(0.0188)(1.217) + (0.0168)(1.2) + (0.023 - 0.020)\cos^2 30°\right]}{(2)(0.01855)} = 1.214$$

The specific rain attenuation is

$$\gamma_{R0.01} = 0.01855(95)^{1.214} = 4.67 \text{ dB/km}$$

Other parameters' values are

$$L_S = \frac{5 - 0.2}{\sin 30°} = 9.6 \text{ km}$$

$$L_O = 35 . e^{-0.015(95)} = 8.42 \text{ km}$$

$$r_{0.01} = \frac{1}{1 + \frac{9.6}{8.42} \cdot \cos 30°} = 0.503$$

The effective length of the rainy path is

$$L_e = 9.6 \cdot 0.503 = 4.83 \text{ km}$$

Then, the rain attenuation for $p = 0.01\%$ is

$$A_{R_{0.01}} = 4.67 \cdot 4.83 = 22.5 \text{ dB}$$

The rain attenuation for $p = 1\%$ can be calculated as

$$A_R(1\%) = 22.5 \cdot 0.12 \cdot (1)^{-(0.546 + 0.043 \cdot \log 1)} = 2.7 \text{ dB}$$

Using (3.54), one obtains

$$\Delta T = 10 \log\left[1 + \frac{240 \cdot \left(1 - 10^{-0.27}\right)}{50 + 290 \cdot \left(10^{0.08} - 1\right)}\right] = 3.06 \text{ dB}$$

The rain margin, using (3.58), is

$$M_R \geq 2.7 + 3.06 = 5.76 \text{ dB}$$

which can be rounded up to 6 dB.

Example 3.13

Repeat Example 3.12 using a 99% worst-month availability.

Solution. The average year outage is (see Appendix B)

$$p = 0.3 \cdot (1)^{1.15} = 0.3\%$$

The rain attenuation is now

$$A_R(\%) = 22.5 \cdot 0.12 \cdot (0.3)^{-(0.546 + 0.043 \cdot \log 0.3)} = 5.07 \text{ dB}$$

The value of ΔT is

$$\Delta T = 10 \log\left[1 + \frac{240\left(1 - 10^{-0.507}\right)}{50 + 290\left(10^{0.08} - 1\right)}\right] = 4.017 \text{ dB}$$

The rain margin is

$$M_R \geq 5.07 + 4.017 = 9.087 \text{ dB}$$

and, finally, $M_R = 10$ dB.

References

[1] Crane, R. K., "Prediction of Attenuation by Rain," *IEEE Trans. on Comm.,* Vol. COM-28, No. 9, September 1980, pp. 1717–1733.

[2] ITU-R, *Rec. ITU-R PN.837-1, Rec. 838, Rec. 839*, 1992–1994.

[3] Dissanayake, A., J. Alnutt, and F. Haidara, "A Prediction Model That Combines Rain Attenuation and Other Propagation Impairments Along Earth-Satellite Paths," *IEEE Trans. on Antennas and Propagation,* Vol. 45, No. 10, October 1997, pp. 1546–1558.

[4] Morgan, W. L., and G. D. Gordon, *Communications Satellite Handbook,* New York: John Wiley and Sons, 1989.

[5] Matricciani, E., and C. Riva, "Evaluation of the Feasibility of Satellite System Design in the 10–100 GHz Frequency Range," *International Journal of Satellite Communications,* Vol. 16, 1998, pp. 237–247.

[6] Sklar, B., *Digital Communications: Fundamentals and Applications,* Englewood Cliffs, NJ: Prentice Hall, 1988, pp. 188–244.

[7] Sklar, B., "Defining, Designing, and Evaluating Digital Communication Systems," *IEEE Communications Magazine,* November 1993, pp. 92–101.