Chapter One

Introduction

1.1 Impulsive and Hybrid Dynamical Systems

Modern complex engineering systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication network constraints. The complexity of modern controlled dynamical systems is further exacerbated by the use of hierarchical embedded control subsystems within the feedback control system, that is, abstract decision-making units performing logical checks that identify system mode operation and specify the continuous-variable subcontroller to be activated. These multiechelon systems (see Figure 1.1) are classified as *hybrid* systems (see [6, 126, 161] and the numerous references therein) and involve an *interacting* countable collection of dynamical systems possessing a hierarchical structure characterized by continuous-time dynamics at the lower-level units and logical decision-making units at the higher level of the hierarchy. The lower-level units directly interact with the dynamical system to be controlled, while the logical decision-making, higher-level units receive information from the lower-level units as inputs and provide (possibly discrete) output commands, which serve to coordinate and reconcile the (sometimes competing) actions of the lower-level units.

The hierarchical controller organization reduces processor cost and controller complexity by breaking up the processing task into relatively small pieces and decomposing the fast and slow control functions. Typically, the higher-level units perform logical checks that determine system mode operation, while the lower-level units execute continuous-variable commands for a given system mode of operation. Due to their multiechelon hierarchical structure, hybrid dynamical systems are capable of simultaneously exhibiting continuous-time dynamics, discrete-time dynamics, logic commands, discrete events, and resetting events. Such systems include dynamical switching systems [29, 101, 140], nonsmooth impact systems [28, 32], biological systems [93], sampled-data systems [71], discrete-event systems [139], intelligent vehicle/highway systems [113], constrained mechanical sys-

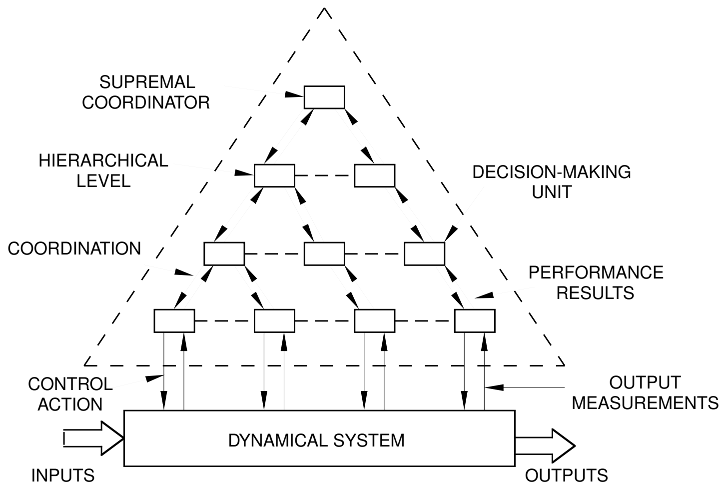

Figure 1.1 Multiechelon dynamical system [87].

tems [28], and flight control systems [158], to cite but a few examples.

The mathematical descriptions of many hybrid dynamical systems can be characterized by impulsive differential equations [12, 14, 79, 93, 148]. Impulsive dynamical systems can be viewed as a subclass of hybrid systems and consist of three elements—namely, a continuous-time differential equation, which governs the motion of the dynamical system between impulsive or resetting events; a difference equation, which governs the way the system states are instantaneously changed when a resetting event occurs; and a criterion for determining when the states of the system are to be reset. Since impulsive systems can involve impulses at variable times, they are in general time-varying systems, wherein the resetting events are both a function of time and the system's state. In the case where the resetting events are defined by a prescribed sequence of times which are independent of the system state, the equations are known as *time-dependent differential equations* [12, 14, 35, 61, 62, 93]. Alternatively, in the case where the resetting events are defined by a manifold in the state space that is independent of time, the equations are autonomous and are known as *state-dependent differential equations* [12, 14, 35, 61, 62, 93].

Hybrid and impulsive dynamical systems exhibit a very rich dynamical behavior. In particular, the trajectories of hybrid and impulsive dynamical systems can exhibit multiple complex phenomena such as *Zeno* solutions, *noncontinuability* of solutions or *deadlock*, *beating* or *livelock*, and *confluence* or merging of solutions. A Zeno solution involves a system trajectory with infinitely many resettings in finite time. Deadlock corresponds to a dynamical system state from which no continuation, continuous or discrete, is possible. A hybrid dynamical system experiences beating when the system trajectory encounters the same resetting surface a finite or infinite number of times in zero time. Finally, confluence involves system solutions that coincide after a certain point in time. These phenomena, along with the breakdown of many of the fundamental properties of classical dynamical system theory, such as continuity of solutions and continuous dependence of solutions on the system's initial conditions, make the analysis of hybrid and impulsive dynamical systems extremely challenging.

The range of applications of hybrid and impulsive dynamical systems is not limited to controlled dynamical systems. Their usage arises in several different fields of science, including computer science, mathematical programming, and modeling and simulation. In computer science, discrete program verification and logic is interwoven with a continuous environment giving rise to hybrid dynamical systems. Specifically, computer software systems interact with the physical system to admit feedback algorithms that improve system performance and system robustness. Alternatively, in mathematical linear and nonlinear optimization with inequality constraints, changes in continuous and discrete states can be computed by a switching dynamic framework. Modeling and simulating complex dynamical systems with multiple modes of operation involving multiple system transitions also give rise to hybrid dynamical systems. Among the earliest investigations of dynamical systems involving continuous dynamics and discrete switchings can be traced back to relay control systems and bang-bang optimal control.

Dynamical systems involving an interacting mixture of continuous and discrete dynamics abound in nature and are not limited to engineering systems with programmable logic controllers. Hybrid systems arise naturally in biology, physiology, pharmacology, economics, biocenology, demography, chemistry, neuroscience, impact mechanics, quantum mechanics, systems with shock effects, and cosmology, among numerous other fields of science. For example, mechanical systems subject to unilateral constraints on system positions give rise to hybrid dynamical systems. These systems involve discontinuous solutions, wherein discontinuities arise primarily from impacts (or collisions) when the system trajectories encounter the unilateral constraints. In physiological systems the blood pressure and blood flow to different tissues of the human body are controlled to provide sufficient oxygen to the cells of each organ. Certain organs such as the kidneys normally require higher blood flows than is necessary to satisfy basic oxygen needs. However, during stress (such as hemorrhage) when perfusion pressure falls, perfusion of certain regions (e.g., brain and heart) takes precedence over perfusion of other regions, and hierarchical controls (overriding controls) shut down flow to these other regions. This shutting down process can be modeled as a resetting event giving rise to a hybrid system. As another example, biomolecular genetic systems also combine discrete events, wherein a gene is turned on or off for transcription, with continuous dynamics involving concentrations of chemicals in a given cell. Even though many scientists and engineers recognize that a large number of life science and engineering systems are hybrid in nature, these systems have been traditionally modeled, analyzed, and designed as purely discrete or purely continuous systems. The reason for this is that only recently has the theory of impulsive and hybrid dynamical systems been sufficiently developed to fully capture the interaction between the continuous and discrete dynamics of these systems.

Even though impulsive dynamical systems were first formulated by Mil'man and Myshkis [123,124],[1] the fundamental theory of impulsive differential equations is developed in the monographs by Bainov, Lakshmikantham, Perestyuk, Samoilenko, and Simeonov [12–14, 93, 148]. These monographs develop qualitative solution properties, existence of solutions, asymptotic properties of solutions, and stability theory of impulsive dynamical systems. In this monograph we build on the results of [12–14, 93, 148] to develop invariant set stability theorems, partial stability, Lagrange stability, boundedness and ultimate boundedness, dissipativity theory, vector dissipativity theory, energy-based hybrid control, optimal control, disturbance rejection control, and robust control for nonlinear impulsive and hybrid dynamical systems.

1.2 A Brief Outline of the Monograph

The main objective of this monograph is to develop a general analysis and control design framework for nonlinear impulsive and hybrid

[1]Mil'man and Myshkis were the first to develop qualitative analysis results for impulsive dynamical systems. However, work on impact and hybrid systems can be traced back to ancient Greek scientists and mathematicians such as Aristotle, Archimedes, and Heron. Problems related to Heron's work on hybrid automata (*Περί αὐτοματοποιητικῆς*) as well as problems on impact dynamics attracted the interest of numerous physicists and mathematicians who followed with relevant contributions made in the last three centuries. Notable contributions include the work of Leibniz, Newton, (Jacob) Bernoulli, d'Alembert, Poisson, Huygens, Coriolis, Darboux, Routh, Appell, and Lyapunov.

dynamical systems. The main contents of the monograph are as follows. In Chapter 2, we establish notation and definitions, and develop stability theory for nonlinear impulsive dynamical systems. Specifically, Lyapunov stability theorems are developed for time-dependent and state-dependent impulsive dynamical systems. Furthermore, we state and prove a fundamental result on positive limit sets for state-dependent impulsive dynamical systems. Using this result, we generalize the Krasovskii-LaSalle invariant set theorem to impulsive dynamical systems. In addition, partial stability, Lagrange stability, boundedness, ultimate boundedness, and stability theorems via vector Lyapunov functions are also established.

In Chapter 3, we extend the notion of dissipative dynamical systems [165,166] to develop the concept of dissipativity for impulsive dynamical systems. Specifically, the classical concepts of system storage functions and system supply rates are extended to impulsive dynamical systems. In addition, we develop extended Kalman-Yakubovitch-Popov conditions in terms of the hybrid system dynamics for characterizing dissipativeness via system storage functions and hybrid supply rates for impulsive dynamical systems. Furthermore, a generalized hybrid energy balance interpretation involving the system's stored or accumulated energy, dissipated energy over the continuous-time dynamics, and dissipated energy at the resetting instants is given. Specialization of these results to passive and nonexpansive impulsive systems is also provided. In Chapter 4, we extend the results of Chapters 2 and 3 to develop stability and dissipativity results for impulsive nonnegative and compartmental dynamical systems.

In Chapter 5, we develop vector dissipativity notions for large-scale nonlinear impulsive dynamical systems. In particular, we introduce a generalized definition of dissipativity for large-scale nonlinear impulsive dynamical systems in terms of a hybrid vector inequality, a vector hybrid supply rate, and a vector storage function. Dissipativity properties of the large-scale impulsive system are shown to be determined from the dissipativity properties of the individual impulsive subsystems making up the large-scale system and the nature of the system interconnections. Using the concepts of dissipativity and vector dissipativity, in Chapter 6 we develop feedback interconnection stability results for impulsive nonlinear dynamical systems. General stability criteria are given for Lyapunov, asymptotic, and exponential stability of feedback impulsive dynamical systems. In the case of quadratic hybrid supply rates corresponding to net system power and weighted input-output energy, these results generalize the positivity and small gain theorems to the case of nonlinear impulsive dynamical systems.

In Chapter 7, we develop a hybrid control framework for impulsive port-controlled Hamiltonian systems. In particular, we obtain constructive sufficient conditions for hybrid feedback stabilization that provide a shaped energy function for the closed-loop system while preserving a hybrid Hamiltonian structure at the closed-loop level. A novel class of energy-based hybrid controllers is proposed in Chapter 8 as a means for achieving enhanced energy dissipation in Euler-Lagrange, port-controlled Hamiltonian, and dissipative dynamical systems. These controllers combine a logical switching architecture with continuous dynamics to guarantee that the system plant energy is strictly decreasing across resetting events. The general framework leads to closed-loop systems described by impulsive differential equations. In addition, we construct hybrid controllers that guarantee that the closed-loop system is consistent with basic thermodynamic principles. In particular, the existence of an entropy function for the closed-loop system is established that satisfies a hybrid Clausius-type inequality. Extensions to hybrid Euler-Lagrange systems and impulsive dynamical systems are also developed.

In Chapter 9, a unified framework for hybrid feedback optimal and inverse optimal control involving a hybrid nonlinear nonquadratic performance functional is developed. It is shown that the hybrid cost functional can be evaluated in closed form as long as the cost functional considered is related in a specific way to an underlying Lyapunov function that guarantees asymptotic stability of the nonlinear closed-loop impulsive system. Furthermore, the Lyapunov function is shown to be a solution of a steady-state, hybrid Hamilton-Jacobi-Bellman equation. Extensions of the hybrid feedback optimal control framework to disturbance rejection control and robust control are addressed in Chapters 10 and 11, respectively.

In Chapter 12, we develop a unified dynamical systems framework for a general class of systems possessing left-continuous flows, that is, left-continuous dynamical systems. These systems are shown to generalize virtually all existing notions of dynamical systems and include hybrid, impulsive, and switching dynamical systems as special cases. Furthermore, we generalize dissipativity, passivity, and nonexpansivity theory to left-continuous dynamical systems. Specifically, the classical concepts of system storage functions and supply rates are extended to left-continuous dynamical systems providing a generalized hybrid system energy interpretation in terms of stored energy, dissipated energy over the continuous-time dynamics, and dissipated energy over the resetting events. Finally, the generalized dissipativity notions are used to develop general stability criteria for feedback interconnections of left-continuous dynamical systems. These results generalize the positivity and small gain theorems to the case of left-continuous and hybrid dynamical systems.

Finally, in Chapter 13 we generalize Poincaré's theorem to dynamical systems possessing left-continuous flows to address the stability of limit cycles and periodic orbits of left-continuous, hybrid, and impulsive dynamical systems. It is shown that the resetting manifold provides a natural hyperplane for defining a Poincaré return map. In the special case of impulsive dynamical systems, we show that the Poincaré map replaces an nth-order impulsive dynamical system by an $(n-1)$th-order discrete-time system for analyzing the stability of periodic orbits.