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Auctions in the Electricity Market

Bidding when Production Capacity Is Constrained

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Chapter 2
Literature Review

Abstract This work focuses on bidding behavior and prices at power exchanges. For this a detailed knowledge of important issues of electricity markets is necessary. Section 2.1 gives a short insight into the electricity market. It also highlights the advantage of using auctions in power exchanges. Auctions are of great use balancing demand and supply of a whole market. The electricity market in England and Wales is the starting point of the model in this work. For this reason, Section 2.2 takes a look at this market. Sections 2.3 and 2.4 deal with approaches, which are often used in the literature analyzing bidding behavior at power exchanges. Critical aspects of these approaches are discussed. Section 2.5 gives an insight into literature primarily using auction theory.

2.1 Auctions in Electricity Markets

Consumers get their electricity out of sockets, which are connected with the electricity grid. This method of transmission is more economical than using storage batteries. But using an electricity grid makes the handling complicated. The reason is that electricity is a flow, which cannot be monitored perfectly. Due to Kirchhoff's Laws, it is impossible to determine which producer injected into the grid the electricity which was withdrawn by a consumer at the other end of the line. Additionally, the power flow must be balanced at every point in time to avoid damage to the grid and all connected devices.[1]

Electricity becomes tradable by creating property rights, which allow electricity to be injected or withdrawn at specific grid nodes. Balancing the electricity grid also requires information about injections and withdrawals as well as data concerning the capacity of the transmission lines. Exceeding these capacities could lead to a blackout. The system operator, who controls the electricity grid, has to respect additionally the loss of electricity that occurs during the transmission. Reasons for

[1] See, e.g. Wilson (2002), pp. 1300–1302.

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the losses are resistance and magnetic induction. Resistance dissipates electrical energy into, e.g. heat. Magnetic induction occurs, for instance, when the voltage of transmission lines is changed in current transformers. Such changes are necessary because transmission at higher voltage is less lossy.[2]

An economic analysis of electricity markets must take into account two limiting factors. The first factor is the capacity of the transmission lines between different nodes in the electricity grid.[3] They must not be exceed otherwise a blackout is possible. The other limit is the aggregated amount of production capacity of electricity. It is technically impossible to produce more. Increasing transmission or production capacities is however time consuming and expensive.[4] This restricts market flexibility dramatically in the short term but leads to flexibility of supply in the long run.[5]

A crucial issue of the electricity market is the balancing of supply and demand or, which is the same, the injections into and withdrawals from the electricity grid. Only one agent should be responsible for this task, because a failure leads to tremendous damage to the economy in the case of a blackout. This person is often the system operator. He has to collect all necessary information without suppressing the competition between producers and consumers. An efficient way of doing this is using auctions to clear the market. Almost all electricity markets use uniform-price auctions, a specific type of auction.[6]

Auctions are of great use. They enforce competition among bidders, who can be either producers or consumers. Bidders in electricity auctions have usually private information about their utility functions. For a producer, it is the profit function. His profit is the result of the price, the amount of electricity sold in the auction and the costs to produce it. Costs are different for each producer and depend on the type of power plant. The efficiency of the power plants and the prices for fuel and other input factors are private information.

The auctioneer uses only submitted bids to determine the payments of each bidder. Hence, bidders incorporate their private information into the bids. The advantage is that no costly procedures must be implemented to guarantee truly submitted information. Additionally, the information of all market participants comes together at one place. The auctioneer has therefore all information to guarantee the safety of the electricity grid. The market can be cleared at lowest costs respecting all restrictions such as the capacity of transmission lines and the loss of electricity.

A uniform-price auction handles all nodes of the electricity grid equally. In cases of congested transmission lines, supply and demand from both sides of a congested line have to be managed differently. This causes a separation of the entire power

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[2] Losses typically count for about 3–5% of the total power production. See Wu and Varaiya (1999), p. 80 and Chao and Peck (1996), p. 39.

[3] See e.g. Hogan (1998), pp. 5–8, where pricing rules for transmission service in competitive electricity markets are discussed.

[4] See Vickers and Yarrow (1991), pp. 189–190. They also give further economic characteristics of electricity supply.

[5] See Wilson (2002), p. 1301.

[6] See Cramton (2003), p. 5.

market and creates different electricity prices. The price difference can be seen as a transmission charge. It could be used as an incentive to expand the existing transmission lines. Such handling raises questions about how to insure against these differences. Solutions are either financial hedges or rights of physical access. The topology of the grid may require many separate markets. Reducing the number of separate markets for reasons of practicality induces false incentives, as has been seen in California.[7]

An electricity market usually consists of different markets. These markets result from different needs of the market participants: the producers, the consumers, and the system operator. Depending on legal restrictions, it may be allowed to run forward markets or day-ahead markets. They take place, for example, one day before the traded electricity is delivered. This gives the system operator the opportunity to detect possible congestion and reduce real-time operations in the spot market, which was discussed above.[8]

The safety of the electricity grid requires that enough production capacity is at hand in case of an unexpected falling out of a power plant. The system operator may use auctions to buy production capacity as reserves.[9] The market of reserves shows that the production capacity of a power plant can either be used to produce electricity directly for consumers or as an option to guarantee the safety of the grid. The various markets in an electricity market are all linked in some ways. This makes the analysis of the entire electricity market very complex. A reduction of the complexity by neglecting some issues is necessary to get answers to different questions.

2.2 England and Wales: The Reference Market

Many countries have deregulated or restructured their electricity markets since the 1990s. The main idea behind this process was the recognition that power production was not a natural monopoly. For a long period of time, the bundling of electricity production and power grid were seen as a natural monopoly[10] because balancing the electricity flow in the grid requires the production capacities of power plants. But a separation of production and transmission grid was feasible as well as the implementation of competition among electricity producers. Chile was a pioneer in this field, beginning the process as early as 1978.[11] Nevertheless, the deregulation process really started in the U.K. with the Electricity Act 1989. The prominent restructured market discussed in the literature was that of England and Wales. Its new structure was introduced on March 31, 1990.[12]

[7] See Wilson (2002), pp. 1321–1325. Chao and Peck (1996) present a market model incorporating electricity losses and transmission congestion.

[8] See Wilson (2002), pp. 1325–1327. Twenty percent of electricity in mature markets is traded day-ahead, less than 10% spot, and the rest long-term. See Wilson (2002), p. 1326.

[9] Singh (1999) discusses e.g. auctions for reserves and other services in an electricity market.

[10] See Joskow (1997), p. 119.

[11] More about the market restructuring can be found in Spiller and Martorell (1996), pp. 106–119.

[12] See Beharrell (1991), p. 45.

The discussion in the literature concerning the electricity market in England and Wales often neglects some features. A short overview is therefore given here. One institution, the Central Electricity Generating Board (CEGB), owned the high-voltage transmission system and most of the power plants before the reform in 1990. It had a monopoly over the wholesale market. Twelve Area Boards with a monopoly for their regions controlled the retail market.[13]

The reform split the CEGB, passing the ownership of power plants to new companies. Nuclear Electric owns all nuclear power plants with a capacity share of about 15%. In 1991 privatization resulted in two new companies, National Power and PowerGen. National Power had a market share of about 52% and PowerGen 33%. The newly established National Grid Company PLC (NGC) owns and manages the power grid. The Area Boards were renamed Regional Electricity Companies (RECs) and privatized in December 1990. They are responsible for the local electricity grid and jointly own the NGC, the system operator.[14]

The sale of electricity to consumers was staged. Customers with a peak demand below 10 MW bought from local distribution companies at regulated tariffs. They received the freedom to choose another power provider depending on their peak demand. In 1998 the retail market was open to full competition. Customers with a peak demand over 10 MW had to pay the competitive prices of the newly created power pool.[15]

The spot market for wholesale electricity, called the power pool, was the key element of the reform. It was designed as a market with 48 separate auctions for each half hour of the next day. Producers had to notify the NGC of their available generation sets and the required prices for each of them. A price bid applied for all 48 auctions. The withdrawal of generation sets was always possible. Information about demand came from RECs and large consumers. Only the latter were allowed to specify a maximum price per half hour. RECs were presumed to be price takers. Based on this information, the NGC prepared a forecast of electricity demand for every 30 minutes of the next day to fulfill its function as the system operator.[16]

The price which producers received for a delivery time period of 30 minutes was the sum of the system marginal price and a capacity payment. The system marginal price was the uniform price of a one-sided auction, with producers as bidders. The auction was one-sided because demand could not be reduced in accordance with those market rules. The capacity payment was introduced, giving an incentive to install production capacity. Reserves, which are unused production capacities, are necessary for the grid's safety. This payment can therefore be viewed as an option price on reserves and underlines the dual nature of power plants. The capacity payment was calculated as the product of the probability of unsatisfied demand and the difference between the value of lost load minus the system marginal price. The probability depended on the situation in each half hour and could be affected, for example, by a failure of a generation set. The value of unsatisfied demand was set at $ 2 per kWh.

[13] See Vickers and Yarrow (1991), p. 191.

[14] See Vickers and Yarrow (1991), pp. 191–192.

[15] See Vickers and Yarrow (1991), pp. 191–194.

[16] See Beharrell (1991), pp. 55–56.

Consumers paid the electricity price, which producers received, plus a payment for ancillary services. Those payments reflect all costs of the system operator to ensure a balanced electricity flow. The NGC had to be compensated for energy losses and, for example, higher costs due to transmission congestion.[17]

A significant part of the literature refers to the electricity market in England and Wales. The attractiveness of the market may arise from the implemented auction system and the effective duopoly of producers. Although three big producers existed, only the privatized companies National Power and PowerGen were influencing the prices. The reason is that Nuclear Electric had the lowest marginal production costs but very high costs to run-up a nuclear generation set. The optimal strategy was to bid zero because demand was always larger than company's production capacity. Hence, another bidder sets the uniform price and Nuclear Electric operated at full capacity.

Wolfram (1998) empirically studies the electricity pool regime in England and Wales. Data cover 6 months in the period 1992–1994. The focus is on the bidding behavior of National Power and PowerGen, the two largest producers, because they very often influenced the market prices. Both producers had power plants located at grid nodes, which were often affected by congestion on the power grid. This made it necessary to call those power plants into action and gave bidders an incentive to submit high bids for these production facilities.[18] An important measure of the study is the mark-up calculated as bid minus marginal costs. Empirical results suggest that the largest producer National Power will bid higher than PowerGen for similar power plants.[19] It shows that the largest company can exercise market power.

Rising prices and investigations showing that producers exercised market power led to a redesign of the electricity market in 2001. Since then, bidders sign bilateral agreements about quantities and prices. The market is therefore similar to a discriminatory auction.[20] Bower and Bunn (2001) study both market designs, before and after 2001, using agent-based computer simulations. The agents of the simulation represent producers of the real electricity market with portfolios of power plants. They have information only about their private portfolio and use internal decision rules to decide about bids for each power plant. Their objectives are profit maximization and a target utilization of the portfolio for each simulated trading day.[21] Consumers are represented by an aggregated demand curve. Demand varies from hour to hour based on real data. Uncertainty is incorporated by adding a random amount of electricity to the basic demand.[22]

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[17] See Vickers and Yarrow (1991), pp. 195–196 and Beharrell (1991), pp. 56–59. Note that the formula given by Beharrell (1991), p. 57, is wrong. Newbery and Green (1996) present the correct formula on p. 62. They also give a good overview of the development of the English electricity industry starting in the 1850s.

[18] See Wolfram (1998), p. 716.

[19] See Wolfram (1998), p. 719.

[20] Thomas (2006) gives an overview of the market development since deregulation in 1990.

[21] See Thomas (2006), pp. 573–575.

[22] See Thomas (2006), pp. 575–576.

Bower and Bunn (2001) analyze a uniform-price auction and a discriminatory auction. Agents submit 24 separate bids for each hour of a day in one setting and one bid for the whole day in another setting. It is presumed that the whole available production capacity of a power plant is offered. The agents are only informed about the success of their own bids.[23] The simulation contains 750 days. Only the final 250 days are used for summary statistics. Peak prices of uniform-price auctions are lower than peak prices of the corresponding discriminatory auction. This relationship remains generally unchanged when comparing the prices for all hours. Power plants with low production costs face a higher probability of being underbid by power plants with higher costs in discriminatory auctions than in uniform-price auctions. That is, resources are less efficiently used in discriminatory auctions. Furthermore, the results of the simulations show that submitting one bid for the whole day induces lower prices compared with bidding separately for each hour of a day.[24]

2.3 The Supply-Function Approach

Klemperer and Meyer (1989) developed an approach, which is often used for analyzing electricity markets. They presume an oligopoly of producers facing uncertain but price-elastic demand.Individual marginal costs increase with the quantity of produced goods. Cost functions are common knowledge. Each producer chooses a supply function, which specifies for all possible prices how much he is willing to sell. All producers do this simultaneously before demand is realized. After the realization, the quantity each producers sells is the result of his supply function evaluated at the uniform price. This price is the market price where supply matches demand.[25]

Authors following this approach restrict their analysis to uniform-price auctions. More importantly, they presume common knowledge about production costs. This assumption is questionable, especially for a competitive electricity market. A lot of this information, such as fuel prices and technical conditions influencing the efficiency of a power plant, is actually private. Auctions do not use private information directly, although it may play a roll in calculating bids. But this is done secretly by each producer bidding in the auction. A main concern for them is how to handle incomplete information about competitors. Hence, the supply-function approach is not an appropriate approach for analyzing electricity auctions because it neglects an important issue of a competitive electricity market.

Bolle (1992) refers to the electricity market in England and Wales. He attempts to assess whether a spot market can create complete competition. The theoretical analysis neglects all technical issues of a power market. The author presents three games, which should clear a market of uncertain aggregated demand. Producers are presumed to have constant marginal costs and offer supply functions to the system

[23] See Thomas (2006), pp. 572–573.

[24] See Thomas (2006), pp. 577–582.

[25] See Klemperer and Meyer (1989), pp. 1245 and 1250.

operator who is responsible for the market. Such a supply function assigns to each price a quantity that the bidder is willing to produce. Demand is presumed to fall linearly with higher prices. This assumption may not reflect the market in England and Wales very well because the fraction of large consumers, who were solely able to submit price-dependent bids, was small compared to RECs.

In the first game proposed by Bolle (1992), the system operator announces a constant sale price for consumers before producers offer their supply functions. He always has to operate at zero expected profits. This game does not have an equilibrium. High profits for producers are possible, which contradicts balanced payments of the system operator. The next game reverses the steps of the first game. One arguable assumption is that now the system operator sets the price for consumers before demand is known but settles all transactions using the realized demand. This game has equilibria. Unfortunately, the supply functions decrease with the price. This is an economic result contrary to intuition.

The last game is based on the supply-function approach of Klemperer and Meyer (1989). The game reflects a spot market better because now consumers and producers face the same price after demand is realized. Results show that the spot price is the lowest for the lowest possible demand and the highest – which is the monopoly price here – for the highest demand. Bolle (1992) concludes that a pure spot market may fail to enforce complete competition. It is in turn an argument for the market design in England and Wales because this was not a pure spot market.

Another work analyzing the electricity market in the U.K. is Green and Newbery (1992). They use the supply-function approach and presume a duopoly reflecting the competitive situation at the time of their analysis. The spot market is modeled ignoring capacity payments. Behavioral incentives from repeated auctions are excluded by focusing on a one-shot game.[26] Supply functions are derived. The first result refers to producers with no capacity constraints. The other result is more relevant for the analyzed electricity market because it takes into account that one of the two producers has a capacity constraint. It reflects the situation that National Power had much more installed production capacity than PowerGen.[27]

In a next step, Green and Newbery (1992) fit their theoretical model to real market data. They compare their derived solution with a linear demand curve and marginal costs using data from 1988–89. It highlights the difference between the modeled spot market duopoly and complete competition. Depending on assumed figures, the loss to society is calculated as 6% of the total market revenue under the marginal-cost pricing regime. It is not surprising that calculated deadweight losses increase when the price elasticity of consumers decreases.[28] The case of entrants is additionally reviewed. Green and Newbery (1992) conclude finally that the government underestimated the exercise of market power. A more important result is that five companies with equal production capacities would have led to a much more competitive market compared to the established one with two unequal companies.

[26] See Green and Newbery (1992), pp. 932–934.

[27] See Green and Newbery (1992), pp. 937–940. It may be that the solution can only be found by numerical integration. See there on p. 940.

[28] See Green and Newbery (1992), pp. 941–946.

Baldick and Hogan (2002) review at the electricity market in England and Wales in 1999. They use the supply-function approach with linear demand and marginal costs similar to the empirical part of Green and Newbery (1992). In contrast to other works, constrained production capacities, price caps and reserve prices are presumed.[29] The first part of the work discusses problems finding non-decreasing supply functions by solving differential equations as proposed in Klemperer and Meyer (1989), Green and Newbery (1992), and Green (1996). Baldick and Hogan (2002) focus hereby on cases of more than two producers, constrained production capacity and asymmetric costs.[30] Multiple equilibria are problematic because only one result is empirical observed. The stability of equilibria is discussed and conditions for unstable supply-function equilibria are presented.[31]

The constraint of non-decreasing supply functions is important in practice. Hence, it is discussed by Baldick and Hogan (2002) in more detail. They show that under certain conditions the non-decreasing constraint must be explicitly modeled.[32] An iterative numerical approach and its results for different markets conditions are finally presented. The range of stable supply-function equilibria is very small under tight conditions and a binding price cap.[33]

Submitting linear or piecewise linear supply functions is analyzed by Baldick, Grant, and Kahn (2004).[34] Similar to Green and Newbery (1992), linear demand and marginal costs are presumed.[35] In the case of unrestricted production capacities, producers reveal the true intercepts of their marginal costs.[36] The case of constrained capacities is also analyzed. Unfortunately, only producers who are price-takers are presumed to have such constraints and bid their marginal costs. Hence, all producers must know in advance the final market price to anticipate the residual demand, which can be divided among the unconstrained producers. Nevertheless, Baldick, Grant, and Kahn (2004) develop a piecewise linear supply function that is non-decreasing. They emphasize that their result cannot represent an equilibrium among nonlinear functions. Additionally, they presume that demand is unlikely to be realized within a certain small range. Despite these problems, the authors explain reasons why the mentioned difficulties are not important for practical use.[37]

They use their model to compare the market situation in England and Wales from 1996 to 1999 when the duopoly changed to an oligopoly with three or five firms. The calculations show that prices declined as more competitors entered the

[29] See Baldick and Hogan (2002), pp. 6–9.

[30] See Baldick and Hogan (2002), pp. 11–28.

[31] See Baldick and Hogan (2002), pp. 29–40.

[32] See Baldick and Hogan (2002), pp. 40–56.

[33] See Baldick and Hogan (2002), pp. 56–110.

[34] Baldick, Grant, and Kahn (2004) assume a strict linear supply function at p. 149. This assumption is relaxed later, see e.g. p. 152.

[35] See Baldick, Grant, and Kahn (2004), p. 148.

[36] See Baldick, Grant, and Kahn (2004), pp. 149–151. Rudkevich (1999) developed a similar model but with zero intercept or marginal costs, see Rudkevich (1999), p. 6.

[37] See Baldick, Grant, and Kahn (2004), pp. 154–156.

market. The predicted prices using the model with positive intercepts of marginal costs are closer to realized market prices than models with zero intercepts such as Green (1999) or Rudkevich (1999).[38]

2.4 Literature Based on von der Fehr/Harbord (1993)

A second approach is based on the work of von der Fehr and Harbord (1993). They presume that a bid consists of a price for an amount of electricity, which is positive and not infinitesimal. The latter is implicitly assumed by the supply-function approach due to modeling differentiable supply functions. This contradicts the bidding rules of the uniform-price auction in the electricity market of England and Wales. A bid in von der Fehr and Harbord (1993) has just one price for a positive quantity. That is, the main difference of both approaches lies in the assumption about the offered amount of electricity.[39]

The focus of von der Fehr and Harbord (1993) is the electricity market of England and Wales a short time after deregulation. Hence, they analyze the bidding behavior of two competitors. Both have constant marginal costs and a production capacity, which cannot be exceeded. The costs and the production capacities are common knowledge. Demand is aggregated from all consumers. It is a random variable with a known lower and upper demand limit. The auction starts with the simultaneous submission of bids for the whole production capacity of each producer. Demand is realized thereafter. Now the auctioneer calculates the lowest price at which supply matches demand. All successful producers receive the same price paid. The authors analyze theoretically the duopoly market by searching for Nash equilibria.[40]

Each producer owns just one power plant with known production capacity. Marginal costs are presumed constant. Demand is uncertain and varies within known ranges. A first result is that pure equilibria exist if the number of bidders needed to match demand is known before the auction.If demand allows only one producer to produce, the paid price is the costs of the competitor and the equilibrium is unique. The highest acceptable price for the auctioneer, the reserve price, is paid in cases where both competitors are needed. Now a continuum of pure equilibria exists because producers only have to bid not higher than the reserve price.[41] Situations, where the number of selected bidders is not certain before the auction takes place, lead to a unique but mixed-strategy Nash equilibrium.[42] The last part of the paper takes a look at real data from July 1990 to April 1991. The authors find changes of bidding behavior over time, which can be seen to conform to the theory. Another

[38] See Baldick, Grant, and Kahn (2004), pp. 159–162.

[39] See von der Fehr and Harbord (1993), p. 532.

[40] See von der Fehr and Harbord (1993), pp. 532–533.

[41] See von der Fehr and Harbord (1993), pp. 533–535.

[42] See von der Fehr and Harbord (1993), p. 536.

explanation takes into account the existence of contracts of differences. They were signed before the deregulation and expired over time.[43]

Brunekreeft (2001) extends the theoretical model of von der Fehr and Harbord (1993). He allows more than two producers owning more than one power plant. Marginal costs are constant. The power plant with the next lowest or highest costs is owned by competitor.[44] Because demand is presumed uncertain, only lower and upper limits for bids are derived.[45] An equilibrium with pure strategies does not exist. It is a similar result to that of von der Fehr and Harbord (1993).[46] The other part of the article deals with real data of the electricity market in England and Wales. The results show that the derived bidding behavior fairly well matches the data provided by von der Fehr and Harbord (1993). Another interesting aspect is that fewer bidders induce higher bids also when the aggregated production capacity remains constant.[47]

Crawford, Crespo, and Tauchen (2007) extend the assumptions of von der Fehr and Harbord (1993). Their goal is the analysis of bidding behavior in a duopoly where producers own many power plants with different costs. All information such as marginal costs is common knowledge. Each producer has the same number of power plants with the capacity of one electricity unit. Marginal costs follow a step function with increasing values. Producers submit bid functions representing the price asked for each power plant.[48] In the analysis, producers are divided either into a price-setter or a non-price-setter. The first is the marginal producer who determines the market price with his bid. The bid of a non-price-setter is lower than the bid of the marginal producer. Hence, it does not affect the market price but allows the non-price-setter to sell electricity. The analysis can only offer bounds for those parts of the bid function which represent utilized power plants not being used for the price calculation. Pure Nash equilibria exist but they do not have to be unique.[49] The theoretical results are used to analyze real data of the electricity market of England and Wales from January 1993 to December 1995. Price-setters show much lower mark-ups than non-price-setters.[50]

The work of Fabra, von der Fehr, and Harbord (2002) is motivated by the discussion of changes in the market structure of England and Wales and problems in the Californian electricity market. The debate refers to the choice of pricing regime similar to the discussion about Treasury bill auctions in the United States.[51] The authors analyze an electricity market with a uniform-price, a discriminatory, and a generalized second-price auction. The last auction type is derived from the idea of

[43] See von der Fehr and Harbord (1993), pp. 539–544.

[44] See Brunekreeft (2001), pp. 104–106.

[45] See Brunekreeft (2001), pp. 107–108.

[46] See Brunekreeft (2001), pp. 108–109.

[47] See Brunekreeft (2001), pp. 109–111.

[48] See Crawford, Crespo, and Tauchen (2007), p. 1239.

[49] See Crawford, Crespo, and Tauchen (2007), pp. 1241–1242.

[50] See Crawford, Crespo, and Tauchen (2007), p. 1255.

[51] The debate on the Treasury Bill auctions started in the 1960s. Institutional aspects are discussed e.g. by Brimmer (1962), Goldstein (1962), and Friedman (1963).

the Vickrey auction.[52] The calculation of the payment for successful bidders forces the producers to bid their true marginal costs in equilibrium. Revealing true costs may be of interest to consumers. The analysis of all three auctions is also done for demand, which is known before calculating bids, as well as for uncertain demand which is realized after submitting bids. All other assumptions follow von der Fehr and Harbord (1993). A similar work to Fabra, von der Fehr, and Harbord (2002) is Fabra, von der Fehr, and Harbord (2006). It comes to similar results but does not analyze the generalized second-price auction.

Results for a market where demand is known before the auction takes places are similar to von der Fehr and Harbord (1993). The uniform-price and discriminatory auction have the same optimal bidding strategy when one producer can fully serve demand. The producer with the lowest costs bids the marginal costs of his competitor. The results for the uniform-price auction are the same as in von der Fehr and Harbord (1993). In the discriminatory auction, no equilibria with pure strategies exist for all other demands. Only unique equilibria with mixed strategies can be derived.[53] Producers in the generalized second-price auction bid their marginal costs independently, whether demand is known before or after the auction. The uniform-price and the discriminatory auction result in equilibria with mixed strategies only for uncertain demand.[54]

Fabra, von der Fehr, and Harbord (2002) and Fabra, von der Fehr, and Harbord (2006) also present bidding strategies as well as a welfare analysis, especially for certain demand. The generalized second-price auction always leads to lowest production costs because the most efficient producer is chosen first to produce electricity. This is reached in the uniform-price or discriminatory auction only if one producer can serve demand. In all other cases, it is possible that either the uniform-price auction or the discriminatory auction results in lower aggregated production costs.[55] The generalized second-price auction weakly dominates the other auction type in terms of consumer surplus if the producer with the lowest marginal costs also has the lowest production capacity. In this case, the uniform-price auction does not lead to better results than the discriminatory auction. The ranking of the auctions varies in the case that the producer with the lowest marginal costs has the same or a higher production capacity compared to his competitor. Only the discriminatory auction leads to an equal or higher consumer surplus compared with the uniform-price auction.[56]

One extension of both works is when bidders do not stick to one bid price for the whole production capacity in the case of certain demand. They are allowed to submit

[52] The Vickrey auction refers to the auction introduced by Vickrey (1961).

[53] See Fabra, von der Fehr, and Harbord (2002), pp. 14–18 and Fabra, von der Fehr, and Harbord (2006), pp. 26–27.

[54] See Fabra, von der Fehr, and Harbord (2002), pp. 24–26 and Fabra, von der Fehr, and Harbord (2006), pp. 34–35.

[55] See Fabra, von der Fehr, and Harbord (2002), pp. 18–20 and Fabra, von der Fehr, and Harbord (2006), pp. 27–28.

[56] See Fabra, von der Fehr, and Harbord (2002), pp. 20–21 and Fabra, von der Fehr, and Harbord (2006), pp. 27–28.

step offer-price functions. The existence of a unique equilibrium for the generalized second-price auction is not surprising. Equilibrium outcomes of the other auctions are independent of the step size of the offered supply functions.[57]

Crampes and Creti (2003) analyze a uniform-price auction with similar assumptions as von der Fehr and Harbord (1993). The authors presume additionally that the producer with the lowest marginal costs has the highest production capacity. The analysis focuses on the decision concerning the availability of production capacity if demand is common knowledge. Producers first decide the amount of electricity they want to sell. The bid price of electricity is chosen thereafter. This is done knowing the decisions of all competitors about their available production capacities.[58]

Crampes and Creti (2003) firstly analyze the competition in the second stage. The results show, similar to von der Fehr and Harbord (1993) and Fabra, von der Fehr, and Harbord (2002), that multiple equilibria are possible.[59] This makes the analysis of the capacity competition at the first stage difficult. Multiple equilibria exist if demand can be served by the producer with the highest capacity. A capacity withholding of at least one producer occurs if demand can be served by the producer with the lowest capacity. Otherwise, a withholding is possible but not always sure. In the case that both producers are necessary to serve demand, a capacity withholding is likely. Demand will always be matched, i.e. possible withholding does not lead to a shortage of electricity.[60]

2.5 Related Literature on Auctions

Various approaches are used in the literature concerning electricity markets, of which auction theory is just one. The advantage of this approach is that the market microstructure of auction systems can be modeled very precisely. The focus of this work is on power exchanges, which use auction systems. Hence, auction theory is a good choice for theoretical analysis.

Many different auctions are discussed in the literature. Based on the properties of electricity discussed in Section 2.1, an electricity auction can be characterized as a multi-unit auction. The traded good is homogeneous and divisible into infinite objects of infinitesimal size. Auction rules may define electricity as multiple objects of finite size. Electricity auctions can also be interpreted as multi-unit auctions with complementarities.

Wilson (1979) studies an auction where fractions of one good are sold. Buyers submit bid functions representing the price for all possible shares of the good. The price for all successful bidders is the same. The result of the common value model

[57] See Fabra, von der Fehr, and Harbord (2002), pp. 22–24 and Fabra, von der Fehr, and Harbord (2006), pp. 30–31.

[58] See Crampes and Creti (2003), pp. 5–11.

[59] See Crampes and Creti (2003), pp. 12–17.

[60] See Crampes and Creti (2003), pp. 17–25.

is that an auctioneer's revenue is much lower in this auction type than in a standard first price auction of the whole good.[61]

The assumptions of Ausubel and Cramton (2002) are similar to Wilson (1979). The main difference is that bidders have private values for all fractions of the good.[62] The authors find that a uniform-price auction does not always lead to efficient equilibria, where bidders with the highest valuations get their optimal amounts, because demand is reduced due to bid shading.[63] On the other hand, Ausubel and Cramton (2002) show that an efficient equilibrium can be achieved with a discriminatory auction in cases where the uniform-price auction fails. But it is also possible that this auction does not have an efficient equilibrium.[64] According to Ausubel and Cramton (2002), a ranking of both auction types is ambiguous in terms of efficiency as well as in terms of revenue.[65]

Bikhchandani (1999) analyzes first price auctions where several heterogeneous objects are sold simultaneously. Bidders have reservation values for all combinations, which increase with the number of bought objects. Bidders are not financially constrained.[66] Bikhchandani (1999) shows that a pure strategy Nash equilibrium exists if, and only if, a Walrasian equilibrium exists.[67]

Parisio and Bosco (2003) analyze an electricity market containing two types of power plants. The baseload type has lower production costs compared to the peak-load type. A bidder can own one power plant of each type. In contrast to many electricity market models, costs are presumed to be private knowledge. A uniform-price auction within two different environments is analyzed focusing on the competition among peak-load power plants only. In the first environment, one large producer owns both power plants while the others run only a peak-load type. A strong assumption for the bidding behavior of the large producer is that his competitors bid their marginal costs. The optimal bid exceeds marginal costs. Hence, efficient production is most likely not reached. The second environment refers to a duopoly where producers own both types of power plants. Parisio and Bosco (2003) present optimal bid functions for certain as well as for uncertain price-independent demand.

The aim of Elmaghraby (2005) is to find an auction format that always supports efficient production in equilibrium. The cost structure as well as the production capacity is presumed to be common knowledge. Costs are composed of ramp-up and constant variable costs. Ramp-up costs arise only when a power plant is started. Electricity auctioned in the model covers fluctuating demand over a defined time period. Typically, demand is divided into parts with constant amounts and shorter duration than the whole time period. Elmaghraby (2005) calls this auction format vertical. The analysis shows that neither a uniform-price nor a discriminatory auction could always ensure lowest production costs.

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[61] See Wilson (1979), pp. 682–684.

[62] See Ausubel and Cramton (2002), p. 7.

[63] See Ausubel and Cramton (2002), p. 11.

[64] See Ausubel and Cramton (2002), p. 18.

[65] See Ausubel and Cramton (2002), p. 19.

[66] See Bikhchandani (1999), pp. 195–196.

[67] See Bikhchandani (1999), p. 203.

A horizontal auction divides the time-dependent demand curve into slots of constant demand with different durations. Each slot is allocated to only one bidder who gets his bid paid. Efficiency is always reached in equilibrium if slots are auctioned according to duration starting with the longest. Additionally, bidders must submit bids, which are binding for all slots. Hence, bid revision based on new information is not possible.