1.

ON SOME ELECTROMAGNETIC PHENOMENA CONSIDERED IN CONNEXION WITH THE DYNAMICAL THEORY.

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It is now some time since general equations applicable to the conditions of most electrical problems have been given, and attempts, more or less complete, have been made to establish an analogy between electrical phenomena and those of ordinary mechanics. In particular, Maxwell has given a general dynamical theory of the electromagnetic field*, according to which he shows the mutual interdependence of the various branches of the science, and lays down equations sufficient for the theoretical solution of any electrical problem. He has also in scattered papers illustrated the solution of special problems by reference to those which correspond with them (at least in their mathematical conditions) in ordinary mechanics. There can be no doubt, I think, of the value of such illustrations, both as helping the mind to a more vivid conception of what takes place, and to a rough quantitative result which is often of more value from a physical point of view, than the most elaborate mathematical analysis. It is because the dynamical theory seems to be far less generally understood than its importance requires that I have thought that some more examples of electrical problems illustrated by a comparison with their mechanical analogues might not be superfluous.

As a simple case, let us consider an experiment first made by De la Rive, in which a battery (such as a single Daniell cell) whose electromotive force is insufficient to decompose water, becomes competent to do so by the intervention of a coil or electromagnet. Thus, let the primary wire of a Ruhmkorff coil be connected in the usual manner with the battery, and the electrodes of the voltameter (which may consist of a test-tube containing dilute sulphuric acid into which dip platinum wires) with the points where

* *Philosophical Transactions* for 1865.

in the ordinary use of the instrument the contact is made and broken. There will thus be always a complete conducting circuit through the voltameter; but when the contact is made the voltameter will be *shunted*, and the poles of the battery joined by metal. Now when the shunt is open the battery is unable to send a steady current through the voltameter, because, as has been shown by Thomson, the mechanical value of the chemical action in the battery corresponding to the passage of any quantity of electricity is less than that required for the decomposition of the water in the voltameter. When, however, the shunt is closed, a current establishes itself gradually in the coil, where there is no permanent opposing electromotive force, and after the lapse of a fraction of a second reaches its full value as given by Ohm's law. If the contact be now broken, there is a momentary current through the voltameter, which causes bubbles of gas to appear on the electrodes, and which is often (but not, I think, well) called the extra current. Allowing the rheotome to act freely we get a steady evolution of gas.

To this electrical apparatus Montgolfier's hydraulic ram is closely analogous. The latter, it will be remembered, is a machine in which the power of a considerable quantity of water falling a small height is used to raise a portion of the water to a height twenty or thirty times as great. The body of water from the reservoir flows down a closed channel to the place of discharge, which can be suddenly closed with a valve. When this takes place, the moving mass by its momentum is able for a time to overcome a pressure many times greater than that to which it owes its own motion, and so to force a portion of itself to a considerable height through a suitably placed pipe. Just as the electromotive force of the battery is unable directly to overcome the opposing polarization in the voltameter, so of course the small pressure due to the fall cannot lift a valve pressed down by a greater. But when an independent passage is opened, the water (or electricity) begins to flow with a motion which continues to accelerate until the moving force is balanced by friction (resistance), and then remains steady. At the moment the discharge-valve is closed (or, in the electrical problem, the shunt-contact is broken), the water, by its inertia, tends to continue moving, and thus the pressure instantly rises to the value required to overcome the weight of the great column of water. The second valve is accordingly opened, and a portion of the water is forced up. Now the electrical current, in virtue of self-induction, can no more be suddenly stopped than the current of water; and so in the above experiment the polarization of the voltameter is instantly overcome, and a quantity of electricity passes.

If no second means of escape were provided for the water in the hydraulic ram, the pipe would in all probability be unable to withstand the shock, and in any case could only do so by yielding within the limits of its elasticity, so as gradually, though of course very quickly, to stop the flow of water. The bursting of the pipe may properly be compared to the passage of a spark at the place where a conductor carrying an electric current is opened. Just as the natural elasticity of the pipe or the compressibility of the air in a purposely connected air-vessel greatly diminishes the strain, so the electrical spark may be stopped by connecting the breaking-points with the plates of a condenser, as was done by Fizeau in the induction-coil. Contrary to what might at first sight have been expected, the fall of the primary current is thus rendered more sudden, and the power of the instrument for many purposes increased. Of course the spark is equally prevented when the breaking-points are connected by a short conducting circuit, as in our experiment by the voltameter. In fact the energy of the actual motion which exists the moment before contact is broken is in the one case transformed into that of the sound and heat of the spark, and in the other has its equivalent partly in the potential energy of the decomposed water, partly in the heat generated by the passage of the momentary current in the voltameter branch.

The experiment will be varied in an instructive manner if we replace the voltameter by a coil (with or without soft iron), according to the resistance and self-induction of the latter. In order to know the result, we must examine closely what takes place at the moment when contact is broken. The original current, on account of its self-induction or inertia, tends to continue. At the same time the inertia in the branch circuit tends to prevent the sudden rise of a current there. A force is thus produced at the breaking-points exactly analogous to the pressure between two bodies, which we will suppose inelastic, one of which impinges on the other at rest. The pressure or electrical tension continues to vary until the velocities or currents become equal. All this time the motion of each body or current is opposed by a force of the nature of friction proportional to the velocity or current. Whether this resistance will affect the common value of the currents (or velocities) at the moment they become equal, will depend on its magnitude as compared with the other data of the problem.

There is for every conducting circuit a certain time-constant which determines the rapidity of the rise or fall of currents, and which is proportional to the self-induction and conductivity of the circuit. Thus, to use Maxwell's notation, if L and R be respectively the coefficient of self-induction and the resistance, the time-constant is $L/R=\tau$. If the current c exist at any moment in the circuit and fall undisturbed by external electromotive force, the value at any time t afterwards is given by $x=c\,.\,e^{-t/\tau}$. Any action which takes place in a time much smaller than τ will be sensibly unaffected by resistance.

We see, then, that we may neglect the effects of resistance during the time of equalization of the currents, provided that the operation is completed in a time much smaller than the time-constants of either circuit. And this I shall suppose to be the case. The value of the common current or velocity at the moment the impact is over will of course be given by the condition that the momentum, electromagnetic or ordinary, is unchanged. If L and N be the coefficients of self-induction for the main and branch circuits respectively, x and X the original and required currents, the analytical expression of the above condition is

$$(L+N)\,X = Lx, \qquad \text{or} \qquad X = \frac{L}{L+N}x.$$

It is here supposed that there is no sensible mutual induction between the two circuits.

The spark is the result of the excess of the one current over the other, and lasts until its cause is removed. Its mechanical value is the difference between that of the original current in the main circuit and that of the initial current in the combined circuit, and is expressed by

$$\tfrac{1}{2}Lx^2 - \tfrac{1}{2}(L+N)\,X^2;$$

or if the value of X be substituted,

$$\tfrac{1}{2}\frac{LNx^2}{L+N}.\text{*}$$

Exactly the same expression holds good for the heat produced during the collision of the inelastic bodies, which is necessarily equal to the loss of ordinary actual energy, at least if the permanent change of their molecular state may be neglected. From the value X the current gradually increases or diminishes to that determined according to Ohm's law, by the resistance of the combined circuit. It may be seen from the expression just found that the resistance of the branch may be varied without affecting the spark, provided always that it is not so great in relation to the self-induction as to make the time-constant comparable in magnitude with the duration of the spark. The spark depends only on the comparative self-induction of the branch circuit, being small when this is small, and when this is great approximating to its full value $\tfrac{1}{2}Lx^2$.

These results are easily illustrated experimentally. I have two coils of thick wire belonging to an electromagnet, which for convenience I will call A and B. Each consists of two wires of equal length, which are coiled together. These may be called A_1 A_2, B_1 B_2. When A_1 A_2 are joined consecutively, so that the direction of the current is the same in the two wires, we have a circuit whose self-induction is four times that of either

* [1898—An erratum is here corrected.]

wire taken singly. But if, on the contrary, the current flows opposite ways in the two wires, the self-induction of the circuit becomes quite insensible.

The main circuit may be composed of the wire A_1 (A_2 remaining open) into which the current from a single Daniell cell is led, and which can be opened or closed at a mercury cup. One end of the branch circuit dips into the mercury while the other communicates with the wire whose entrance or withdrawal from the cup closes or opens the main circuit. In this way the coils of the branch may be said to be *thrown in* at the break.

If the branch is open, we obtain at break the full spark, whose value is $\frac{1}{2}Lx^2$. If the wire B_1 be thrown in, the spark is still considerable, having approximately the value $\frac{1}{4}Lx^2$, for $N=L$. And if B_1 B_2 are thrown in, so that the currents are parallel, the spark is still greater and is measured by $\frac{1}{2}Lx^2 \times \frac{4}{5}$. But if the currents are opposed, the spark disappears, because now $N=0$; so that the addition of the wire B_2, whereby the resistance of the branch is doubled, diminishes the spark. It is true that to this last case our calculation is not properly applicable, inasmuch as the time-constant of the branch is so exceedingly small. But it is not difficult to see that in such a case (where the self-induction of the branch may be neglected) the tension at the breaking-points, or more accurately the difference of potential between them, cannot exceed that of the battery more than in the proportion of the resistances of the branch and main circuits, so that it could not here give rise to any sensible spark. Soft iron wires may be introduced into the coils in order to exalt the effects; but solid iron cores would allow induced currents to circulate which might interfere with the result.

In this form of the experiment there was no sensible mutual induction between the coils A and B. Should there be such, the result may be considerably modified. For instance, let the wire A_2 be thrown at the break into the circuit of A_1 and the battery. This may happen in two ways. If the connexions are so made that the currents are parallel in A_1 A_2, there will be no sensible spark; but if the directions of the currents are opposed, the spark appears equal to the full spark $\frac{1}{2}Lx^2$.

And this is in accordance with theory. The current X is given by the same condition as before, which leads to the equation

$$Lx + Mx = (L + 2M + N)X,$$

M being the coefficient of mutual induction between the two circuits. The spark is therefore

$$\tfrac{1}{2}Lx^2 - \tfrac{1}{2}(L + 2M + N)X^2 = \frac{x^2}{2}\frac{L-M}{2}, \quad \text{as } N = L.$$

Now in the first-mentioned connexion $M = L$ very nearly, and in the second $M = -L$; so that the observed sparks are just what theory requires.

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Excerpt
More information

With regard to those electrical phenomena which depend on the mutual induction of two circuits, it may be remarked that it is not easy to find exact analogues in ordinary mechanics which are sufficiently familiar to be of much use as aids to conception. A rough idea of the reaction of neighbouring currents may be had from the consideration of the motion of a heavy bar to whose ends forces may be applied. If when the bar is at rest one end is suddenly pushed forwards in a transverse direction, the inertia of the material gives the centre of gravity in some degree the properties of a fulcrum, and so the other end begins to move backwards. This corresponds to the inverse wave induced by the rise of a current in a neighbouring wire. If the motion be supposed infinitely small, so that the body never turns through a sensible angle, the kinetic energy is proportional to

$$\tfrac{1}{2}(a^2+k^2)\,x^2+\tfrac{1}{2}(b^2+k^2)\,y^2+(ab-k^2)\,xy,$$

where a and b are the distances of the driving-points (whose velocities are x and y) from the centre of gravity, k^2 the radius of gyration about the latter point. This corresponds to the expression for the energy of the electromagnetic field due to two currents,

$$\tfrac{1}{2}Lx^2+Mxy+\tfrac{1}{2}Ny^2;$$

and if we imagine the motion of the driving-points to be resisted by a frictional force proportional to the velocity, we get a very tolerable representation of the electrical conditions.

Or we may take an illustration, which is in many respects to be preferred, from the disturbance of a perfect fluid, by the motion of solid bodies in its interior. Thus if in an infinite fluid two spheres move parallel to each other and perpendicularly to the line joining them, and with such small velocities that their relative position does not sensibly change, the kinetic energy may as usual be expressed by

$$\tfrac{1}{2}Lx^2+Mxy+\tfrac{1}{2}Ny^2,$$

x, y denoting the velocities of the two spheres, and L, M, N being approximately constants*. When the spheres move in the same direction, the reaction of the fluid tends to press them together; but if the motions are opposed, the force changes to a repulsion. We see here the analogues of the phenomena of attraction and repulsion discovered by Ampère. If when all is at rest a given velocity is impulsively impressed on one sphere, the other immediately starts backwards, and, as Thomson† has shown, with such velocity that the energy of the whole motion is the least possible under the given condition.

This theorem is general, and leads directly to the solution of a large class of electrical problems connected with induction; for whenever a current is

* Thomson and Tait's *Natural Philosophy*, §§ 331, 332.
† Thomson and Tait, § 317.

suddenly generated in one of the circuits of a system, the initial currents in all the others are to be determined so as to make the energy of the field a minimum. These initial currents are formed unmodified by resistance whenever the electromotive impulses to which they owe their existence last only for a time which may be regarded as vanishingly small compared with the time-constants of the circuits. The sudden fall of a current when a circuit is opened generates the same currents, except as to sign, in neighbouring circuits as those due to a rise of the first current, and the condition as to sufficient suddenness is more generally fulfilled; at the same time it is more convenient in explaining the theory to take the case of the establishment of the primary current.

Suppose, then, that in the wire A_1 of our coil a current x is suddenly generated, while the ends of A_2 are joined by a short wire. The condition of minimum energy is obviously fulfilled if there arise in A_2 a current represented by $-x$; for then the energy of the field is approximately zero. But if the self-induction of the wire joining the ends of A_2 be sensible, the annihilation of the energy can no longer be perfect. Thus, let the circuit of A_2 be completed by B_1 B_2, then the general expression for the energy of two currents becomes in this case

$$\tfrac{1}{2}Lx^2 + Lxy + \tfrac{1}{2}Ly^2 \times (5 \text{ or } 1),$$

according to the connexions; and the value of y for which this is a minimum is $-x\,(1 \text{ or } \frac{1}{5})$. In the first case, the exterior part of the induced circuit having no sensible self-induction, takes away nothing from the initial current; but in the second there is a reduction to one-fifth. On the other hand, it makes no difference to the total current $(-xM/S)$*, as measured by the deflection of the galvanometer-needle, which way the connexion is made; for the smaller initial current, in virtue of its greater inertia, sustains itself proportionally longer against the damping action of resistance, which is the same in the two cases. The heating-power and the effect on the electro-dynamometer, which depend on the integral of the square of the current while it lasts $(\frac{1}{2}x^2M^2/NS)$, will be different; but the easiest proof of the diversity of the currents is to be had by comparing their powers of magnetizing steel.

Thus, if we include in the induced circuit a magnetizing spiral in which is placed a new sewing-needle, we shall find an immense difference in the magnetization produced by a break-induced current, according as its direction is the same or otherwise in the wires B_1 B_2. In the actual experiment the diluted current was unable, even after several repetitions, to give the needle any considerable magnetization (the vibrations were only about three per minute), while after one condensed current the needle gave sixteen, raised

* R, S are the resistances of the primary and secondary circuits respectively.

by repetition to nineteen*. A new needle submitted to the action of several condensed currents also gave nineteen per minute. The magnetic moments, which are as the squares of these numbers, show a still greater disproportion.

The truth seems to be that the time required for the permanent magnetization of steel is so small as compared even with the duration of our induced currents, that the amount of acquired magnetism depends essentially on the initial or maximum current without regard to the time for which it lasts.

The increased heating-effect when the two parts of the current in B are opposed in direction is, of course, at the expense of the spark in the mercury-cup. The mechanical value of the spark is the difference between the values of the currents which exist at the moments before and after the breaking of the contact, and

$$= \tfrac{1}{2}Lx^2 - \tfrac{1}{2}Ny^2 = \tfrac{1}{2}x^2\left(L - \frac{M^2}{N}\right) = \tfrac{1}{2}x^2\left(L - \frac{L^2}{N}\right) \text{ nearly.}$$

Now, according to the connexions, $N = L$ or $5L$; and so in the first case the spark disappears, while in the second it falls short of the full spark by only one-fifth.

While considering the dynamics of the field of two currents, I noticed that the initial induced current due to a sudden fall of a given current in the primary wire is theoretically greater the smaller the number of terms of which the secondary consists; for in calculating the energy of the field, it makes no difference whether we have a current of any magnitude in a doubled circuit, or twice that current in a single circuit. The same conclusion may be arrived at by the consideration of the analytical expression for the initial induced current

$$y_0 = -\frac{M}{N}x;$$

for if the secondary circuit consists essentially of a single coil of n terms, we have, *cœteris paribus*, $M \propto n$, while $N \propto n^2$, so that $y_0 \propto 1/n$. The whole induced current $\int y\,dt \propto M \propto n$. Intermediate to these is the heating-effect $\int y^2 dt$, which $\propto M^2/N$, and is therefore independent of n. Thus it was evident that neither the galvanometer nor electrodynamometer was available for the verification of this rather paradoxical deduction from theory, at least without commutators capable of separating one part of the induced current from the rest. On the other hand, it appeared probable that the smaller total current, in virtue of its greater maximum, might be the most powerful in its magnetizing action on steel.

* These were *complete* vibrations.

With the view of putting this idea to the test of experiment, I bound three wires of ·001 inch diameter, and about 20 feet long, together into a coil whose opening was sufficient to allow it to pass over the coil A. The ends of the wires were free, so that they could be joined up in any order into one circuit, which was also to contain the magnetizing spiral. It is evident that if the currents are parallel in the three wires (an arrangement which I will call a), then

$$M = 3M_0, \quad N = 9N_0,$$

M_0 N_0 being the values of the induction-coefficients for *one* wire; while if in the two wires the current flows one way round and in the third the opposite (b), we shall have $M = M_0$, $N = N_0$. Inasmuch as the self-induction of the magnetizing spiral was relatively very small, these may be regarded as the induction-coefficients for the secondary circuit as a whole. This arrangement was adopted in order that there might be no change in the resistance in passing from one case to the other. The primary current was excited by a Daniell cell in the two wires of A arranged collaterally, and was interrupted at a mercury-cup. The needle was submitted to the *break* induction-currents only—although the make currents had no perceptible magnetizing-power, on account of the relatively large time-constant of the primary circuit, and the consequent slow rise of its current to the maximum.

On actually submitting a new needle to the current a, I obtained after one discharge 12 vibrations (complete) per minute, a number raised after several discharges to 15. On the other hand, a new needle after one discharge b gave only 5 per minute, and was not much affected by repetition. The last needle being now submitted to discharge a gave $8\frac{1}{2}$, and after several 12. Other trials having confirmed these results, there seemed to be no doubt that the current a was the most efficient magnetizer. There remained, however, some uncertainty as to whether the time-constant, especially in b, was sufficiently large relatively to the time for which the spark at the mercury-cup lasted to allow of the initial current being formed undiminished by resistance. In order to make the fall of the primary current more sudden, I connected the breaking-points with the plates of a condenser belonging to a Ruhmkorff coil, and now found but little difference between the magnetizing-powers of a and b. Seeing that the theoretical condition had not been properly fulfilled, I prepared another triple coil of much thicker wire, and, for greater convenience, arranged a mercury-cup commutator, by means of which it was possible to pass at once from the one mode of connexion to the other. The magnetizing spiral was still of fine wire coiled, without any tube, closely over the needle, and its ends were soldered to the thicker wire of the triple coil.

The experiment was now completely successful. Out of the large number of results obtained, the following are selected as an example. A new needle was submitted to the break discharge of arrangement b, and gave,

After	1	discharge,	19	per minute.
„	3	„	23	„
„	6	„	24	„

Another needle was now taken and magnetized by discharge a. It gave,

After	1	discharge,	11	per minute.
„	3	„	12	„
„	10	„	$12\frac{1}{2}$	„

On submitting this needle, which had received all the magnetism that a could give it, to current b, I obtained,

After	1	discharge,	21	per minute.
„	3	„	24	„

In fact it was the general result of the experiments that more magnetism is always given to the needle by arrangement b than by a. In order, however, that the difference may be striking, it is advisable not to approach too nearly the point of magnetic saturation. The numbers quoted were obtained with the condenser, which was still necessary, in order to make the break sufficiently sudden. I have no doubt, however, that it might have been dispensed with had the triple coil consisted of a larger number of turns.

The circumstances of this experiment are in some degree represented by supposing, in the hydrodynamical analogue, one of the balls to vary in size. When a given motion is suddenly impressed on the other ball, the corresponding velocity generated in the first would vary inversely with its magnitude; for the larger the ball the greater hold, as it were, would it have on the fluid.

It is interesting also to examine the influence of neighbouring soft iron on the character of the induced current. This influence is of two sorts; but I refer here to the modifications produced by the magnetic character of iron. The circulation of induced currents in its mass may generally be prevented from exercising any injurious influence on the result by using only wires, or fragments of small size. The proximity of soft iron always increases the coefficient of self-induction N, while M may be either increased or diminished. The latter statement is true also for the initial current y_0, which is proportional to M/N. For the two wires of the coil A, however, it is easy to see that M and N are approximately equal, whether there be soft iron in their neighbourhood or not. Thus, if A_1 be connected with a Daniell cell while the circuit of A_2 is completed by the magnetizing spiral,