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Jerry H. Ginsberg
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CHAPTER 1

Basic Considerations

1.1 Introduction

The subject of dynamics is concerned with the relationship between the forces acting on a physical object and the motion that is produced by the force system. Our concern in this text shall be situations in which the classical laws of physics (i.e., Newtonian mechanics) are applicable. For our purposes, we may consider this to be the case whenever the object of interest is moving much more slowly than the speed of light. In part, this restriction means that we can use the concept of an absolute (i.e. fixed) frame of reference, which will be discussed shortly.

A study of dynamics consists of two phases: kinematics and kinetics. The objective of a kinematical analysis is to describe the motion of the system. It is important to realize that this type of study does not concern itself with what is causing the motion. A kinematical study might be needed to quantify a nontechnical description of the way a system moves, for example, finding the velocity of points on a mechanical linkage. In addition, some features of a kinematical analysis will always arise in a kinetics study, which analyzes the interplay between forces and motion. A primary objective will be the development of procedures for applying kinematics and kinetics principles in a logical and consistent manner, so that one may successfully analyze systems that have novel features. Particular emphasis will be placed on three-dimensional systems, some of which feature phenomena that you might not have encountered in your studies thus far. This is particularly the case if your prior experiences in the area of dynamics were limited to planar motion problems. As we proceed, you might recognize several topics from your earlier courses, both in engineering and in mathematics. Those topics are treated again here because of their importance, and also in order to gain greater understanding and rigor.

1.2 Newton's Laws

A fundamental aspect of the laws presented by Sir Isaac Newton is the concept of an absolute reference frame, which implies that somewhere in the universe there is an object that is stationary. This concept was discarded in modern physics (relativity theory), but the notion of a fixed reference frame introduces negligible errors for slowly moving objects. The corollary of this concept is the dilemma of what object should be considered to be fixed. Once again, negligible errors are usually produced if one considers the sun to be fixed. However, in most engineering situations it is preferable to use the earth as our reference frame. The primary effect of the earth's motion in most cases is to modify the (in vacuo) free-fall acceleration g resulting from the gravitational attraction between an object and the earth. Other than that effect, it is usually permissible to consider the earth to be an absolute reference frame. (A more careful treatment of the effects of the earth's motion will be part of our study of motion relative to a moving reference frame.)

For the purpose of formulating principles and solving problems, the fixed reference frame will be depicted as a set of coordinate axes, such as xyz. It is important to realize that coordinate axes are also often used to represent the directions for the component description of vectorial quantities. The two uses for a coordinate system are not necessarily related. Indeed, we will frequently describe a kinematical quantity relative to a specified frame of reference in terms of its components along the coordinate axes associated with a different frame of reference.

A remarkable feature of Newton's laws is that they address only objects that can be modeled as a single particle, that is, a body whose mass occupies a single point. Bodies of finite dimension are not formally covered by these laws. The three kinematical quantities for a particle with which we are primarily concerned are position, velocity, and acceleration. By definition, a particle occupies only a single point in space. As time evolves, the point occupies a succession of positions. The locus of all positions occupied by the point is its *path*.

The position of a point, as well as the velocity and acceleration, may be described mathematically by giving three independent coordinate values. Such a description is said to be *extrinsic,* because it does not rely on knowledge of the path. In contrast, an *intrinsic* kinematical description defines position, velocity, and acceleration in terms of the properties of the path.

In either case, the *position* of the point may be depicted by a vector arrow extending from some reference location, such as the origin of the fixed frame of reference, to the point of interest. *We shall always use an overbar to denote a vector quantity.* (A more common notation uses boldface to denote a vector, but the overbar has the advantage of being simpler for handwritten work.) Also, we employ subscripts to denote the point of interest and the reference point. For example, $\bar{r}_{P/O}$ denotes the position of point P with respect to point O (the slash, /, may be translated to mean "with respect to"). A typical position vector is shown in Figure 1.1.

The position changes as time goes by, so $\bar{r}_{P/O}$ is a vector function of time. The rules for differentiation of a vector are the same as those for differentiation of a scalar, except that the order of multiplication cannot be changed in treating cross

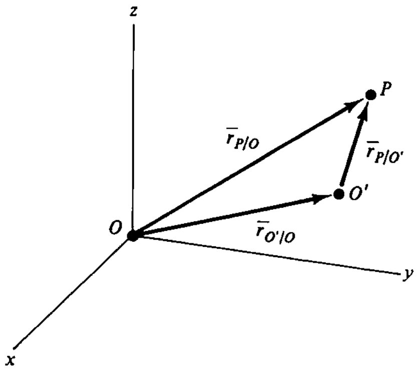

Figure 1.1 Position vectors.

products. The time derivative of the position is called the *velocity*. It is conventional to use one overdot to denote each time derivative. Thus

$$\blacklozenge \qquad \bar{v} = \frac{d\bar{r}_{P/O}}{dt} = \dot{\bar{r}}_{P/O}. \tag{1.1}$$

Two aspects are notable here. First, no subscripts have been used to denote the velocity vector. If there is any ambiguity as to the point whose velocity is under consideration, the subscript will match that point. It is never necessary to indicate the reference point in the description of velocity, because the velocity is the same as seen from all locations in an absolute frame of reference. This may be proved from Figure 1.1. If points O and O' are both fixed, then the difference in the position of point P relative to these points is constant, that is, $\bar{r}_{O'/O}$ is constant. The derivative of a constant is obviously zero. In Chapter 3 we will treat moving reference frames, in which case we will be interested in the motion relative to that reference frame. Equation (1.1) defines the *absolute* velocity, whereas the velocity seen from a moving reference frame is a relative velocity. The same terminology applies to the description of acceleration, whose definition follows. If it is not specified otherwise, the words velocity and acceleration should be understood to mean the absolute quantities.

Because velocity is a vector, it has an associated magnitude and direction. The magnitude is called the *speed*,

$$v = |\bar{v}|, \tag{1.2}$$

and the direction of $\bar{v}$ tells us the *heading*. Both of these properties are particularly important for formulations using intrinsic (path-related) variables.

Acceleration needs to be considered because it is the only motion parameter that arises in Newton's laws. The basic relation for this quantity is

$$\blacklozenge \qquad \bar{a} = \dot{\bar{v}} = \ddot{\bar{r}}_{P/O}. \tag{1.3}$$

It might be argued that our senses are accurately attuned to acceleration only when we are experiencing it – it is difficult to judge the acceleration of an object that we are passively observing. Indeed, the time derivative of $\bar{a}$, which is called the *jerk*, occurs primarily in considerations of ride comfort for vehicles.

Newton's laws have been translated in a variety of ways from their original statement in the *Principia* (1687), which was in Latin. We shall use the following version.

First Law
The velocity of a particle can only be changed by the application of a force.

Second Law
The resultant force (that is, the sum of all forces) acting on a particle is proportional to the acceleration of the particle. The factor of proportionality is the mass.

$$\blacklozenge \qquad \sum \bar{F} = m\bar{a}. \tag{1.4}$$

Third Law
The forces acting on a body result from an interaction with another body such that there is a reactive force (that is, reaction) applied to the other body. The action–

reaction pair consists of forces having the same magnitude, and acting along the same line of action, but having opposite direction.

We realize that the first law is included in the second, but we retain it primarily because it treats systems in static equilibrium without the need to discuss acceleration. The second law is quite familiar, but it must be emphasized that it is a vector relation. Hence, it can be decomposed into as many as three scalar laws, one for each component. The third law is very important to the modeling of systems. The "models" that are created in a kinetics study are free-body diagrams, in which the system is isolated from its surroundings. Careful application of the third law will assist identification of the forces exerted on the body.

The conceptualization of the first and second laws can be traced back to Galileo. Newton's revolutionary idea was the recognition of the third law and its implications for the first and second laws. An interesting aspect of the third law is that it excludes the concept of an inertial force, $-m\bar{a}$, which is usually associated with d'Alembert, because there is no corresponding reactive body. It is for that reason that we shall employ the inertial force concept in Chapter 6 only to develop the principles of analytical mechanics. (D'Alembert employed the artifice of an inertial force as a way of converting dynamic systems into static ones, in order to employ the principle of virtual work. This is the initial step in deriving Lagrange's equations in Chapter 6.)

It is also worth noting that the class of forces described by the third law is limited – any force obeying this law is said to be a *central force*. An example of a noncentral force arises from the interaction between moving electric charges. Such forces have their origin in relativistic effects. Strictly speaking, the study of *classical mechanics* is concerned only with systems that fully satisfy all of Newton's laws. However, many of the principles and techniques are applicable either directly, or with comparatively minor modifications, to relativity theory.

We should note that the acceleration to be employed in Newton's second law is relative to the hypothetical absolute reference frame. However, the same acceleration can also be observed from a variety of moving reference frames, all of which are translating (that is, the reference directions are not rotating) at a constant velocity relative to the fixed reference. Such reference frames are said to be *inertial*. The fact that Newton's laws are valid in any inertial reference frame is the *principle of Galilean invariance*, or the principle of *Newtonian relativity*.

1.3 Systems of Units

Newton's second law brings up the question of the units to be used for describing the force and motion variables. Related to that consideration is the dimensionality of a quantity, which refers to the basic measures that are used to form the quantity. In dynamics, the basic measures are time T, length L, mass M, and force F. The law of dimensional homogeneity requires that these four quantities be consistent with the second law. Thus,

$$F = ML/T^2. \tag{1.5}$$

It is clear from this relation that only three of the four basic measures are independent. Measures for time and length are easily defined, so this leaves the question of whether mass or force is the third independent quantity. When a system of units is defined such that the unit of mass is fundamental, the units are said to be *absolute*. In contrast, when the force unit is fundamental and the mass unit is given by $M = FT^2/L$, the units are said to be *gravitational*. This terminology stems from the relation among the weight w, the mass m, and the free-fall acceleration.

The only system of units to be employed in this text are SI (Standard International), which is a metric MKS (meter–kilogram–second) system, with standardized prefixes for powers of 10 and standard names for derived units. Newton's law of gravitation† states that the magnitude of the attractive force exerted between the earth and a body of mass m is

$$F = \frac{GMm}{r^2}, \tag{1.6}$$

where r is the distance between the centers of mass, G is the universal gravitational constant, and M is the mass of the earth,

$$G = 6.6732(10^{-11})\ \text{m}^3/\text{kg}\cdot\text{s}^2, \qquad M = 5.976(10^{24})\ \text{kg}.$$

The weight w of a body usually refers to the gravitational attraction of the earth when the body is near the earth's surface. When a body near the earth's surface is falling freely in a vacuum, the gravitational attraction is the mass of the body multiplied by the free-fall acceleration, that is,

$$w = mg. \tag{1.7}$$

Matching Eq. (1.6) at the earth's surface to Eq. (1.7) leads to

$$g = \frac{GM}{r_e^2}, \tag{1.8}$$

where r_e is the radius of the earth, $r_e = 6371$ km.

The relationship between g and the gravitational pull of the earth is actually far more complicated than Eq. (1.8). In fact, g depends on the location along the earth's surface. One reason for such variation is the fact that the earth is not perfectly spherical, which means that r_e is not actually constant. Variation in the value of g also arises because the earth is not homogeneous. In addition to these deviations in gravitational force, the value of g is influenced by the motion of the earth, because g is an acceleration measured relative to a noninertial reference frame. (This issue is discussed in Section 3.7.) Consequently, it is not exactly correct to employ Eq. (1.8).

The mass of a particle is constant (assuming no relativistic effects), so defining mass as a fundamental parameter yields an absolute system of units whose definition is not dependent on position; SI units constitute an absolute system. Prior to adoption of the SI standard, many individuals used a gravitational metric system in which grams or kilograms were used to specify the weight of a body. In the SI system, where mass is basic, any body should be described in terms of its mass in kilograms.

† It is implicit to this development that the inertial mass in Newton's second law be the same as the gravitational mass appearing in the law of gravitation. This fundamental assumption, which is known as the *principle of equivalence,* actually is owed to Galileo, who tested the hypothesis with his experiments on various pendulums. Subsequent, more refined, experiments have continued to verify the principle.

Its weight in newtons ($1\ \mathrm{N} \equiv 1\ \mathrm{kg} \times 1\ \mathrm{m/s^2} = 1\ \mathrm{kg\text{-}m/s^2}$) is mg, where g for a standard location on the earth's surface may be taken as

$$g = 9.807\ \mathrm{m/s^2}.$$

The system now known as U.S. customary is another gravitational system. Its basic unit is force, measured in pounds (lb). The body whose weight is defined as a pound must be at a specified location. If that body were moved to a different place then the gravitational force acting on it, and hence the units of force, might be changed. The ambiguity as to a body's weight is one source of confusion in U.S. customary units. Another stems from early usage of the pound as a mass unit. If one also employs a pound-force unit, such that 1 lbf is the weight of a 1-lbm body at the surface of the earth, then $f = ma$ is not satisfied unless the acceleration is measured in multiples of g. This is an unnecessary complication that has been abandoned in most scientific work.

Even when one recognizes that mass is a derived unit in U.S. customary units, the mass unit is complicated by the fact that two length units, feet and inches, are in common use. Practitioners working in U.S. customary units use the standard values

$$g = 32.17\ \mathrm{ft/s^2} \quad \text{or} \quad g = 386.0\ \mathrm{in./s^2}.$$

Hence, computing the mass as $m = w/g$ will give a value for m that depends on the length unit in use. The *slug* is a standard name for the U.S. customary mass unit, with

$$1\ \mathrm{slug} = 1\ \mathrm{lb}/(1\ \mathrm{ft/s^2}) = 1\ \mathrm{lb\text{-}s^2/ft}.$$

This mass unit is not applicable when inches is the length unit. In order to emphasize this matter, it is preferable for anyone using U.S. customary units to make it a standard practice to give mass in terms of the basic units. For example, a mass might be listed as 5.2 lb-s^2/ft, or a moment of inertia might be 125 lb-s^2-in.; SI units avoid all of these ambiguities.

1.4 Vector Calculus

It is assumed here that you are familiar with the basic laws and techniques for the algebra of vectors. Specifically, you should be able to represent a vector in terms of its components and perform calculations such as addition, dot products, and cross products using that component representation. If you feel uncertain about your current proficiency, it is strongly recommended that you take some time to review the appropriate topics. Much of the mathematical software in current use is equipped to carry out these operations.

As mentioned earlier, most of the laws for calculus operations are the same as those for scalar variables. It is only necessary to remember to keep the overbar on vector quantities and to remember that the order in which a cross product is taken is not commutative. In the following, $\bar{A}$ and $\bar{B}$ are time-dependent vector functions, and c and α are scalar functions of time.

Definition of a Derivative

$$\dot{\bar{A}} = \lim_{\Delta t \to 0} \frac{\bar{A}(t+\Delta t) - \bar{A}(t)}{\Delta t}. \tag{1.9}$$

Definite Integration
Let $\bar{B}=\dot{\bar{A}}$. Then

$$\bar{A}(t)=\bar{A}(0)+\int_0^t \bar{B}(\tau)\,d\tau. \tag{1.10}$$

Derivative of a Sum

$$\frac{d}{dt}(\bar{A}+\bar{B})=\dot{\bar{A}}+\dot{\bar{B}}. \tag{1.11}$$

Derivative of Products

$$\frac{d}{dt}(c\bar{A})=\dot{c}\bar{A}+c\dot{\bar{A}}, \tag{1.12}$$

$$\frac{d}{dt}(\bar{A}\cdot\bar{B})=\dot{\bar{A}}\cdot\bar{B}+\bar{A}\cdot\dot{\bar{B}}, \tag{1.13}$$

$$\frac{d}{dt}(\bar{A}\times\bar{B})=\dot{\bar{A}}\times\bar{B}+\bar{A}\times\dot{\bar{B}}. \tag{1.14}$$

Chain Rule for Differentiation
Let $\bar{A}$ be a function of some parameter α and let α be a function of time. (This obviously means that $\bar{A}$ is an implicit function of time.) Then

$$\dot{\bar{A}}=\frac{d\bar{A}}{d\alpha}\frac{d\alpha}{dt}=\dot{\alpha}\frac{d\bar{A}}{d\alpha}. \tag{1.15}$$

These rules will be used in the next chapter to treat the component representation of vectors with respect to various triads of directions.

1.5 Energy and Momentum

A basic application of the calculus of vectors in dynamics is the derivation of energy and momentum principles, which are integrals of Newton's second law. These integrals represent standard relations between velocity parameters and the properties of the force system. We will derive these laws for particle motion here; the corresponding derivations for a rigid body appear in Chapter 5.

Energy principles are useful when we know how the resultant force varies as a function of the particle's position – in other words, when $\sum\bar{F}(\bar{r})$ is known. The *displacement* of a point is intimately associated with energy principles. The displacement is defined as the change in the position occupied by a point at two instants,

$$\Delta\bar{r}=\bar{r}(t+\Delta t)-\bar{r}(t). \tag{1.16}$$

To obtain a differential displacement $d\bar{r}$, we let Δt become the infinitesimal interval dt. A dot product of Newton's second law with a differential displacement of a particle yields

$$\sum\bar{F}\cdot d\bar{r}=m\bar{a}\cdot d\bar{r}. \tag{1.17}$$

Multiplying and dividing the right side by dt, and then using the definition of velocity and acceleration, leads to

$$\sum \bar{F}\cdot d\bar{r} = m\bar{a}\cdot\frac{d\bar{r}}{dt}\,dt = m\frac{d\bar{v}}{dt}\cdot\bar{v}\,dt = \frac{1}{2}m\frac{d}{dt}(\bar{v}\cdot\bar{v})\,dt$$
$$= \frac{1}{2}m\,d(\bar{v}\cdot\bar{v}) = \frac{1}{2}m\,d(v^2) \tag{1.18}$$

The right side is a perfect differential, and the left side is a function of position only owing to the assumed dependence of the resultant force. Hence, we may integrate the differential relation between the two positions. The evaluation of the integral of the left side must account for the variation of the resultant force as the position changes when the particle moves along its path; this is called a *path integral.* We therefore find that

$$\oint_1^2 \sum \bar{F}\cdot d\bar{r} = \frac{1}{2}m(v_2^2 - v_1^2), \tag{1.19}$$

where the subscripts indicate that the speed should be evaluated at either the initial position 1 or the final position 2. The *kinetic energy* is defined as

♦ $$T = \tfrac{1}{2}mv^2, \tag{1.20}$$

and the path integral is the *work done by the force* in moving the particle,

$$W_{1\to 2} = \oint_1^2 \sum \bar{F}\cdot d\bar{r}. \tag{1.21}$$

The subscript notation for W indicates that the work is done in going from the starting position 1 to the end position 2 along the particle's path. The corresponding form of Eq. (1.19) is

$$T_2 = T_1 + W_{1\to 2}; \tag{1.22}$$

this is known as the *work–energy principle.*

The operation of evaluating the work is depicted in Figure 1.2, where a differential amount of work done by the resultant force in moving the particle may be considered in either of two ways. It is the product of the differential distance the particle moves and the component of the resultant force in the direction of movement, or equivalently, the product of the magnitude of the resultant force and the projection of the

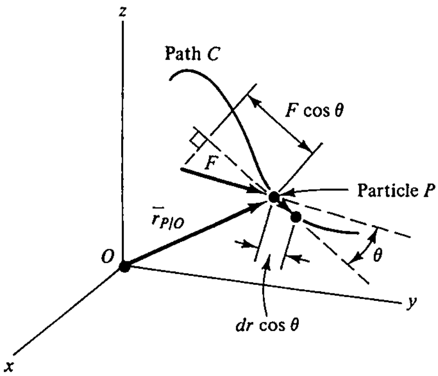

Figure 1.2 Work done by a force.

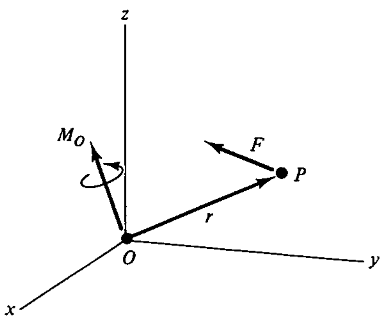

Figure 1.3 Moment of a force.

displacement in the direction of the force. Only in the simple case where the force has a constant component in the direction of the displacement does the work reduce to the simple expression "force multiplied by distance displaced." Evaluation of work is a major part of formulating the work–energy principle. We will find in Chapter 5 that this task is alleviated by introducing the concept of potential energy.

Two momentum principles follow from Newton's second law. The linear impulse–momentum principle is an immediate result when the resultant force is given as a function of time. Because acceleration is the time derivative of velocity, multiplying the second law by dt and integrating over an interval $t_1 \le t \le t_2$ leads to

$$\int_{t_1}^{t_2} \sum \bar{F}\,dt = \int_{t_1}^{t_2} m\bar{a}\,dt = m(\bar{v}_2 - \bar{v}_1). \tag{1.23}$$

The quantity $m\bar{v}$ is the *momentum* of the particle, which we shall denote by the symbol $\bar{P}$. Thus, we have

$$\bar{P} = m\bar{v}, \qquad \bar{P}_2 = \bar{P}_1 + \int_{t_1}^{t_2} \sum \bar{F}\,dt. \tag{1.24}$$

The time integral of the resultant force is the *impulse*. More precise names for the terms appearing in Eq. (1.24) are the linear momentum and linear impulse, because they are associated with the movement of a particle along a (possibly curved) line. The primary utility of the linear impulse–momentum principle is to treat systems excited by impulsive forces – that is, forces that impart a very large acceleration to a body over a very short time interval. Otherwise, the principle is an obvious consequence of knowing the resultant force as a function of time.

The angular momentum principle is associated with the moment the resultant force exerts. Let us evaluate the moment $\bar{M}_O$ about origin O of the fixed reference frame in Figure 1.3. Using the position $\bar{r}$ to form the lever arm leads to

$$\bar{M}_O = \bar{r} \times \sum \bar{F} = \bar{r} \times m\bar{a} = \bar{r} \times m\frac{d\bar{v}}{dt}. \tag{1.25}$$

We now take the time derivative outside the cross product by compensating the equation with an appropriate term to maintain the identity; specifically,

$$\bar{M}_O = \frac{d}{dt}(\bar{r} \times m\bar{v}) - m\frac{d\bar{r}}{dt} \times \bar{v} = \frac{d}{dt}(\bar{r} \times m\bar{v}) - \bar{v} \times m\bar{v}. \tag{1.26}$$

The last term vanishes because the momentum $m\bar{v}$ is parallel to the velocity. The remaining term on the right side of the equation is the time derivative of the moment about origin O of the linear momentum of the particle. We refer to this term as the *angular momentum,* denoted $\bar{H}_O$, because a moment is associated with a rotational tendency. Thus,

♦ $$\bar{H}_O = \bar{r} \times m\bar{v}. \tag{1.27}$$

Substitution of $\bar{H}_O$ onto Eq. (1.26) leads to the derivative form of the *angular impulse–momentum principle,*

♦ $$\bar{M}_O = \frac{d\bar{H}_O}{dt} \equiv \dot{\bar{H}}_O. \tag{1.28}$$

Multiplying the relation by dt and integrating over a time interval $t_1 \le t \le t_2$ leads to

$$(\bar{H}_O)_2 = (\bar{H}_O)_1 + \int_{t_1}^{t_2} \sum \bar{M}_O \, dt, \tag{1.29}$$

where the time integral of the moment is called the *angular impulse* of the resultant force.

Situations where the angular impulse–momentum principle, Eq. (1.29), are needed to study the motion of a particle are few. As is the case for its linear analog, the angular momentum principle might be useful to treat an impulsive force. Also, when the moment of the resultant force about an axis $\bar{e}$ is zero, the principle yields a conservation principle: $\bar{H}_O \cdot \bar{e}$ is constant. The primary utility of the angular momentum principle lies in the application of the derivative form, Eq. (1.27), to a rigid body. We will find in Chapter 5 that the angular momentum of a body is related to the rotation of the body. The study of orbital motion in a gravitational field is another notable application of the principle.

1.6 Brief Biographical Perspective

As we proceed through the various topics, the names of some early scientists and mathematicians will be encountered in a variety of contexts. The magnitude of the contribution of these pioneers cannot be overstated. Indeed, it is a testimonial to their ingenuity that we continue to use so much of their work. A view of the historical relationship between these researchers can greatly enhance our insight. The following is an informal chronological survey of a few individuals who have made key contributions to classical, as opposed to relativistic, physics. More details may be found in the list of references for this chapter.

Galileo, Galilei (1564–1642)
Galileo is best known for experiments on gravity at the leaning tower of Pisa, in his native country, Italy, but there is no conclusive evidence that those experiments actually occurred. From his measurements of the motion of pendulums, which led him to propose the use of a pendulum to provide the time base for a clock, he deduced that gravitational and inertial mass are identical. He refuted Aristotle's ancient statements by observing that the state of motion can only be altered by the presence of other bodies, and that there is no unique inertial reference frame. In astronomy, he developed the astronomical telescope, and used it for many pioneering observations. His last eight years were spent under house arrest for advocating the Copernican view of the solar system, which held that the sun, rather than the earth, is the center of the solar system.

Newton, Sir Isaac (1642–1727)
Newton was a professor of mathematics at Cambridge University whose inspiring work leads many to regard him as one of the two most revolutionary figures in science (Albert Einstein being the other). Newton pursued his studies of physics in England, aware of scientific developments flowering throughout Europe. The foundation for our study of mechanics was laid out by him in *Principia Mathematica Philosophiae Naturalis* (1687). In addition to his basic laws governing the movement