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Chapter 2

Co-seismic Gravity Changes Computed for a Spherical Earth Model Applicable to GRACE Data

W. Sun, G. Fu, and Sh. Okubo

Abstract Dislocation theories were developed conventionally for a deformed earth surface because most traditional gravity measurements are performed on the terrain surface. However, through development of space geodetic techniques such as the satellite gravity missions, co-seismic gravity changes can be detected from space. In this case, the conventional dislocation theory cannot be applied directly to the observed data because the data do not include surface crustal deformation (the free air gravity change). Correspondingly, the contribution by the vertical displacement part must be removed from the traditional theory. This study presents the corresponding expressions applicable to space observations. In addition, a smoothing technique is necessary to damp the high-frequency contribution so that the theory can be applied reasonably. As examples, the Sumatra earthquakes (2004, 2007) are considered and discussed.

2.1 Introduction

Numerous studies have been undertaken by many scientists to study co-seismic deformation in a half-space Earth model, a spherical earth model, and even a 3D earth model. For a half-space earth model, Steketee (1958), Maruyama (1964), and Okada (1985), etc. presented analytical expressions for calculating the

surface displacement, tilt, and strain resulting from various dislocations. Especially, Okada (1985) summarized previous studies and presented a complete set of analytical formulae for calculating these geodetic deformations. Okubo (1992) proposed closed-form expressions to describe potential and gravity changes resulting from dislocations. Because of their mathematical simplicity, these dislocation theories (e.g., Okada, 1985; Okubo, 1992) have been applied widely to study or invert seismic faults. However, the validity of these theories is strictly limited to a near field because Earth's curvature and radial heterogeneity are ignored. Modern geodesy can detect and observe far-field crustal deformation. Consequently, even a global so-seismic deformation, a dislocation theory for a more realistic Earth model, is demanded to interpret far-field deformation.

Efforts to develop formations for such a spherical Earth model have been advanced through numerous studies (e.g., Ben-Menahem and Singh, 1968; Smylie and Mansinha, 1971). Such studies have revealed that Earth's curvature effects are negligible for shallow events, although vertical layering might impart considerable effects on deformation fields. However, Sun and Okubo's (2002) study comparing discrepancies between a half-space and a homogeneous sphere and between a homogeneous sphere and a stratified sphere indicates that both curvature and vertical layering strongly affect co-seismic deformation.

Stratified spherical model such as the PREM model (Dziewonski and Anderson, 1981) is the most realistic: it reflects both sphericity and Earth's stratified structure. For such an Earth model, Pollitz (1992) solved the problem of regional displacement and strain fields induced by dislocation in a

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viscoelastic, non-gravitational model. Sun and Okubo (1993) presented methods to calculate co-seismic gravity changes in spherically symmetric Earth models. Piersanti et al. (1995) and Sabadini et al. (1995) studied displacement and its rates induced by dislocation in viscoelastic stratified Earth models, accounting for sphericity and self-gravitation using a self-consistent approach. Tanaka et al. (2006) computed co-seismic gravity changes for a visco-elastic earth model using an integrating technique to avoid intrinsic numerical difficulties. Fu and Sun (2008) presented a new theory for computing co-seismic gravity changes in a three-dimensional inhomogeneous earth model. In addition, Rundle (1982) studied deformations by a rectangular thrust fault in a gravitating model consisting of an elastic layer over a viscoelastic half-space.

All of the theories explained above were developed for a deformed earth surface because most traditional gravity measurements are performed on the earth surface. However, advances in modern geodetic techniques, such as GPS, InSAR, altimetry, and GRACE enable better detection of co-seismic deformations such as displacement, gravity change, and strain. For example, the co-seismic gravity change caused by the 2004 Sumatra earthquake was detected by GRACE (Gross and Chao, 2001; Sun and Okubo, 2004; Han et al., 2006). Han et al. (2006) calculated the gravity changes caused by the earthquake, and interpreted the gravity changes using a very simple method based on a half-space earth model. In this case, a more reasonable dislocation theory must be used instead. However, the conventional dislocation theory cannot be applied directly to the observed data because the theory includes contributions from the surface crustal deformation, although the GRACE data do not include it. Correspondingly, the contribution by the vertical displacement part must be removed from the traditional theory. For this purpose, in this study, we present the formulas applicable to the space observation. In addition, a smoothing technique, e.g., a Gaussian filter, is necessary to damp the high-frequency contribution, so that the theory can be applied reasonably. As an example, the 2004 Sumatra earthquake is considered and investigated. More case studies are made to observe whether or not the co-seismic gravity changes for a smaller earthquake (e.g., M8.0) are detectable from space.

2.2 Dislocation Theory Applicable in Satellite Data

If a dislocation is considered in a spherical Earth model (Fig. 2.1), such as a homogeneous sphere or a spherically symmetric, non-rotating, perfectly elastic and isotropic Earth (SNREI), the excited vertical displacement is represented as $u_r(a, \theta, \varphi)$ (radius, co-latitude and longitude) (Sun and Okubo, 1993). The co-seismic gravity change $\delta g(a, \theta, \varphi)$ on the deformed earth surface ($r = a + u$) is expressed as

$$\delta g(a, \theta, \varphi) = \Delta g(a, \theta, \varphi) - \beta u_r(a, \theta, \varphi) \quad (1)$$

where the first term $\Delta g(a, \theta, \varphi)$ of the right-hand-side of (1) is the gravity change at a fixed-space point ($r = a$) and β is the free-air gravity gradient, which can be expressed as (Sun and Okubo, 1993) $\beta = 2g(a)/a$, where $g(a)$ is the mean gravity on the earth surface. The last term $-\beta u_r(a, \theta, \varphi)$ gives the free-air correction caused by the vertical displacement on the earth surface. This free-air correction is considered to convert the gravity change from the fixed-space point to the deformed earth surface. This correction is necessary if a theoretical co-seismic gravity change is computed to compare with the observed gravity change on the deformed earth surface. However, if one wants to study the co-seismic gravity change observed in space, such

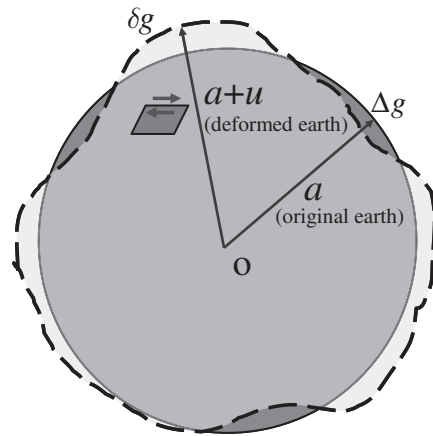


Fig. 2.1 Illustration of an original (un-deformed $r = a$) and a deformed ($r = a + u$) earth, and corresponding gravity changes

as by GRACE, Eq. (1) is not applicable, and the free-air correction term must be eliminated because the satellite gravity mission does not “see” this part.

We need only consider the first term on the right-hand-side in (1) to interpret the gravity change observed from space. This term $\Delta g(a, \theta, \varphi)$ is actually the gravity change at a fixed-space point on the undeformed earth (see level) surface $r = a$. The gravity change $\Delta g(a, \theta, \varphi)$ can be decomposed into two terms (Sun and Okubo, 1993) as

$$\Delta g(a, \theta, \varphi) = - \left. \frac{\partial \psi(r, \theta, \varphi)}{\partial r} \right|_{r=a}, \quad (2)$$

$$+ 4\pi G \rho u_r(a, \theta, \varphi)$$

where $\psi(r, \theta, \varphi)$ is the co-seismic potential change caused by the mass redistribution of the whole earth, G is Newton’s gravitational constant, and ρ is the density. The last term $4\pi G \rho u_r(a, \theta, \varphi)$ is the Bouguer gravity correction attributable to the deformation of the earth surface, i.e., the vertical displacement. It implies that the surface deformation is considered in $\Delta g(a, \theta, \varphi)$, but the free-air correction is eliminated. On the other hand, the first term on the right-hand-side in (2) also contains a Bouguer layer term, but it is opposite in sign.

$$- \left. \frac{\partial \psi}{\partial r} \right|_{r=a} = \frac{g}{a^3} \sum_{n,m}^{\infty} (n+1) k_{nm}^{ij} Y_n^m(\theta, \varphi) \quad (3)$$

$$\cdot v_i n_j U dS - 4\pi G \rho u_r(a, \theta, \varphi)$$

As a result, the Bouguer terms related with the vertical displacement $u_r(a, \theta, \varphi)$ cancel each other. Finally, the gravity change $\Delta g(a, \theta, \varphi)$ is obtainable as

$$\Delta g(a, \theta, \varphi) = \frac{g}{a^3} \sum_{n,m}^{\infty} (n+1) k_{nm}^{ij} Y_n^m(\theta, \varphi) \quad (4)$$

$$\cdot v_i n_j U dS$$

where U denotes the dislocation slip, v_i, n_j respectively indicate the slip and normal components, and Y_n^m is the spherical function. The variable k_{nm}^{ij} is the dislocation Love number of the potential change. Its numerical computation can be made similarly to that in Sun and Okubo (1993).

2.3 Co-seismic Gravity Changes on the Deformed Earth Surface

The gravity changes on the deformed earth surface (1) and at a space-fixed point (4) are used respectively for surface and space measured gravity. They are different in distribution and opposite in sign. We consider the 2004 Sumatra earthquake to demonstrate the difference between the two gravity changes (Banerjee et al., 2005; Ammon et al., 2005). The seismic slip model (Fig. 2.2a) used in this study includes two seismic events: the Sumatra earthquake, which occurred on December 26, 2004; and the Nias earthquake, which occurred on March 28, 2005. Digital data are provided by Chen Ji and by Han et al. (2006).

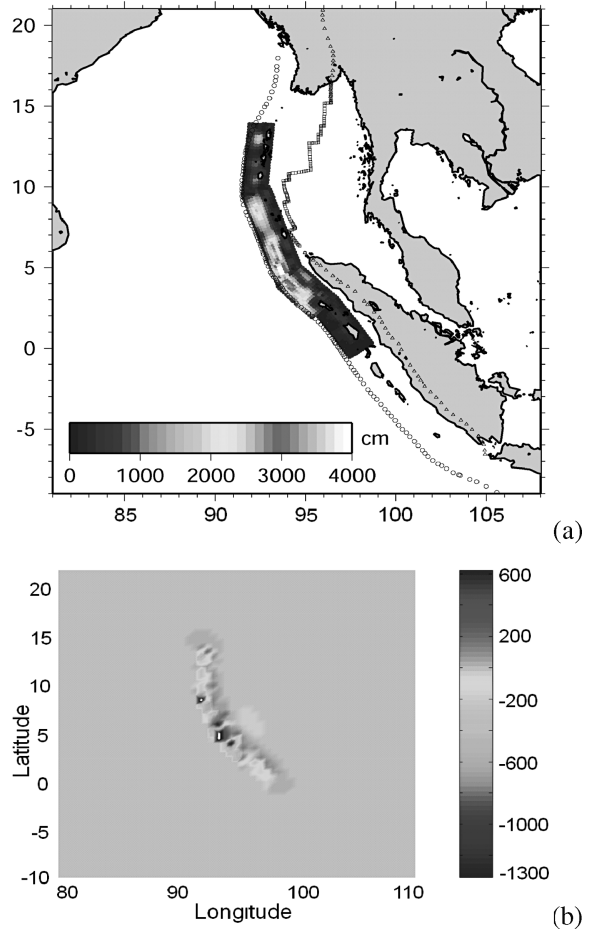


Fig. 2.2 (a) Fault slip distribution of the 2004 Sumatra earthquake, which includes seven fault planes (after Han et al., 2006). (b) Computed co-seismic gravity change (unit: μGal) on a deformed earth surface caused by the earthquake

Using the seismic fault model described above in Fig. 2.2a and Eq. (1), we calculate the co-seismic gravity changes $\delta g(a, \theta, \varphi)$ on the deformed earth surface and plot them in Fig. 2.2b. Results show that the co-seismic gravity changes vary from $-1,250 \mu\text{Gal}$ to $+600 \mu\text{Gal}$. Generally, the gravity changes appear positive in the land side (northeast direction), and negative in the ocean side (the South-West direction) near the fault area. The distribution pattern appears positive and negative values mixed together and the positive-negative boundary is not clear. This phenomenon is attributable to the sparse computing points. The cell size of the computing points is about $50 \times 50 \text{ km}$, i.e., the gravity change is calculated for 3,840 points in all in the plotted area shown in Fig. 2.2b. Actually, for a simple seismic slip model, e.g., one with few pieces of sub-tangential faults, these computing points are sufficiently dense to reflect the distribution of the gravity changes. However, because the Sumatra earthquake is a large event and the seismic slip distribution is complicated (Fig. 2.2a), the computing cell size of $50 \times 50 \text{ km}$ seems too large to visualize the phenomenon. This phenomenon is true not only for spherical dislocation theory but also for applying the half-space dislocation theory, such as Okubo (1992). In the following, we will make a computation for a smaller geometrical cell size to show the difference in distribution pattern. The rough results presented here were produced for two reasons: one is to illustrate that computation for large cells saves much computing time; the other is to clarify the following comparison.

2.4 Co-seismic Gravity Changes at Space-Fixed Point

We next compute the co-seismic gravity changes $\Delta g(a, \theta, \varphi)$ at the space-fixed point $r = a$ in the same computing scheme as $\delta g(a, \theta, \varphi)$, but using Eq. (4). Results of $\Delta g(a, \theta, \varphi)$ are depicted in Fig. 2.3. Comparing $\Delta g(a, \theta, \varphi)$ (Fig. 2.3) and $\delta g(a, \theta, \varphi)$ (Fig. 2.2b) reveals a great difference in amplitude and sign. The co-seismic gravity changes $\Delta g(a, \theta, \varphi)$ vary from $-410 \mu\text{Gal}$ to $+640 \mu\text{Gal}$; they appear negative on the land side, and positive in ocean side. For comparison, we also calculate the gravity changes using the half-space dislocation theory (Okubo, 1992) with

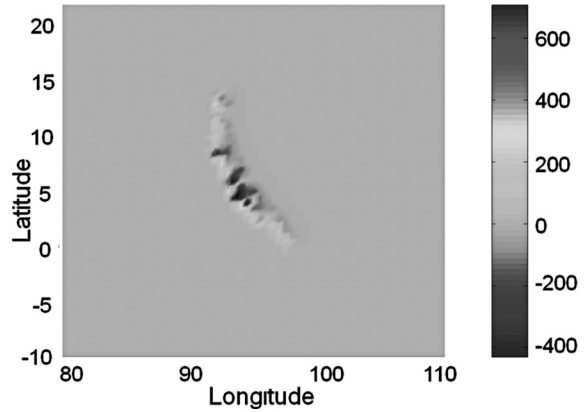


Fig. 2.3 Calculated co-seismic gravity changes (unit: μGal) at space-fixed point for the spherical earth model

elimination of the free-air correction. Comparison of the results shows that, in the near field, the gravity changes calculated using spherical and half-space theories are fundamentally identical, but with some differences in detail and in the far field.

The satellite (GRACE) is known to observe only the low-frequency gravity change because of the attenuation of the signals; the accuracy of the high-frequency signals is low. In practical applications of satellite data, a filter is usually used for damping the error in the high-frequency part. For example, Han et al. (2006) adopted the Gaussian filter with smoothing radius of 300 km, which corresponds to the spherical harmonic degree of 60. The same filter is expected to be used in the theoretical computation to compare the observed gravity changes with theoretically predicted ones. In this case, the geometrical cell size of $50 \times 50 \text{ km}$, as used above, is expected to be sufficiently small because the high-frequency contribution (less than 300 km) is expected to be filtered out. However, our investigation below shows that the geometrical cell size of $50 \times 50 \text{ km}$ is insufficient. We consider the more detailed geometrical cell size of $1 \times 1 \text{ km}$ to observe the effect of geometrical cell size on computing co-seismic gravity changes. The computed co-seismic gravity changes are depicted in Fig. 2.4. Comparison of the results in Figs. 2.3a and 2.4 shows that the gravity varies smoothly and the positive-negative boundary becomes clear and reasonable if the computing cells are sufficiently small. Results also show that the maximum amplitude of the gravity changes for small cells ($1 \times 1 \text{ km}$) size become larger than those of big cells ($50 \times 50 \text{ km}$).

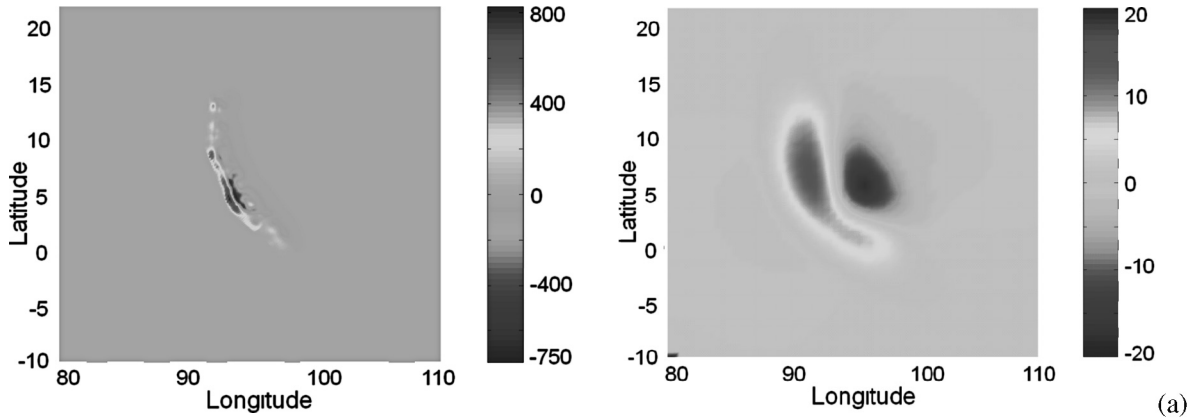


Fig. 2.4 The same as Fig. 2.3, but with small computing cell size of 1×1 km

2.5 Co-seismic Gravity Changes by Damping High-Frequency Part

As described above, to apply the theoretical prediction to satellite observed data, a filter must be used in computations. The smoothed gravity changes include only the low frequency part and become smaller in amplitude, as presented in Fig. 2.5, when the isotropic Gaussian filter ($R = 300$ km) is applied to the results shown above in Figs. 2.3a and 2.4. Figure 2.5a, b represent co-seismic gravity changes for a space-fixed point, but with cell size of 50×50 km and 1×1 km, respectively. They show different distribution patterns and amplitudes. The discrepancy is attributable entirely to the different computing cell sizes. The latter (Fig. 2.5b) looks reasonable and close to the GRACE observed one (Han et al., 2006). It implies that the computing cell size is sufficiently small to obtain reasonable results.

2.6 Are Co-seismic Gravity Changes Detectable for a M8.4 Earthquake?

As indicated by Han et al. (2006), GRACE can detect co-seismic gravity changes for a huge earthquake such as the 2004 Sumatra earthquake (M9.3). The magnitude of the gravity change is about ± 15 μgal after the Gaussian filter ($R=300$ km) is used (Fig. 2.5b). In this section, we investigate co-seismic gravity changes caused by a smaller earthquake to see whether they are

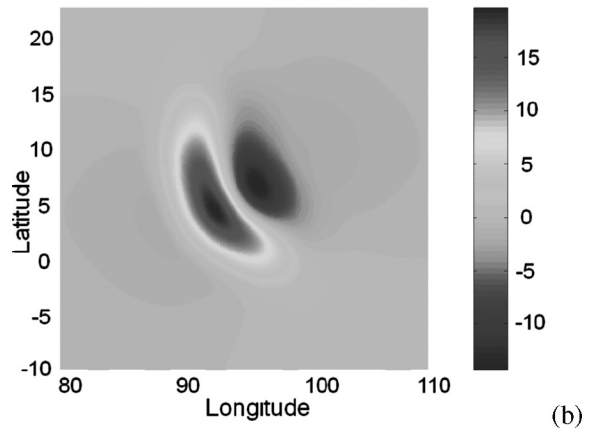


Fig. 2.5 Theoretical co-seismic gravity changes (unit: μGal) calculated for space-fixed point (Gaussian filter with $R = 300$ km): (a) for cell size of 50×50 km; (b) for cell size of 1×1 km

detectable by GRACE. For this purpose, we consider the 2007 Southern Sumatra earthquake. That earthquake includes several large shocks that occurred on September 12, 2007. Here we respectively consider the largest shock that occurred, with seismic magnitude of M8.4. The fault slip distribution is depicted in Fig. 2.6 (Chen, 2007). Then the co-seismic gravity changes caused by the earthquake are calculated. The results before the filter is used, as portrayed in Fig. 2.7a, show that the gravity changes were about -80 to $+120$ μGal . Finally, we apply the Gaussian filter with smoothing radius of $R=300$ km to the gravity changes in Fig. 2.7 and thereby obtain the smoothed gravity changes (Fig. 2.7b). The results show that the smoothed gravity changes become smooth but that the amplitude is smaller than that without a filter. However, the gravity changes still reach about -1.5 to

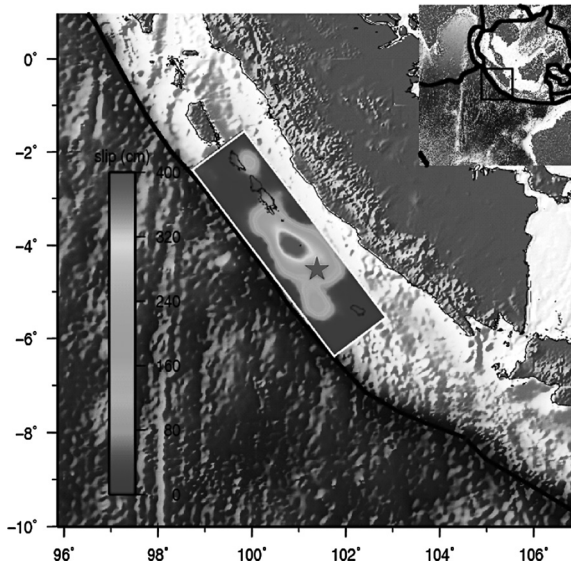


Fig. 2.6 Slip distribution on fault of the Southern Sumatra earthquake (2007, M8.4) (after Chen, 2007)

+2.5 μGal . According to the detection capability of GRACE (Sun and Okubo, 2004), the gravity changes are detectable. This conclusion is to be confirmed using actual GRACE data.

2.7 Summary

This study presented expressions of co-seismic gravity changes for a spherical earth model, applicable to satellite observed data. The difference between the co-seismic gravity changes for deformed earth surface and space-fixed point are compared and discussed. Results show that the two kinds of gravity changes are entirely different in both magnitude and sign. The effect of geometrical cell size on computation accuracy is investigated. Results show that to guarantee an accurate result of co-seismic gravity change, a small geometrical computing cell size is necessary. The co-seismic gravity change calculated by the present theory seems reasonable and coincides with the observed one. The gravity change for a smaller earthquake (M8.4) is also investigated; the results show that they are detectable by GRACE.

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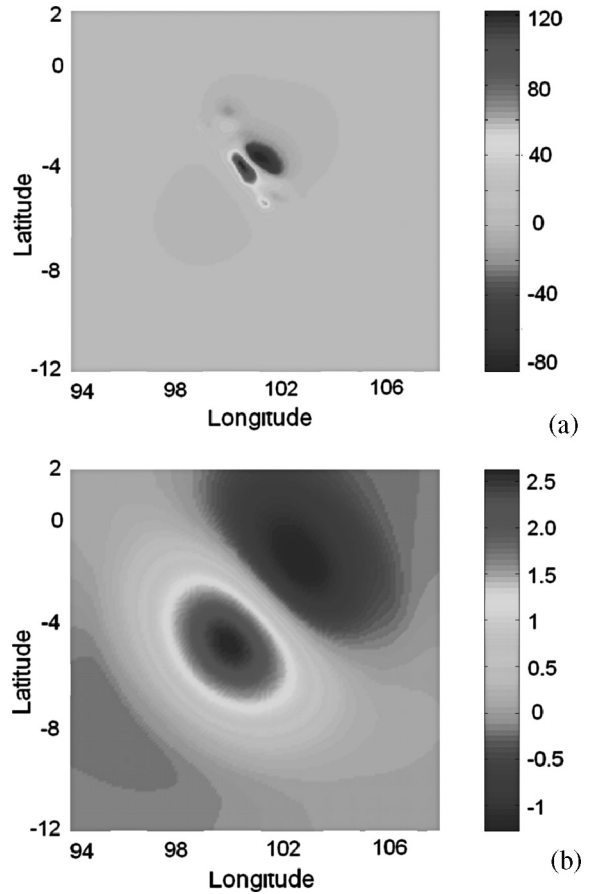


Fig. 2.7 Co-seismic gravity changes caused by the 2007 Sumatra earthquake (M8.4). (a) without filter; (b) with filter $R=300$ km (unit: μGal)

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