# Travelling Mathematics - The Fate of Diophantos' Arithmetic

Bearbeitet von
Ad Meskens

1. Auflage 2010. Buch. x, 210 S. Hardcover
ISBN 978 3 0346 0642 4
Format (B x L): 15,5 x 23,5 cm
Gewicht: 537 g

# Chapter 2

# Alexandria ad Aegyptum

## 2.1 The capital of memory

From the time of Alexander the Great, *Alexandria ad Aegyptum*[1], the Hellenic capital of Egypt designed by Deinokrates, was a centre of learning.
It was situated at the westernmost tributary of the Nile, along its widest channels, and seemed destined after Alexander's destruction of Tyre to become the dominant centre of trade between the Mediterranean, the Nile Valley, Arabia and India. Ideally located at the crossroads of these cultures, it attracted people from everywhere and could rightly be called the world's first cosmopolitan metropolis. The city was oblong-shaped: about 4 miles at its longest and about a mile wide[2]. The streets were laid out in a checkerboard pattern, with two large thoroughfares that almost bisected the city. Off the mainland, in the harbour of Alexandria, lay the isle of Pharos with its famous lighthouse. The island was almost connected to the shore by a long finger of land, a promontory known as Lochias, which stretched out towards the east. A causeway called the Heptastadium was built from the mainland, thus closing the bay toward the west.

The tongue of land on which the city stood made it singularly adapted to its purpose as a commercial and military centre. Lake Mareotis, which bounded the city to the south, served both as a wet dock and as a general harbour for inland navigation along the Nile valley. Economically, it was the largest market in the inhabited world[3] and the commercial hub of the Eastern Mediterranean. In the pre-Roman era, the city's economy declined temporarily, but it would soon recover and prosper again under the Romans. Egypt's agrarian economy was focused on Alexandria, with its stockpiles of grain destined for export. Other export products

[1] Alexandria *by* Egypt, not *in* Egypt.
[2] Pliny, *The Natural History*, 5.11 and Strabo, Geography, XVII, I, 8.
[3] Strabo, *Geography*, XVII, I.13.

A. Meskens, *Travelling Mathematics - The Fate of Diophantos' Arithmetic*, Science Networks. Historical Studies 41, DOI 10.1007/978-3-0346-0643-1_2, © Springer Basel AG 2010

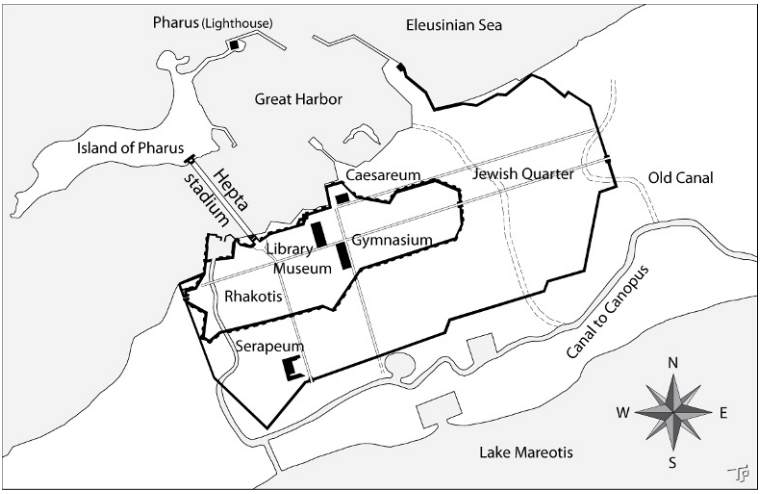


**Figure 2.1** *Ancient Alexandria*

included papyrus, book scrolls, glasswork, jewelry, fragrance and medicine.
The harbour also offered military benefits: it was large enough to accommodate large fleets, while its narrow entrance made it easy to defend.

According to Strabo, Alexandria's famed salubrious atmosphere was attributable to its location, in between the sea and Lake Mareotis, which was filled annually by waters from the Nile[4]. Galen also believed that the regular replenishment of the lake prevented pestilences, which he associated with marshes of stagnant waters. However, the waters would eventually recede, creating a fertile breeding ground for disease-carrying mosquitoes. Obviously there is no reason to imagine that the autumnal outbreaks of *pestilential fevers* (= malaria) described by sixteenth-century local physicians, and which were associated with the propinquity of Lake Mareotis, might have been absent in Antiquity[5].

The population of Alexandria is assumed to have been close to half a million[6]. The city was divided into three districts: the Jewish quarter, the Greek quarter or *Brucheion*, and the Egyptian quarter or *Rhakotis*. After 31 B.C., the Romans, who occupied the Greek quarter, became the fourth ethnic group. The Jewish quarter had its own walls and gates, as hostilities between Greeks and Jews were common.

[4]Strabo, *Geography*, XVII, I.7 "When a large quantity of moisture is exhaled from swamps, a noxious vapour rises, and is the cause of pestilential disorders. But at Alexandreia, at the beginning of summer, the Nile, being full, fills the lake also, and leaves no marshy matter which is likely to occasion malignant exhalations".
[5]W. SCHEIDEL(2001), p.79.
[6]W. SCHEIDEL(2001), p.184.

The Royal or Greek quarter contained the most important public buildings, lush gardens and the Royal Palaces. It is here that the far-famed Library and Museum were located.

The Museum, or Temple of the Muses, is believed to have been erected under Ptolemy I (367-283 B.C.)[7]. It was a semi-religious research institution based on the model of the Athenian Academy and Lyceum and devoted to the cult of the Muses. Contrary to the Academy and the Lyceum, however, it was under government control. The Museum was apparently located within the palace walls[8]. Not all members of the Museum are known, nor whether the institution had a continuous existence. However, we do encounter some famous names among its librarians: Apollonios of Samothrace, Eratosthenes (the mathematician and scientist), Aristophanes, and Aristarchos of Rhodes[9]. The members were not obliged to teach, but it seems likely that most were surrounded by a group of students. It was at the Museum that the first attempts at literary criticism were made. The librarians were painfully aware that the texts they received were imperfect in various ways. Therefore, they indicated possible corruptions, standardized texts, and developed a number of reading aids, such as punctuation and accentuation. From 38 onward, membership was a kind of award bestowed upon civil servants and high-ranking officers[10]. There is no detailed account of daily life at the Museum at any time in its history. Indirect evidence, mostly relating to Kings and Emperors[11], comes from authors such as Strabo and Athenaeus, or from imperial biographies.
A covered marble colonnade connected the Museum with an adjacent stately building: the famous Library known as the *Alexandrina*[12].

[7] E.A. PARSONS(1952), pp.83 ff., P.M. FRASER(1972), pp.312 ff.

[8] "The Museum is also part of the Royal Quarters, having a public walk (*peripaton*), seating chamber (*exedra*), and a large building containing the dining-hall of the men of learning (*philologon andron*) who participate in the Museum. This group of men have common property as well as a priest in charge of the Museum, appointed in former times by the Kings, and nowadays by the Emperor." Strabo, *Geography*, XVII, I.8.

[9] E.A. PARSONS(1952), p.116, L. REYNOLDS & N. WILSON(1974), p.8, R. MacLEOD(2000), p.6.

[10] See for example N. LEWIS(1963).

[11] Emperor Claudius (10 B.C.-54 A.D.), for example, had a new wing added to the institution: "At last he [=Claudius] even wrote historical works in Greek, twenty books of *Etruscan History* and eight of *Carthaginian History*. Because of these works there was added to the old Museum at Alexandria a new one called after his name, and it was provided that in the one his *Etruscan History* should be read each year from beginning to end, and in the other his *Carthaginian*, by various readers in turn, in the manner of public recitations." Suetonius, *Life of Claudius*, 42.

[12] For an account of the history of the Library, see E.A. PARSONS(1952). Excellent brief overviews can be found in D. DELIA(1992), M. EL-ABBADI(1990/92) and K. STAIKOS & T. CULLEN(2004), pp.157-245. Other works, some of which tend to adopt a strong literary perspective, include L. CANFORA et al (1988).

The library of Alexandria was supposedly erected by Ptolemy II (309-247 B.C.), although no ancient account of its establishment exists[13]. Estimates of the size of the library collection vary from 400000 to 700000 volumes, though the distinction between books, scrolls and chapters is not always entirely clear[14]. The Library is said to have held Greek, Roman, Jewish, Persian, Ethiopian, Babylonian, Phoenician and Indian writings, and its collection seems to have grown so rapidly that part of it had to be housed in the temple of Serapis or Serapeum, in the Rhakotis district[15].

However, R. Bagnall [16] feels these numbers are grossly exaggerated. We know of only about 450 Greek authors, which, at an average of fifty scrolls each, makes 31250 scrolls. The cited figures are ten times greater, implying that we know just 10% of the classical authors. This, argues Bagnall, seems highly unlikely. On the other hand, the suggested physical space needed to store the number of scrolls is quite reasonable[17]. Also, we know that Seneca reproached Livy for showing regret

[13]Diogenes Laertius 4.1, 5.51. The Alexandrian library was certainly not the first of its kind. Assurbanipal, for example, had previously established a library in Niniveh, which at its height seems to have held some 30000 tablets. See also D.T. POTTS in R. MacLEOD(2000). Pergamum is said to have had a library once comparable in size to the Alexandrina, possibly containing up to 200000 scrolls. It is believed by some that competition with the library of Pergamum led to a veritable bookhunt. The Seleucids built a library in Antioch about which little is known. E.A. PARSONS(1952), pp.29 & 49.

[14]Flavius Josephus, *Antiquities of the Jews*, 12.11, Aulus Gellius, *Attic Nights*, 7.17.

[15]Tacitus, *History*, 4.84, Tertullian, *Apologeticum*, 18, Epiphanius, *De Mensuris et Ponderibus*, 11, J.S. McKENZIE, S. GIBSON & A.T. REYES(2004), pp.99-100.

[16]R. BAGNALL(2002)

[17]We can estimate the physical size of the Library from these numbers. Suppose that a papyrus scroll has a length of 10 m and a width of 30 cm. Further suppose that, when rolled up, the 'empty' part in the middle has a radius of 1 cm, and that the thickness of the papyrus is 1 mm. To simplify calculations, let us assume that the spiral formed by the papyrus can be approximated by a series of cylinders for which $R_{n+1} = R_n + 1$ [mm]. A scroll would then have a diameter of about 114 mm, or just over 10 cm. The surface area of the base of the cylinder formed by the rolled-up scroll is 0.0102 $m^2$. Assuming that about 70% of the storage space is actually taken in by the scrolls, then each scroll occupies an average of 0.0145 $m^2$ in space. Further assuming there are 400000 scrolls, we find the storage space would have to have an area of 5800 $m^2$. Imagine that the scrolls were stored on shelving measuring 10 m in length and 3 m in height, or 30 $m^2$. Dividing 5800 by 30, we arrive at 193 shelving units, which we round off to 200. Finally, assuming that a combination of two shelving units occupy a breadth of about 1 m (including sufficient space between them to allow access), we arrive at a floor area of 1000 $m^2$ or 10 m by 100 m.
By comparison, at the presumed site of the Serapeum, 19 shelving rooms of about 4 m by 3 m were found. Under the same assumptions, we have $4 \times 2$ shelves, 3 metres deep. Multiplied by 19, we arrive at 1368. Divide this number by 0.0145 and we find nearly 95000 scrolls. Even if the figure was only about half this estimate, that is still a sizable collection. If the Serapeum was indeed a daughter library, then the number of volumes in the collection of the library as a whole may have run into six figures.
The figure of 50000 suggested here is very close to that mentioned by John Tzestzes, a twelfth-century Byzantine scholar, in a scholion to Plautus. He claims there were 42800 volumes in the outer library, and that the palace library contained 400000 mixed volumes [=unsorted] and 90000 volumes and digests. Tzestzes repeated these numbers in *Prolegomena to Aristophanes*. See E.A. PARSONS(1952), pp.106 ff.

at the destruction of 40000 (forty thousand) volumes in the Alexandrine wars[18], which suggests that the total collection must have been considerably greater. In 47 B.C., Caesar ordered the torching of the Egyptian fleet. Nearby warehouses and the (or a) library also caught fire and perished[19]. Mark Anthony allegedly had 200000 books transferred from the Pergamum library as compensation[20]. Estimates for the third century, when a large part of the library was possibly destructed, vary from between 200000 and 400000[21].

Whatever the size of the collection of the Library, its grandeur must have been awe-inspiring. Agents of Ptolemy III scoured the Mediterranean for books, an enterprise which would be repeated by Baghdad caliphs and Roman pontiffs alike. It made Alexandria pivotal in the ancient scholarly world, giving its scientist and literati an unparallelled access not only to Greek books, but also to Babylonian, Jewish and Egyptian writings. Among the scholars reputed to have visited or to have been invited to either the Museum or the Library are Euclid, Archimedes, Eudoxos, Aristarchos of Samos (the astronomer) and Hipparchos.
Its reputation was still intact in the second century, when emperor Domitian sent scribes to Alexandria to copy books that had been lost in the Roman library[22].

Contrary to popular myth, the Alexandrian library was not destroyed by the Arabs. It was rather destroyed and rebuilt on several occasions. The most destructive event arguably took place in 272, when the civil tension that had always been present in the city escalated and turned violent. Alexandria's walls were torn down and the Greek quarter, with its Library and Museum, was left in ruins. The

[18]R.S BAGNALL(1993), p.351; "Forty thousand books were burned at Alexandria; let someone else praise this library as the most noble monument to the wealth of kings, as did Titus Livius, who says that it was the most distinguished achievement of the good taste and solicitude of kings." Seneca, *De tranquilitate animi*, 9.5.

[19]Amminianus informs us in his *Roman History* (written around 353-378) that 70000 books perished. Amminianus Marcellinus, *Historiae* XXII, 16.13. E. A. PARSONS(1952), pp.304-307, suggests that it was not the library that went up in flames but a warehouse, where 40000 scrolls had been stored for shipment to Rome.

[20]A. MEASSON(1994), pp.32-36. "Again, Calvisius, who was a companion of Caesar, brought forward against Antony the following charges also regarding his behaviour towards Cleopatra: he had bestowed upon her the libraries from Pergamum in which there were two hundred thousand volumes." Plutarch, *Antony*, 58.4.

[21]"So being asked in our presence how many myriads there are of books, he answered—-'Over twenty myriads, O king: and I shall endeavour to have the rest made up to fifty myriads in a short time." Eusebius of Caesarea, *Praeparatio Evangelica*, 350b, "Ptolemaeus was a great lover of literature. With the help of Demetrius of Phalerum and other distinguished men, he used the royal funds to buy books from all over the world, and gathered them in two libraries in Alexandria. The outer library had 42,800 volumes; the library inside the palace complex had 400,000 mixed volumes, and 90,000 unmixed single volumes. Callimachus later compiled a catalogue of these books." Johannes Tzetzes, *Prologomena de Comoedia*, 20.

[22]"At the beginning of his rule he neglected liberal studies, although he provided for having the libraries, which were destroyed by fire, renewed at very great expense, seeking everywhere for copies of the lost works, and sending scribes to Alexandria to transcribe and correct them." Suetonius, *The Life of Domitian*, 20.

library's deathblow seems to have been dealt during the troubles of 391[23].
Unlike the rest of Egypt, which has a dry climate, Alexandria's climate is Mediterranean. As we have previously mentioned, papyrus does not preserve well in such conditions. Had the papyri not been replaced with copies on a regular basis, texts from the age of the Ptolemies would not have survived until the seventh century, or they would, at the very least, have deteriorated beyond repair or legibility. It is more likely that climate, bugs, mice and deterioration acted as a *slow fire* that gradually consumed the library holdings than that they were destroyed by Christian mobs or Arab conquerors.
Moreover, according to written sources, no fewer than twenty-three earthquakes struck the Egyptian coast between the years 320 and 1303, including a particularly severe one during the summer of 365. Over time, the harbour floor dropped more than 20 feet, so that the Royal Quarters, where the Museum was located, effectively collapsed and sunk beneath the waves.

Whether or not the Library was ever as grand as some ancient writers would have it, its fabled existence alone was enough to feed the imagination of notable book collectors during the Renaissance. It was this imagination that laid the foundation for the literary rebirth of many ancient writers, Diophantos included.

## 2.2 Diophantos' Alexandria

The only thing we can be reasonably sure of in the case of Diophantos is that he lived in Alexandria under Roman rule, most probably somewhere during the third century, though we cannot date him even to within several decades. Consequently, we do not know either whether he was ever required to pay the high taxes imposed by Augustus or whether he lived through or died during the pestilence that raged during the reign of Emperor Gallienus.

After Octavian had defeated Mark Anthony's forces at Actium and reconquered Egypt, he founded a new town in the Nile Delta, just east of Alexandria. It was called Nicopolis. Having bitter memories of Alexandria and Egypt, he imposed high taxes on their populations. He also put Egypt under direct imperial supervision, so that he controlled the food supply to Rome. His successors however would impose less harsh regimes. Under the Caesars, Alexandria was actually leniently governed, for it was in their interest to be popular in the city that commanded the largest granaries of Rome. The canal between the Nile and the Red Sea, which served a similar purpose as the present day Suez Canal, was redug. Goods from Asia were transported along the Nile to Alexandria, making it the world's principal commodities market. Most of the Caesars had some kind of relationship with Alexandria[24]. The first important change in their polity was introduced by the

[23]See also D.H. FOWLER(1987) and C. JACOB(1998).

[24]The city was favoured by Claudius, who added a wing to the Museum (see footnote 11). Claudius also had to deal with animosity between Greek and Jewish inhabitants of Alexandria.

Emperor Severus in 196. The Alexandrian Greeks were no longer formidable, and Severus accordingly restored their senate and municipal government. He also ornamented the city with a temple of Rhea and with a public bath, the *Thermae Septimianae*.

Alexandria did suffer terribly, though, after a visit from Caracalla in 215. Although he was greeted with hecatombs, he ordered the massacre of Alexandria's youth of military age in retribution for the fact that he had been mocked in the city in previous years [25].

Under Roman authority, Alexandria had previously enjoyed peace and stability. However, as imperial authority became more and more fragmented in the mid-third century, political stability in the city deteriorated. In the last quarter of the third century, Alexandria lost its predominance in Egypt. The native Egyptian population, reinforced by Arabian immigrants, had become a turbulent force.
Diocletian's siege and subsequent capture of Alexandria in 298 seems to have been a watershed in the city's history. Throughout the autumn and Spring of 297/8, Diocletian, rather uncharacteristically, laid siege to the city in an attempt to crush the Egyptian rebellion centred around the cities of Alexandria and Coptos, who had backed the usurper Lucius Domitius Domitianus and his successor Aurelius Achilleus[26].
During the reign of Gallienus, Alexandria appears to have been struck by a pestilence[27], but it is not clear whether this was a particularly fierce outbreak of malaria or another infectious disease to which Egypt, and Alexandria in particular, was prone because of its position at the crossroads of civilizations.

Religiously, Alexandria was a curious mix, where Eastern and Western faiths met, crashed or blended[28]. Alongside Judaism, the cult of Serapis was widespread in Alexandria and indeed throughout Egypt. It was in itself an amalgam of religious practices, originating in the need to make Egyptian religious traditions more accessible to the Greeks. According to this cult, the sacred bull Apis, after its death, merged its divine characteristics with those of the god Osiris. In Alexan-

---

He warned them of the possible consequences if they forced the benevolent ruler to take action (H. BOTERMANN(1996), pp.107-114, B. LEVICK (1990), pp.89 and 182-185.). Nero intended to visit Alexandria, but never set sail, because of an ominous portent (Suetonius, *Nero*, 19). Alexandria also served as the headquarters of Vespasian (C. Tacitus, *Historiae*, 3.48) during the civil wars that preceded his accession. Struck by a dearth, the city was supplied with corn by Trajan (C. Plinius the Younger, *Panegyricus*, 31). And in 122, Alexandria was visited by Hadrian, who provided a graphic picture of the population (Vopiscus, *Saturninus*, 8).

[25]"Then he betook himself to Alexandria, and here he called the people together into the gymnasium and heaped abuse on them; he gave orders, moreover, that those who were physically qualified should be enrolled for military service. But those whom he enrolled he put to death,...", Spartianus, *Caracalla* 6, see also Dion. Cassius LXVIII.22 and Herodianus IV.8-9.

[26]Eutropius IX.22

[27]Eusebius, *Historia Ecclesiastica* XXII, (ca. 263) W. SCHEIDEL(2001) does not record any particular pestilence for this period.

[28]On religion in Roman Egypt, see D. FRANKFURTER (1998).

dria, as we have seen, the temple of Serapis also served as a book repository for the main Library[29]. Although it remained an impressive structure, it endured alternating periods of prosperity and neglect. Around 181, the temple was destroyed by fire, in just one incident during which many manuscripts must have perished. It was later rebuilt on an even grander scale.
The presence of so many religions meant that unrest was never far away in this, arguably the most polyglot of Roman cities. Things did not improve when a new and unsettling religion made its appearance in Alexandria: Christianity. Christianity came to Alexandria relatively early, under the influence of, among other things, the presence of a large Hellenized Jewish community. It was supposedly introduced by St. Mark. From the time of Nero onwards, Christians would have to endure sporadic local and sometimes Empire-wide persecution. Alexandria, true to its reputation, saw the emergence of the first centres of Christian learning, such as the Cathechetical School, which –despite the persecutions– would continue to gain in influence in subsequent centuries[30]. The Christian Church began to thrive from around the mid-third century[31], but it was not until Constantine emerged as sole ruler that Christianity truly won the day.

Alexandria's position as the capital of the East was undermined when, in 324, Emperor Constantine decided to found a new city bearing his name. Constantinople would become the new seat of power in the Roman Empire. Alexandria's grain ships would no longer feed Rome, but the new capital.

So, assuming that Diophantos lived in the third century, he may well have been a witness to Caracalla's cruel treatment of Alexandria, the rise of Christianity, Diocletian's siege and the persecution of the Christians.

---

[29]On the cult of Serapis, other pagan religions and early Christianity see A.K. BOWMAN(1986), 167ff.

[30]P. SCHAFF, H. WACE & A.C. McGIFFERT (s.d.), p.345, footnote 1506.

[31]In 200, Severus' imperial edict forbade all subjects in the Empire to "make Jews or Christians" (i.e. to convert people to either Judaism or Christianity). After his death (211), the persecutions stopped and the Church grew in numbers and in wealth. Under Decius (249-251), the profession of Christianity was denounced as incompatible with the requirements of the state. The persecutions were put to an end after his death, only to resurface again under Valerian (257-261). After Valerian's capture by the Persians, his son Gallienus issued an Edict of Toleration (F. CONYBEARE(1914), see also Eusebius, *Historia Ecclesiastica*, III. 17 (Domitian), VI.1 (Severus), VI.28 (Maximinus), VII.1 (Decius and Gallus) VII.10 (Valerian), VII.13 &23 (Lucinius)).
Alexandria entered the late Roman period as the centre of a concerted rebellion against imperial authority, which had to be suppressed by the emperor Diocletian (284-306) himself. The beginning of the fourth century witnessed the start of Diocletian's 'Great Persecution' of Christians. It seems Egypt may have suffered more heavily than other areas, as one of the most fanatical anti-Christian persecutors, Sossianus Hierocles, held the office of prefect of Egypt (A.K. BOWMAN(1986), p. 45.). After Diocletian had retired from public life, a tetrarchy was organized, with ultimately Maximinus and Constantine as Augusti and Galienus and Lucinius as Caesars.

## 2.3 Education and the culture of *paideia*

Diophantos was undoubtedly part of, or at least very well acquainted with, the culture of *paideia*. Originally, the term meant "child-rearing", but it eventually became synonymous with "culture", the purely intellectual maturation and assimilation of the educational values acquired through schooling[32]. *Paideia* became a much more embracing concept, that could be understood as a code of behaviour, a way of life and of networking. It was acquired through an education that not only taught literature, but also allowed men of culture to master a behavioural code[33]. Regardless of their religious allegiance, men of standing were expected to participate in political and cultural life. Alexandria offered both, and at a high level of sophistication. The Museum and the Library naturally attracted prominent intellectuals, including scholars and authors, and many others used to send epistles or treatises to their friends and peers in the city. While these were not always intended for general circulation, it was common for the recipient to have them copied and sent to his circle of friends[34].
This culture did not change significantly after Christianity had come to prominence[35], but the new faith did add an ethos of hope.

Our general picture of education in Graeco-Roman Egypt is relatively clear[36]. However, the aspect of science and mathematics teaching is frequently ignored by authors on this topic.
For those not fortunate enough to receive private tuition, education began at an elementary school, which would not necessarily have been housed in a building. It may well have been in open air in the shadow of a tree. The goals of this education were modest, focusing on basic reading and writing skills. Arithmetic teaching followed the same basic structure it had done for over two thousand years. Pupils were familiarized with the basic operations: adding, subtracting, multiplying and dividing. Adding may have proceeded orally, by finger reckoning or by means of an abacus[37].
In specialized scribal schools, the acquisition of a deeper knowledge of multiplication and division was an integral part of the curriculum, as one might expect. Just as in Babylon, students also acquired metrological expertise: they were familiarized with weights, measures conversions and the monetary system. A book such as Heron's *Definitions* fits into this picture very neatly.

[32] R. CRIBIORE(2001), pp. 243-244.
[33] See E.J. WATTS(2006), p.2, pp.12-19.
[34] E.J. WATTS(2006), p.154.
[35] S. RAPPE(2001) on the incorporation of pagan elements into Christian education.
[36] This description is largely based on R. CRIBIORE(2001).
[37] Pupils were often required to recite simple additions in monotonous chants. G. CRIBIORE(2001), p.181. On finger reckoning, see B. WILLIAMS & R.S. WILLIAMS(1995); on the abacus, see R. NETZ(2002b).

Secondary education for the rich seems to have had a rhetorical focus. Students were taught the virtues and ideals to which one was expected to aspire. These were embodied in Greek and Roman literature alike. The Trojan hero Aeneas, for example, embodied the Roman ideals of duty and patriotism. The attempt of Greek Alexandria to emulate the Athenian model of education may have provided an incentive for the study of mathematics.
Higher education – at least during the Empire– seems to have been organized in the Musea. Although, as previously noted, members of the Alexandrian Museum were not obliged to teach, they did have students. Other Musea existed outside Alexandria, including at Ephesus and Smyrna. By the end of the fourth century, the word Museum had become synonymous with school[38].

Strikingly, most Greek mathematicians lived and worked in Alexandria: Euclid, Erasthotenes, Heron, Ptolemy, Pappos, Theon ... This may give rise to the rather misleading perception that there was a veritable concentration of mathematicians in the city. After all, we must not forget that we are looking at a timeframe of many centuries. Moreover, there is no evidence to suggest that there was any such thing as an Alexandrian school of mathematics[39]. It may just be one of those ironies of history that only texts by Alexandrian mathematicians did fortuitously survive[40].
While mathematicians enjoyed a high degree of social visibility, be it as land surveyors, artisans or indeed astrologers, they –and hence their texts– were equally clearly influenced by general social and cultural trends. One of these trends was that intellectuals and authors were becoming increasingly interested in classifications and rearrangements of previous knowledge, and they developed a predilection for commentaries and epitomes[41].

Despite the fact that many scholars believe that, by the fourth century, the teaching of mathematics had become either non-existent or limited to an elementary curriculum that was in every way subordinate to philosophy[42], it always remained part of the ideal programme of general culture. Indeed, the picture that emerges from the history of science is that, far from being invisible, mathematics was held in high esteem. The rise of Christianity did not change that. On the contrary, the new faith incorporated maths into its own educational programme. The theologian and teacher Origen (185-ca. 254), at the Cathechetical School, is known to have taught a Christian interpretation of physics, astronomy and geometry, whatever that may have encompassed.

[38]H.-I. MARROU(1948), pp.285-287.
[39]See for example G. ARGOUD(1994).
[40]See B.VITRAC(2008), p.531.
[41]S. CUOMO(2000), pp.48-56 and (2001), pp.249 ff.
[42]See for example M.L. CLARKE(1971) and D. PINGREE(1994). As already noted by S. CUOMO(2000), p.46, neither Euclid nor Nicomachus are elementary texts, yet they were part of the curriculum.

The teaching profession was not exclusively in male hands. Women in Hellenistic Egypt participated openly in society and tried to make a name for themselves in various professions. Some of the women known to have taught at the higher levels of education are Hermione, Agallis and Hypatia[43].

## 2.4 Heron of Alexandria: a Diophantine precursor?

Heron of Alexandria is one of the few known applied mathematicians of Antiquity. The name Heron was however rather common, so that it is hard to tell precisely which references are actually to Heron the mathematician. This makes him a rather elusive figure. Moreover, much of his work does not survive in its original form. It has been edited, altered and compiled so often that it is extremely difficult to distinguish Heron's hand from others, not to speak of the derivative or imitative works that are often attributed to him. This has resulted in an intricate web of more or less genuine and spurious manuscripts, and hence it should come as no surprise that the authorship of many of these works is disputed. To complicate matters further, some of his treatises have not been passed on to us in Greek. *Mechanica*, for example, survives only in an Arabic translation, while *Optica* is known to us only in Latin.

It is equally difficult to date Heron. Otto Neugebauer has argued that the 'recent eclipse', which Heron refers to, may be that observed in Alexandria on 13 March 62[44]. From this he concludes that Heron must have lived in the second half of the first century. This is corroborated by Dimitros Sakalis's research, which contains an in-depth study of words and phraseology used by Heron[45]. He also provides some indirect evidence, based on other sources. Galen seems to criticize Heron's work, without referring to him, but in the same phraseology[46]. Vitruvius (first century) does not mention Heron, although he refers to quite a few mathematicians and engineers. Moreover, he refers to mechanisms that are inferior to Heron's contraptions[47]. The oldest Hebrew geometry, the *Mishnat ha-Middot*, dating from the middle of the second century, was strongly influenced by the Heronian corpus[48]. Lastly, Proklos mentions that Heron was younger than Menelaos, who lived ca. 100.
In his works, Heron uses graecicized Latin words that only began to appear in the

[43]R. CRIBIORE(1996), pp.22-23.
[44]O. NEUGEBAUER(1938). However, N. SIDOLI (2005), pp. 250-252 puts it that Heron may have used a hypothetical eclipse. He argues that two other eclipses observed in Alexandria namely, in 133 B.C. and 3 B.C. tie in better with Heron's data.
[45]D. SAKALIS(1972).
[46]D. SAKALIS(1972), pp. 11-15.
[47]D. SAKALIS(1972), pp. 15-25.
[48]D. SAKALIS(1972), pp. 158-26.

first century[49], which reinforces the argument that he lived either in the second half of the first century or at the beginning of the second[50].
Heron's writings fall into many categories. He is, for example, one of our most important ancient sources on pneumatics, instruments and war engines. However, we shall restrict ourselves to his mathematical work[51], which, unlike his work on pneumatics, has received relatively little in-depth scholarly attention.
His writings reveal him to have been a well-educated mathematician, although his theoretical explanations are sometimes weak. But despite this shortcoming, he is an essential figure in the practical mathematics tradition that started in Babylonia. Furthermore, it would be a mistake to assume that practical mathematics was not an essential part of Greek mathematics as a whole.

*Metrika* is Heron's most important work. It is an introduction to practical geometry and measurement. Book I deals with plane geometry and builds freely on Euclid and Archimedes. The book is essentially constructed around three formulae: an iterative algorithm for the calculation of the square root of a number, the so-called Heronic formula[52] for the area of a triangle and the property that the area of a circle segment is larger than four thirds of the area of the inscribed triangle with the same base and height. Book II elaborates on the volumes of cones, cylinders, parallelepipeds, pyramids, frusta and Platonic solids. The volume of the sphere is determined to be two-thirds of the circumscribed cylinder. Book III discusses how figures can be divided into figures of a given ratio.
*Definitions* contains 133 definitions on geometrical terms, beginning with points and lines etc.
*Geometrika* would appear to be a different version of the first chapter of *Metrika*, founded entirely on exercises. Although it is clearly based on Heron's work, it is doubtful whether he was in fact the author.
*Stereometrika* deals with three-dimensional objects and is at least based on the second chapter of *Metrika*. Again, though, the original text is believed to have been altered considerably by later editors. Moreover, its two constituting books would appear to be different versions of the same work.
*Mensurae* is concerned with the measurement of a variety of objects. It is related to both *Metrika* and *Stereometrika*, but this book, too, is thought to be mainly

[49]For instance, πάσσον for *passus* and μίλιον for *milia*. See D. SAKALIS(1972), p.160.

[50]However, to illustrate the difficulty in dating Heron as a result of later additions, we also refer to the following examples. In *Geometrika* 21.26 and *Stereometrika* 1.21.3, a certain *Patricius* is mentioned. Patricius is identified as the Lydian expert in divination who was killed by Valens ca. 371 (T. MARTIN(1854), p. 220). In *Stereometrika* 2.54 we read: 'These [= the measures] were fixed under Modestus, who was praetorian prefect at the time.". S. CORCORAN(1995) identifies Modestus as Domitius Modestus, who was praetorian prefect of the East under Valens from 369 to 377.

[51]The following description of the mathematical works is based on J.J. O'CONNOR & E.F. ROBERTSON (1999g).

[52]If $A$ is the area of triangle with sides $a, b$ and $c$ and $s = \dfrac{a+b+c}{2}$ then $A^2 = s(s-a)(s-b)(s-c)$.

the work of a later editor.

The *Definitions*, like Diophantos' *Arithmetika*, are dedicated to a certain Dionysios, a very common name in Antiquity. Paul Tannery believed the two Dionysii to be one and the same person[53], but this was before Heron could be dated to the first century. Bearing this in mind, Markus Asper[54] concludes that Heron's Dionysios may be identified as Dionysios Glaukon. According to the Suida, this Dionysios was a student of Chaedemon, the Alexandrian librarian, whom he would succeed[55]. Dionysios would become a companion to all emperors from Nero to Trajan. He became the director of libraries and secretary responsible for correspondence, embassies and rescripts.

Of course, this identification will only stand if one accepts that *Definitions* was, at least in part, written by Heron. This attribution was already called into question by Hultsch and is further contested by Knorr[56]. On the grounds of style and genre, as well as the shared dedication to Dionysios, Knorr concludes that *Definitions* is closely associated with or may even have been written by ... Diophantos!
Indeed, most Heronian writings have prefaces that basically follow a consistent format. The preface to *Definitions* diverges from this pattern. However, like *Arithmetika*, it deals with the order of exposition and the pedagogical strategy[57].

Consider, for example, the closing thoughts of both texts (Knorr's translation):

> Definitions: in this wise (houtôs) the subject matter will be well surveyable (eusynoptoi) for you.
> Arithmetika: in this wise (houtôs) the elements will be well traversable (euodenta) for beginners.

If we accept Knorr's attribution and add to this Lucio Russo's thesis[58], the whole history of Euclid's *Elements* may be shattered. According to Russo, Euclid did not include the first seven definitions in his treatise, leaving fundamental entities undefined. In the Imperial Age, Euclid's choice could not be understood and the absence of definitions seemed to be a lacuna. As a remedy, Russo suggests, Heron first wrote his schoolbook *Definitions* and subsequently a list of Heron's work was compiled and inserted into Euclid's text. However, this would mean, at least in Knorr's view, that the Euclidean definitions are in fact ... Diophantine!

[53]P. TANNERY et al.(1912-1940)II, pp.535-538.
[54]M. ASPER(2001).
[55]Suida delta 1173, translated by Malcolm Heath.http://www.stoa.org/sol/
[56]W. KNORR(1993), esp. pp. 184-188.
[57]See also J. MANSFELD(1998), pp.55-57.
[58]L. RUSSO(2004), pp.320-327, esp. p.324.

It is in the Heronian corpus that we find some interesting indeterminate problems. These were not written by Heron, but added subsequently to *Metrika*. Heiberg included this collection of problems in his edition of *Geometrika*, creating the impression that they are genuine Heronian problems[59].

*Find two rectangular areas such that the area and the perimeter are three times as large.*

The problem is equivalent to $\left\{ \begin{array}{rcl} u+v & = & n(x+y) \\ xy & = & n.uv \end{array} \right.$

| | |
|---|---|
| I do it like this, the cube of 3 is 27 | $n^3$ |
| which taken twice is 54 | $2n^3$ |
| If I deduct unity I find 53 | $2n^3-1$ |
| The first side thus is 53 feet | $x = 2n^3-1$ |
| the other 54 feet | $y = 2n^3$ |
| For the other rectangle 53 plus 54 is 107 | |
| which multiplied by 3 [= 321, | $n(x+y)$ |
| 321 - 3] which is 318 feet | $n(x+y)-n = u$ |
| One of the sides therefore is 318 feet | $u = 2n(2n^3-1)$ |
| the other 3 feet | $v = n$ |
| The area of the first is 954 feet | |
| of the other 2862 feet | |

A possible explanation (as attributed to H. Zeuthen by T.L. Heath[60]) for this procedure is as follows: the problem is indeterminate, so start with a hypothesis, e.g. $v = n$.
Then $n(x+y) = n+u$, so $u$ is a multiple of $n$, say $nz$ and $n(x+y) = n+nz$ or $x+y = 1+z$.
The second equation of the system yields:

$$\begin{array}{rcl} xy & = & n^3 z \\ xy & = & n^3(x+y) - n^3 \\ xy - n^3x - n^3y & = & -n^3 \\ xy - n^3x - n^3y + n^6 & = & n^6 - n^3 \\ (x-n^3)(y-n^3) & = & n^3(n^3-1) \end{array}$$

with an obvious solution $x - n^3 = n^3 - 1$ and $y - n^3 = n^3$,
which yields the solution:

$$\left\{ \begin{array}{rcl} x & = & 2n^3-1 \\ y & = & 2n^3 \end{array} \right. \text{ and } \left\{ \begin{array}{rcl} u & = & 2n(2n^3-1) \\ v & = & n \end{array} \right.$$

[59] Unfortunately, *Geometrika* was published in volume IV of Heron's works, while Heiberg explains his editorial method in volume V, thus adding to the confusion.
[60] T.L. HEATH (1981), pp.444-445.

Although algebraically correct, one may wonder whether this method was ever used by pre-Heronian mathematicians. It presupposes a large theoretical algebraic knowledge. Therefore, we suggest that it was found by means of another, most probably empirical, method.