

Contents

1	Introduction	1
2	Hopf Bifurcation and Normal Form Computation	7
2.1	Hopf Bifurcation	7
2.2	Computation of Normal Forms	10
2.2.1	Semisimple Case	16
2.2.2	A Double-Zero Eigenvalue	20
2.2.3	1:1 Resonant Hopf Bifurcation	25
2.3	A Perturbation Method Based on Multiple Timescales	38
2.3.1	Basic Idea of the MTS	38
2.3.2	Hopf and Generalized Hopf Bifurcations	42
2.4	Efficient Computation	51
2.4.1	Theoretical Analysis	52
2.4.2	Symbolic Computation	56
3	Comparison of Methods for Computing Focus Values	59
3.1	The Poincaré Method	60
3.2	A Perturbation Technique	63
3.3	The Singular Point Value Method	64
3.4	Illustrative Examples	67
3.4.1	The Brusselator Model	67
3.4.2	A Cubic-Order System with Z_2 Symmetry	69
3.4.3	A Symmetric Liénard Equation	72
3.4.4	A Numerical Example with 10 Small Limit Cycles	74
3.5	Remarks on the Comparison of Different Methods	76
4	Application (I)—Hilbert’s 16th Problem	81
4.1	Z_q -Equivariant Planar Vector Fields	84
4.2	Third-Order Vector Fields with Z_q Symmetry	86
4.2.1	$q = 1$	87
4.2.2	$q = 2$	97
4.2.3	$q = 3$	124

4.2.4	$q = 4$	126
4.2.5	$q \geq 5$	128
4.3	Fourth-Order Vector Fields with Z_q Symmetry	129
4.3.1	$q = 5$	129
4.3.2	$q \geq 7$ (Odd)	133
4.4	Fifth-Order Vector Fields with Z_q Symmetry	133
4.4.1	$q = 5$	134
4.4.2	$q = 6$	140
4.4.3	$q \geq 7$	149
4.5	The Liénard System	150
4.5.1	Generalized Liénard Systems	152
4.5.2	Liénard Equation with Z_2 Symmetry	166
4.6	Critical Periods	183
4.6.1	Computation of Critical Periods	184
4.6.2	Cubic-Order Planar Reversible Systems	186
4.6.3	Cubic-Order Planar Hamiltonian Systems	201
5	Application (II)—Practical Problems	213
5.1	An Electrical Circuit	213
5.2	A Double Pendulum System	216
5.2.1	Hopf Bifurcation	220
5.2.2	A Double-Zero Singularity	223
5.2.3	Hopf-Zero and Double-Hopf Singularities	227
5.3	Induction Machine Model	229
5.4	An HIV-1 Model	238
5.4.1	Equilibrium and Stability Analysis	239
5.4.2	Hopf Bifurcation	242
5.4.3	Numerical Illustrations	245
5.5	Hopf Bifurcation Control	251
5.5.1	Feedback Control Using a Polynomial Function	253
5.5.2	The Lorenz System	254
6	Fundamental Theory of the Melnikov Function Method	261
6.1	Definition and Lemmas	263
6.2	Main Theorem and Corollaries	266
6.3	An Illustrative Example	269
7	Limit Cycle Bifurcations Near a Center	271
7.1	Normalized Hamiltonian Function	271
7.2	Computation of the Melnikov Function, M	273
7.3	Determining the Number of Limit Cycles Near a Center	284
7.4	Computation of the b_j Coefficients of M	292
7.5	A Generalization of Theorem 7.5	297
7.6	Maple Programs	298
7.7	Application	305
7.7.1	Case $a \neq 0$	306
7.7.2	Case $a = 0$	312

8	Limit Cycles Near a Homoclinic or Heteroclinic Loop	315
8.1	Bifurcation of Limit Cycles Near a Homoclinic Loop	315
8.1.1	Single Homoclinic Loop	315
8.1.2	Computation of the Melnikov Function, M	316
8.1.3	Computation of the c_j Coefficients of M	319
8.1.4	Alternative Formulas for the c_j Coefficients for Different Hamiltonian Functions	322
8.1.5	Double Homoclinic Loops	324
8.1.6	Symmetric Double Homoclinic Loops	325
8.1.7	Nonsymmetric Double Homoclinic Loops	326
8.2	Bifurcation of Limit Cycles Near a Heteroclinic Loop	329
8.2.1	Computation of the Melnikov Function, M	330
9	Finding More Limit Cycles Using Melnikov Functions	335
9.1	Basic Idea and Formulation	335
9.2	A Generalized Theorem	337
9.3	Bifurcation of Limit Cycles in Some Polynomial Systems	339
9.3.1	System 1: 7 Limit Cycles	339
9.3.2	System 2: 5 Limit Cycles	342
10	Limit Cycle Bifurcations in Equivariant Systems	353
10.1	Symmetry of a Vector Field	353
10.2	S -Symmetrical Quadratic Systems	354
10.3	Z_q -Equivariant Vector Fields	355
10.4	Cubic Z_q -Equivariant Systems	358
10.5	Reversing Symmetry and Z_q -Reversible-Equivariant Vector Fields	359
10.6	Cubic Z_3 -Equivariant Hamiltonian Systems	361
	References	383
	Index	393