## Contents

**Preface**

1. Introduction
   1.1 The Spectrum of Applications .......................... 1
   1.2 Brief Historical Perspective ........................... 4
   1.3 Local Averages and Their Extremes ..................... 9

2. Fundamentals of Analysis of Random Fields .................. 13
   2.1 Types of Random Fields ................................ 13
   2.2 Basic Probabilistic Description ........................ 18
   2.3 Expectation and Conditional Expectation ................. 31
   2.4 Characteristic Functions ................................ 39
   2.5 Gaussian and Related Probability Distributions .......... 42
   2.6 Optimal Linear Prediction and Updating ................. 53
   2.7 Purely Random Fields and Markov Processes .............. 59
   2.8 New Quantum-Physics-Based Probability Distributions .... 74

   3.1 Preview, Definitions, and Notation ..................... 85
   3.2 Correlation Function of a Homogeneous Random Field .... 87
   3.3 Spectral Representation of Random Processes
       on the Line ........................................... 92
   3.4 Spectral Analysis of Homogeneous Random Fields ....... 101
   3.5 Input-Output Relations for Invariant Linear Systems ...... 114
   3.6 Derivatives and Local Integrals of Random Fields ....... 119
   3.7 Moving Average and Autoregressive Models ............... 129
3.8 Space-Time Correlation Structure: Basic Relations

4. Spectral Parameters, Level Crossings, and Extremes
   4.1 Spectral Moments and Related Parameters
   4.2 Statistics of Partial Derivatives
   4.3 Basic Envelope Statistics
   4.4 Threshold-Crossing Statistics and Extremes
   4.5 Expected Size of Regions of Excursion
   4.6 Statistics of Level Excursions and Extremes
   4.7 Spectral Parameters of Common Correlation Models
   4.8 Some Extensions to Nonhomogeneous Random Fields

5. Local Average Processes on the Line
   5.1 Variance Function and Scale of Fluctuation
   5.2 Scale of Fluctuation: Frequency-Domain Interpretation
   5.3 Covariance of Local Integrals or Local Averages
   5.4 Mean Square Derivative and Spectral Moments
   5.5 Level-Crossing and Extreme-Value Statistics
   5.6 Invariant and Regenerative Properties
   5.7 Parallel Results for Random Series and Point Processes
   5.8 Role of the Scale of Fluctuation in Optimal Sampling
   5.9 Composite Random Processes and the Scale Spectrum

6. Two-Dimensional Local Average Processes
   6.1 Variance Function and Measure of Correlation
   6.2 Important Special Cases
   6.3 Conditional Variance Functions and Scales of Fluctuation
   6.4 Covariance of Local Averages
   6.5 Statistics of Level Excursions and Extremes
   6.6 Invariants for 2-D Homogeneous Random Fields
   6.7 Space-Time Processes: Frequency-Dependent Scale of Fluctuation
   6.8 Space-Time Processes: Frequency-Dependent Variance Function

7. Multi-Dimensional Local Average Processes
   7.1 Variance Function and Correlation Measures
   7.2 Conditional Variance Functions and Correlation Measures
Contents

7.3 Frequency-Dependent Spatial Random Variation . . . . . 298
7.4 Some Tractable Space-Time Correlation Models . . . . . 305
7.5 Covariance of Local Averages . . . . . . . . . . . . . . . 309
7.6 Stochastic Finite Element Analysis . . . . . . . . . . . . 311
7.7 Partial Derivatives of Local-Average Fields . . . . . . . 315
7.8 Statistics of High-Level Excursions and Extremes . . . . 318

8. Overview of Findings 323

Bibliography 337

Index 347