

# Contents

|                                                         |           |
|---------------------------------------------------------|-----------|
| <b>List of Figures</b>                                  | <b>xi</b> |
| <b>1 Introduction</b>                                   | <b>1</b>  |
| 1.1 Examples . . . . .                                  | 1         |
| 1.2 How to control a system? . . . . .                  | 6         |
| 1.3 Exercises . . . . .                                 | 10        |
| 1.4 Notes and references . . . . .                      | 12        |
| <b>2 State Space Representation</b>                     | <b>13</b> |
| 2.1 State space models . . . . .                        | 13        |
| 2.2 Solutions of the state space models . . . . .       | 19        |
| 2.3 Port-Hamiltonian systems . . . . .                  | 21        |
| 2.4 Exercises . . . . .                                 | 24        |
| 2.5 Notes and references . . . . .                      | 25        |
| <b>3 Controllability of Finite-Dimensional Systems</b>  | <b>27</b> |
| 3.1 Controllability . . . . .                           | 27        |
| 3.2 Normal forms . . . . .                              | 33        |
| 3.3 Exercises . . . . .                                 | 36        |
| 3.4 Notes and references . . . . .                      | 38        |
| <b>4 Stabilizability of Finite-Dimensional Systems</b>  | <b>39</b> |
| 4.1 Stability and stabilizability . . . . .             | 39        |
| 4.2 The pole placement problem . . . . .                | 40        |
| 4.3 Characterization of stabilizability . . . . .       | 44        |
| 4.4 Stabilization of port-Hamiltonian systems . . . . . | 47        |
| 4.5 Exercises . . . . .                                 | 48        |
| 4.6 Notes and references . . . . .                      | 49        |
| <b>5 Strongly Continuous Semigroups</b>                 | <b>51</b> |
| 5.1 Strongly continuous semigroups . . . . .            | 51        |
| 5.2 Infinitesimal generators . . . . .                  | 57        |

|           |                                                                        |            |
|-----------|------------------------------------------------------------------------|------------|
| 5.3       | Abstract differential equations . . . . .                              | 61         |
| 5.4       | Exercises . . . . .                                                    | 62         |
| 5.5       | Notes and references . . . . .                                         | 63         |
| <b>6</b>  | <b>Contraction and Unitary Semigroups</b>                              | <b>65</b>  |
| 6.1       | Contraction semigroups . . . . .                                       | 65         |
| 6.2       | Groups and unitary groups . . . . .                                    | 73         |
| 6.3       | Exercises . . . . .                                                    | 75         |
| 6.4       | Notes and references . . . . .                                         | 77         |
| <b>7</b>  | <b>Homogeneous Port-Hamiltonian Systems</b>                            | <b>79</b>  |
| 7.1       | Port-Hamiltonian systems . . . . .                                     | 79         |
| 7.2       | Generation of contraction semigroups . . . . .                         | 84         |
| 7.3       | Technical lemmas . . . . .                                             | 92         |
| 7.4       | Exercises . . . . .                                                    | 93         |
| 7.5       | Notes and references . . . . .                                         | 96         |
| <b>8</b>  | <b>Stability</b>                                                       | <b>97</b>  |
| 8.1       | Exponential stability . . . . .                                        | 97         |
| 8.2       | Spectral projection and invariant subspaces . . . . .                  | 101        |
| 8.3       | Exercises . . . . .                                                    | 108        |
| 8.4       | Notes and references . . . . .                                         | 109        |
| <b>9</b>  | <b>Stability of Port-Hamiltonian Systems</b>                           | <b>111</b> |
| 9.1       | Exponential stability of port-Hamiltonian systems . . . . .            | 111        |
| 9.2       | An example . . . . .                                                   | 118        |
| 9.3       | Exercises . . . . .                                                    | 120        |
| 9.4       | Notes and references . . . . .                                         | 122        |
| <b>10</b> | <b>Inhomogeneous Abstract Differential Equations and Stabilization</b> | <b>123</b> |
| 10.1      | The abstract inhomogeneous Cauchy problem . . . . .                    | 123        |
| 10.2      | Outputs . . . . .                                                      | 130        |
| 10.3      | Bounded perturbations of $C_0$ -semigroups . . . . .                   | 132        |
| 10.4      | Exponential stabilizability . . . . .                                  | 133        |
| 10.5      | Exercises . . . . .                                                    | 139        |
| 10.6      | Notes and references . . . . .                                         | 140        |
| <b>11</b> | <b>Boundary Control Systems</b>                                        | <b>143</b> |
| 11.1      | Boundary control systems . . . . .                                     | 143        |
| 11.2      | Outputs for boundary control systems . . . . .                         | 147        |
| 11.3      | Port-Hamiltonian systems as boundary control systems . . . . .         | 148        |
| 11.4      | Exercises . . . . .                                                    | 154        |
| 11.5      | Notes and references . . . . .                                         | 155        |

|                                                                |            |
|----------------------------------------------------------------|------------|
| <b>12 Transfer Functions</b>                                   | <b>157</b> |
| 12.1 Basic definition and properties . . . . .                 | 158        |
| 12.2 Transfer functions for port-Hamiltonian systems . . . . . | 163        |
| 12.3 Exercises . . . . .                                       | 167        |
| 12.4 Notes and references . . . . .                            | 169        |
| <b>13 Well-posedness</b>                                       | <b>171</b> |
| 13.1 Well-posedness for boundary control systems . . . . .     | 171        |
| 13.2 Well-posedness for port-Hamiltonian systems . . . . .     | 181        |
| 13.3 $P_1\mathcal{H}$ diagonal . . . . .                       | 186        |
| 13.4 Proof of Theorem 13.2.2 . . . . .                         | 189        |
| 13.5 Well-posedness of the vibrating string . . . . .          | 191        |
| 13.6 Exercises . . . . .                                       | 193        |
| 13.7 Notes and references . . . . .                            | 195        |
| <b>A Integration and Hardy Spaces</b>                          | <b>197</b> |
| A.1 Integration theory . . . . .                               | 197        |
| A.2 The Hardy spaces . . . . .                                 | 202        |
| <b>Bibliography</b>                                            | <b>209</b> |
| <b>Index</b>                                                   | <b>215</b> |