

Contents

1	Introduction	1
1.1	Background and Motivation	1
1.2	Biological Snakes	5
1.2.1	The Anatomy of Snakes	5
1.2.2	The Locomotion of Snakes	7
1.3	Previous Work on Modelling, Mechatronics, and Control of Snake Robots	10
1.3.1	Previous Work on Modelling and Analysis of Snake Robots	10
1.3.2	Previous Work on Implementation of Physical Snake Robots	16
1.3.3	Previous Work on Control of Snake Robots	22
1.4	The Scope of This Book	27
1.4.1	An Analytical Approach	27
1.4.2	Snake Robots Without a Fixed Base	27
1.4.3	A Planar Perspective	27
1.4.4	Locomotion Without Sideslip Constraints	28
1.4.5	Motion Based on Lateral Undulation	28
1.5	An Outline of This Book	28
1.5.1	Outline of Part I—Snake Robot Locomotion on Flat Surfaces	29
1.5.2	Outline of Part II—Snake Robot Locomotion in Cluttered Environments	32
1.6	Publications Underlying This Book	34

Part I Snake Robot Locomotion on Flat Surfaces

2	A Complex Model of Snake Robot Locomotion on Planar Surfaces	39
2.1	The Relation Between This Chapter and Previous Literature	40
2.2	Basic Notation	40
2.3	The Parameters of the Snake Robot	40
2.4	The Kinematics of the Snake Robot	42

2.5	The Ground Friction Models	45
2.5.1	The Friction Models and Their Role in This Book	45
2.5.2	A Coulomb Friction Model	46
2.5.3	A Viscous Friction Model	47
2.6	The Dynamics of the Snake Robot	48
2.7	Separating Actuated and Unactuated Dynamics	50
2.8	Partial Feedback Linearisation of the Model	52
2.9	Chapter Summary	54
3	Development of a Mechanical Snake Robot for Motion Across Planar Surfaces	55
3.1	The Relation Between This Chapter and Previous Literature	55
3.2	The Joint Actuation Mechanism	56
3.3	The Passive Wheels	58
3.4	The Power and Control System	59
3.5	The Experimental Setup of the Snake Robot	59
3.6	Chapter Summary	61
4	Analysis and Synthesis of Snake Robot Locomotion	63
4.1	The Relation Between This Chapter and Previous Literature	64
4.2	Introduction to Nonlinear Controllability Analysis	65
4.3	Stabilisability Properties of Planar Snake Robots	67
4.4	Controllability Analysis of Planar Snake Robots	68
4.4.1	Controllability with Isotropic Viscous Friction	69
4.4.2	Controllability with Anisotropic Viscous Friction	69
4.5	Analysis of Propulsive Forces During Snake Locomotion	74
4.6	Synthesis of Propulsive Motion for the Snake Robot	76
4.7	The Gait Pattern Lateral Undulation	80
4.8	The Control System of the Joints	81
4.8.1	A Simple Joint Controller	81
4.8.2	An Exponentially Stable Joint Controller	82
4.9	Analysis of Turning Motion During Lateral Undulation	82
4.10	Analysis of Relative Motion Between Consecutive Links During Lateral Undulation	85
4.11	Chapter Summary	86
5	Path Following Control and Analysis of Snake Robots Based on the Poincaré Map	89
5.1	The Relation Between This Chapter and Previous Literature	90
5.2	Introduction to Poincaré Maps	91
5.2.1	General Description of Poincaré Maps	91
5.2.2	Practical Application of Poincaré Maps	93
5.3	Straight Line Path Following Control of Snake Robots	94
5.3.1	Control Objective	94
5.3.2	The Straight Line Path Following Controller	95
5.4	Stability Analysis of the Path Following Controller Based on the Poincaré Map	96

5.4.1	Converting the Snake Robot Model to a Time-Periodic Autonomous System	97
5.4.2	Specification of the Poincaré Section for the Snake Robot	97
5.4.3	Stability Analysis of the Poincaré Map	98
5.5	Simulation Study: The Performance of the Path Following Controller	100
5.6	Chapter Summary	101
6	A Simplified Model of Snake Robot Locomotion on Planar Surfaces	103
6.1	The Relation Between This Chapter and Previous Literature	104
6.2	Overview of the Modelling Approach	104
6.3	The Kinematics of the Snake Robot	107
6.4	The Ground Friction Model	109
6.5	The Dynamics of the Snake Robot	112
6.5.1	The Translational Dynamics of the Snake Robot	112
6.5.2	The Rotational Dynamics of the Snake Robot	114
6.6	The Complete Simplified Model of the Snake Robot	115
6.7	Discussion of the Simplified Model	116
6.7.1	Applications of the Simplified Model	116
6.7.2	Accuracy Issues of the Simplified Kinematics	116
6.7.3	Accuracy Issues of the Ground Friction Model	117
6.7.4	Accuracy Issues of the Rotational Dynamics	118
6.8	Stabilisability Analysis of the Simplified Model	118
6.9	Controllability Analysis of the Simplified Model	119
6.10	Simulation Study: Comparison Between the Complex and the Simplified Model	122
6.10.1	Simulation Parameters	122
6.10.2	Relationship Between the Joint Coordinates in the Complex and Simplified Models	123
6.10.3	Comparison of Straight Motion	123
6.10.4	Comparison of Turning Motion	126
6.11	Chapter Summary	128
7	Analysis of Snake Robot Locomotion Based on Averaging Theory	131
7.1	The Relation Between This Chapter and Previous Literature	132
7.2	Introduction to Averaging Theory	132
7.3	The Velocity Dynamics During Lateral Undulation	133
7.4	The Averaged Velocity Dynamics During Lateral Undulation	135
7.5	The Steady-State Behaviour of the Velocity Dynamics During Lateral Undulation	136
7.6	Relationships Between the Gait Parameters and the Forward Velocity During Lateral Undulation	138
7.7	Simulation Study: Comparison Between the Original and the Averaged Velocity Dynamics	139
7.7.1	Simulation Parameters	139
7.7.2	Simulation Results	140

7.8	Simulation Study: Investigation of the Relationships Between Gait Parameters and Forward Velocity	142
7.8.1	Simulation Parameters	142
7.8.2	Simulation Results	143
7.9	Experimental Study: Investigation of the Relationships Between Gait Parameters and Forward Velocity	146
7.9.1	Layout of the Experiment	147
7.9.2	Experimental Results	149
7.10	Chapter Summary	151
8	Path Following Control of Snake Robots Through a Cascaded Approach	153
8.1	The Relation Between This Chapter and Previous Literature	154
8.2	Mathematical Preliminaries	154
8.3	Straight Line Path Following Control of Snake Robots	157
8.3.1	Control Objective	157
8.3.2	Assumptions	158
8.3.3	Model Transformation	158
8.3.4	The Straight Line Path Following Controller	160
8.3.5	The Stability Properties of the Path Following Controller	163
8.3.6	Proof of Theorem 8.2	164
8.4	Path Following Control of Snake Robots Along Curved Paths	167
8.4.1	Comments on the Curved Path Following Controller	167
8.4.2	The Curved Path Following Controller	168
8.5	Waypoint Guidance Control of Snake Robots	169
8.5.1	Description of the Approach	169
8.5.2	The Waypoint Guidance Strategy	170
8.6	Simulation Study: The Performance of the Straight Line Path Following Controller	171
8.6.1	Simulation Parameters	171
8.6.2	Simulation Results	172
8.7	Experimental Study: The Performance of the Straight Line Path Following Controller	172
8.7.1	Implementation Issues	173
8.7.2	Implementation of the Path Following Controller of the Physical Snake Robot	174
8.7.3	Experimental Results	175
8.8	Simulation Study: The Performance of the Waypoint Guidance Strategy	181
8.8.1	Implementation of the Guidance Strategy with the Simplified Model	182
8.8.2	Implementation of the Guidance Strategy with the Complex Model	182
8.8.3	Simulation Results	183
8.9	Chapter Summary	184

Part II Snake Robot Locomotion in Cluttered Environments

9	Introduction to Part II	189
10	A Hybrid Model of Snake Robot Locomotion in Cluttered Environments	193
10.1	The Relation Between This Chapter and Previous Literature	194
10.2	Hybrid Dynamical Systems and Complementarity Systems	195
10.2.1	Modelling of Hybrid Dynamical Systems	195
10.2.2	Complementarity Systems	196
10.3	The Dynamics of the Snake Robot Without Obstacles	197
10.3.1	The Ground Friction Model	198
10.3.2	The Equations of Motion Without Obstacles	199
10.4	Overview of the Contact Modelling Approach	200
10.5	Detection of Obstacle Impacts and Detachments	203
10.6	The Continuous Dynamics of the Snake Robot During Constrained Motion	204
10.6.1	The Unilateral Constraints from the Obstacles	205
10.6.2	The Constrained Dynamics of the Snake Robot Without Obstacle Friction	206
10.6.3	The Constrained Dynamics of the Snake Robot with Obstacle Friction	209
10.7	The Discontinuous Dynamics of the Snake Robot During Obstacle Impacts and Detachments	211
10.7.1	The Discontinuous Dynamics of the Snake Robot During Obstacle Impacts	211
10.7.2	The Discontinuous Dynamics of the Snake Robot During Obstacle Detachments	212
10.8	The Complete Hybrid Model of the Snake Robot in an Obstacle Environment	213
10.8.1	The Jump Set	213
10.8.2	The Jump Map	214
10.8.3	The Flow Set	214
10.8.4	The Flow Map	215
10.8.5	Summary of the Complete Hybrid Plant	215
10.9	Simulation Study: Comparison of the Hybrid Model with Previous Experimental and Simulation Results	215
10.10	Chapter Summary	216
11	Development of a Mechanical Snake Robot for Obstacle-Aided Locomotion	221
11.1	The Relation Between This Chapter and Previous Literature	221
11.2	Overview of the Snake Robot Design	222
11.3	The Exterior Gliding Surface	223
11.4	The Contact Force Measurement System	224
11.4.1	Assumptions Underlying the Sensor System	224

11.4.2	The Sensor System Setup	225
11.4.3	Calculation of Contact Forces	227
11.5	The Power and Control System	228
11.5.1	The Power System	228
11.5.2	The Control System	230
11.6	The Performance of the Snake Robot	231
11.6.1	Experimental Validation of the Contact Force Measurement System	231
11.6.2	Demonstration of Motion Patterns	232
11.7	The Experimental Setup of the Snake Robot	233
11.8	An Alternative Approach for Measuring External Contact Forces	235
11.9	Chapter Summary	237
12	Hybrid Control of Obstacle-Aided Locomotion	239
12.1	The Relation Between This Chapter and Previous Literature	240
12.2	Preliminary Note on Hybrid Controllers	241
12.3	Control Objective	242
12.4	Notation and Basic Assumptions	242
12.5	The Hybrid Controller for Obstacle-Aided Locomotion	244
12.5.1	The Leader-Follower Scheme	244
12.5.2	The Jam Detection Scheme	246
12.5.3	The Jam Resolution Scheme	246
12.5.4	The Joint Angle Controller	248
12.5.5	The Complete Hybrid Controller	248
12.6	Summary of the Closed-Loop System	251
12.7	Simulation Study: The Performance of the Hybrid Controller	252
12.7.1	Simulation Parameters	252
12.7.2	Attempting Lateral Undulation in Open-Loop in a Structured Obstacle Environment	253
12.7.3	Hybrid Controller in an Obstacle Environment	253
12.8	Experimental Study: The Performance of the Hybrid Controller	255
12.8.1	Experimental Setup	256
12.8.2	Experimental Results	256
12.9	Chapter Summary	263
13	Path Following Control of Snake Robots in Cluttered Environments	265
13.1	The Relation Between This Chapter and Previous Literature	266
13.2	A Controller Framework for Snake Robot Locomotion	266
13.3	Straight Line Path Following Control in Cluttered Environments	268
13.3.1	Control Objective	268
13.3.2	Notation and Basic Assumptions	269
13.3.3	The Body Wave Component	271
13.3.4	The Environment Adaptation Component	272
13.3.5	The Heading Control Component	273
13.3.6	The Joint Angle Controller	274
13.3.7	Summary of the Path Following Controller	274

- 13.4 Waypoint Guidance Control in Cluttered Environments 275
- 13.5 Simulation Study: The Performance of the Path Following
Controller 276
 - 13.5.1 Simulation Parameters 276
 - 13.5.2 Simulation Results 277
- 13.6 Experimental Study: The Performance of the Environment
Adaptation Strategy 279
 - 13.6.1 Experimental Setup 280
 - 13.6.2 Experimental Results 280
- 13.7 Chapter Summary 285
- 14 Future Research Challenges of Snake Robot Locomotion 287**
 - 14.1 Control Design Challenges 287
 - 14.2 Hardware Design Challenges 289
- Appendix A Proof of Lemma 8.2 293**
- Appendix B Proof of Lemma 8.3 295**
- Appendix C Low-Pass Filtering Reference Models 297**
 - C.1 A 2nd-Order Low-Pass Filtering Reference Model 297
 - C.2 A 3rd-Order Low-Pass Filtering Reference Model 298
- Glossary 301**
- References 303**
- Index 313**