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HYDRODYNAMICS AND SOUND

This book is designed for a first graduate course in fluid dynamics. It focuses on knowledge and methods that find application in most branches of fluid mechanics and aims to supply a theoretical understanding that will permit sensible simplifications to be made in the formulation of problems and enable the reader to develop analytical models of practical significance. The study of simplified model problems can be used to guide experimental and numerical investigations. The first part (Chapters 1–4) is concerned entirely with the incompressible flow of a homogeneous fluid. Chapters 5 and 6 deal with dispersive waves and acoustics.

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M. S. HOWE

Boston University



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In memoriam James Lighthill

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Preface

Fluid mechanics impinges on practically all areas of human endeavour. But it is not easy to grasp its principles and ramifications in all of its diverse manifestations. Industrial applications usually require the numerical solution of the equations of motion of a fluid on a very large scale, perhaps coupled in a complicated manner to equations describing the response of solid structures in contact with the fluid. There has developed a tendency to regard the subject as defined solely by its governing equations whose treatment by numerical methods can furnish the solution of any problem.

There are actually many practical problems that are not yet amenable to full numerical evaluation in a reasonable time, even on the fastest of present-day computers. It is therefore important to have a proper theoretical understanding that will permit sensible simplifications to be made when formulating a problem. As in most technical subjects such understanding is acquired by detailed study of highly simplified ‘model problems’. Many of these problems fall within the realm of classical fluid mechanics, which is often criticised for its emphasis on ideal fluids and potential flow theory. The criticism is misplaced, however: For example, potential flow methods provide a good first approximation to airfoil theory, and ‘free-streamline’ theory (pioneered in its modern form by Chaplygin) permits the two-dimensional modelling of complex flows involving separation and jet formation.

There is a certain body of knowledge and methods that finds application in most branches of fluid mechanics. This book aims to supply this basic material and to present the most important theoretical methods that will enable the reader to develop analytical models of practical significance. Such analyses can be used to guide more detailed experimental and numerical investigations. The first part (Chapters 1–4) is concerned entirely with the incompressible flow of a homogeneous fluid. It was written for the Boston University introductory graduate-level course ‘Advanced Fluid Mechanics’. The remaining chapters, 5 and 6, deal with dispersive waves and acoustics and are unashamedly inspired by James Lighthill’s masterpiece *Waves in Fluids*.

M. S. Howe