

## Solution Sets for Differential Equations and Inclusions

Bearbeitet von

Smaïl Djebali, Lech Górniewicz, Abdelghani Ouahab

1. Auflage 2012. Buch. XIX, 472 S. Hardcover

ISBN 978 3 11 029344 9

Format (B x L): 17 x 24 cm

Gewicht: 926 g

[Weitere Fachgebiete > Mathematik > Mathematische Analysis >](#)  
[Differentialrechnungen und -gleichungen](#)

schnell und portofrei erhältlich bei



Die Online-Fachbuchhandlung beck-shop.de ist spezialisiert auf Fachbücher, insbesondere Recht, Steuern und Wirtschaft. Im Sortiment finden Sie alle Medien (Bücher, Zeitschriften, CDs, eBooks, etc.) aller Verlage. Ergänzt wird das Programm durch Services wie Neuerscheinungsdienst oder Zusammenstellungen von Büchern zu Sonderpreisen. Der Shop führt mehr als 8 Millionen Produkte.

# Contents

|   |     |
|---|-----|
| Preface   | vii |
| Notations   | xi  |
| <b>1 Topological structure of fixed point sets</b>                | 1   |
| 1.1 Case of single-valued mappings .....                          | 1   |
| 1.1.1 Fundamental fixed point theorems .....                      | 1   |
| 1.1.1.1 Banach's fixed point theorem .....                        | 1   |
| 1.1.1.2 Brouwer's fixed point theorem .....                       | 5   |
| 1.1.1.3 Schauder's fixed point theorem .....                      | 7   |
| 1.1.2 Approximation theorems .....                                | 11  |
| 1.1.3 Browder–Gupta theorems .....                                | 13  |
| 1.1.4 Acyclicity of the solution sets of operator equations ..... | 20  |
| 1.1.5 Nonexpansive maps .....                                     | 23  |
| 1.1.5.1 Existence theory .....                                    | 23  |
| 1.1.5.2 Solution sets .....                                       | 26  |
| 1.2 The case of multi-valued mappings .....                       | 27  |
| 1.2.1 Approximation of multi-valued maps .....                    | 27  |
| 1.2.2 Fixed point theorems .....                                  | 30  |
| 1.2.3 Multi-valued contractions .....                             | 33  |
| 1.2.4 Fixed point sets of multi-valued contractions .....         | 35  |
| 1.2.5 Fixed point sets of multi-valued nonexpansive maps .....    | 38  |
| 1.2.6 Fixed point sets of multi-valued condensing maps .....      | 39  |
| 1.2.6.1 Measure of noncompactness .....                           | 39  |
| 1.2.6.2 Condensing maps .....                                     | 43  |
| 1.3 Admissible maps .....   | 44  |
| 1.3.1 Generalities .....  | 44  |
| 1.3.2 Fixed point theorems for admissible multi-valued maps ..... | 53  |
| 1.3.3 The general Brouwer fixed point theorem .....               | 58  |
| 1.3.4 Browder–Gupta type results for admissible mappings .....    | 60  |
| 1.3.5 Topological dimensions of solution sets .....               | 62  |

|          |   |     |
|----------|---|-----|
| 1.4      | Topological structure of fixed point sets of inverse limit maps . . . . . | 65  |
| 1.4.1    | Definition . . . . .  | 65  |
| 1.4.2    | Basic properties . . . . .  | 66  |
| 1.4.3    | Multi-maps of inverse systems . . . . .                                   | 67  |
| <b>2</b> | <b>Existence theory for differential equations and inclusions</b>         | 72  |
| 2.1      | Fundamental theorems . . . . .  | 72  |
| 2.1.1    | Existence and uniqueness results . . . . .                                | 72  |
| 2.1.2    | Picard–Lindelöf theorem . . . . .   | 73  |
| 2.1.2.1  | Maximal solutions . . . . .   | 75  |
| 2.1.3    | Peano and Carathéodory theorems . . . . .                                 | 77  |
| 2.1.3.1  | Peano theorem . . . . .   | 77  |
| 2.2      | The extendability problem . . . . .                                       | 79  |
| 2.2.1    | Global existence theorems . . . . .                                       | 79  |
| 2.2.2    | Existence results on noncompact intervals . . . . .                       | 82  |
| 2.2.2.1  | The Lipschitz case . . . . .  | 82  |
| 2.2.2.2  | The Lipschitz–Nagumo case . . . . .                                       | 83  |
| 2.2.2.3  | The Nagumo case . . . . .   | 86  |
| 2.2.3    | A boundary value problem on the half-line . . . . .                       | 88  |
| 2.3      | The case of differential inclusions . . . . .                             | 94  |
| 2.3.1    | Initial value problems . . . . .  | 94  |
| 2.3.1.1  | A Nagumo type nonlinearity . . . . .                                      | 94  |
| 2.3.1.2  | A Lipschitz nonconvex nonlinearity . . . . .                              | 97  |
| 2.3.2    | Boundary value problems . . . . .   | 99  |
| 2.3.2.1  | The convex case . . . . .   | 100 |
| 2.3.2.2  | The nonconvex case . . . . .  | 103 |
| <b>3</b> | <b>Solution sets for differential equations and inclusions</b>            | 105 |
| 3.1      | General results . . . . .   | 105 |
| 3.1.1    | Kneser–Hukuhara theorem . . . . .   | 105 |
| 3.1.2    | Problems on bounded intervals . . . . .                                   | 108 |
| 3.1.3    | Problems on unbounded intervals . . . . .                                 | 109 |
| 3.1.4    | Second-order differential equations . . . . .                             | 111 |
| 3.1.5    | Abstract Volterra equations . . . . .                                     | 113 |
| 3.1.6    | Aronszajn type results for differential inclusions . . . . .              | 114 |
| 3.2      | Second-order differential inclusions . . . . .                            | 122 |
| 3.2.1    | The convex case . . . . .   | 122 |
| 3.2.2    | The nonconvex case . . . . .  | 127 |
| 3.2.3    | Solution sets . . . . .   | 130 |

|          |   |            |
|----------|---|------------|
| 3.3      | Higher-order differential inclusions . . . . .                        | 134        |
| 3.4      | Neutral differential inclusions . . . . .                             | 135        |
| 3.4.1    | The convex case . . . . .   | 136        |
| 3.4.2    | The nonconvex case . . . . .  | 142        |
| 3.4.3    | Solutions sets . . . . .  | 146        |
| 3.5      | Nonlocal problems . . . . .   | 146        |
| 3.5.1    | Main results . . . . .  | 147        |
| 3.5.2    | A viability problem . . . . .   | 149        |
| 3.6      | Hyperbolic differential inclusions . . . . .                          | 154        |
| 3.6.1    | Existence results . . . . .   | 155        |
| 3.6.1.1  | The convex case . . . . .   | 155        |
| 3.6.1.2  | The nonconvex case . . . . .  | 159        |
| 3.6.2    | Solution sets . . . . .   | 160        |
| <b>4</b> | <b>Impulsive differential inclusions: existence and solution sets</b> | <b>163</b> |
| 4.1      | Motivation . . . . .  | 163        |
| 4.1.1    | Ecological model with impulsive control strategy . . . . .            | 163        |
| 4.1.2    | Leslie predator-prey system . . . . .                                 | 164        |
| 4.1.3    | Pulse vaccination model . . . . .                                     | 165        |
| 4.2      | Semi-linear impulsive differential inclusions . . . . .               | 166        |
| 4.2.1    | Existence results . . . . .   | 166        |
| 4.2.1.1  | The convex case . . . . .   | 167        |
| 4.2.1.2  | The nonconvex case . . . . .  | 181        |
| 4.2.2    | Structure of solution sets . . . . .                                  | 186        |
| 4.3      | A periodic problem . . . . .  | 197        |
| 4.3.1    | Existence results: $1 \in \rho(T(b))$ . . . . .                       | 198        |
| 4.3.2    | The convex case: a direct approach . . . . .                          | 199        |
| 4.3.3    | The convex case: an MNC approach . . . . .                            | 207        |
| 4.3.4    | The nonconvex case . . . . .  | 212        |
| 4.3.5    | The parameter-dependant case . . . . .                                | 215        |
| 4.3.5.1  | The convex case . . . . .   | 215        |
| 4.3.5.2  | The nonconvex case . . . . .  | 217        |
| 4.3.6    | Filippov's Theorem . . . . .  | 220        |
| 4.3.7    | Existence of solutions: $1 \notin \rho(T(b))$ . . . . .               | 230        |
| 4.3.7.1  | A nonlinear alternative . . . . .                                     | 230        |
| 4.3.7.2  | A Poincaré translation operator . . . . .                             | 233        |
| 4.3.7.3  | The MNC approach . . . . .  | 233        |

|          |  |     |
|----------|--|-----|
| 4.4      | Impulsive functional differential inclusions . . . . .       | 236 |
| 4.4.1    | Introduction . . . . .                                       | 236 |
| 4.4.2    | Existence results . . . . .                                  | 237 |
| 4.4.3    | Structure of the solution set . . . . .                      | 246 |
| 4.5      | Impulsive differential inclusions on the half-line . . . . . | 250 |
| 4.5.1    | Existence results and compactness of solution sets . . . . . | 250 |
| 4.5.1.1  | The convex <i>u.s.c.</i> case . . . . .                      | 251 |
| 4.5.1.2  | The nonconvex Lipschitz case . . . . .                       | 258 |
| 4.5.1.3  | The nonconvex <i>l.s.c.</i> case . . . . .                   | 262 |
| 4.5.2    | Topological structure via the projective limit . . . . .     | 265 |
| 4.5.2.1  | The nonconvex case . . . . .                                 | 266 |
| 4.5.2.2  | The convex case . . . . .                                    | 271 |
| 4.5.2.3  | The terminal problem . . . . .                               | 274 |
| 4.5.3    | Using solution sets to prove existence results . . . . .     | 283 |
| <b>5</b> | <b>Preliminary notions of topology and homology</b>          | 288 |
| 5.1      | Retracts, extension and embedding properties . . . . .       | 288 |
| 5.2      | Absolute retracts . . . . .                                  | 294 |
| 5.3      | Homotopical properties of spaces . . . . .                   | 296 |
| 5.4      | Čech homology (cohomology) functor . . . . .                 | 304 |
| 5.5      | Maps of spaces of finite type . . . . .                      | 306 |
| 5.6      | Čech homology functor with compact carriers . . . . .        | 313 |
| 5.7      | Acyclic sets and Vietoris maps . . . . .                     | 315 |
| 5.8      | Homology of open subsets of Euclidean spaces . . . . .       | 319 |
| 5.9      | Lefschetz number . . . . .                                   | 323 |
| 5.10     | The coincidence problem . . . . .                            | 330 |
| <b>6</b> | <b>Background in multi-valued analysis</b>                   | 337 |
| 6.1      | Continuity of multi-valued mappings . . . . .                | 339 |
| 6.1.1    | Basic notions . . . . .                                      | 339 |
| 6.1.2    | Upper semi-continuity . . . . .                              | 341 |
| 6.1.2.1  | Generalities . . . . .                                       | 341 |
| 6.1.2.2  | $\varepsilon - \delta$ <i>u.s.c.</i> mappings . . . . .      | 344 |
| 6.1.2.3  | <i>U.s.c.</i> maps and closed graphs . . . . .               | 345 |
| 6.1.3    | Lower semi-continuity . . . . .                              | 346 |
| 6.1.3.1  | Generalities . . . . .                                       | 346 |
| 6.1.3.2  | $\varepsilon - \delta$ <i>l.s.c.</i> mappings . . . . .      | 349 |
| 6.1.4    | Hausdorff continuity . . . . .                               | 350 |

|                 |   |            |
|-----------------|---|------------|
| 6.2             | The selection problem . . . . .   | 354        |
| 6.2.1           | Michael's selection theorem . . . . .   | 355        |
| 6.2.2           | Michael's family of subsets . . . . .   | 358        |
| 6.2.3           | $\sigma$ -selectionable mappings . . . . .  | 362        |
| 6.2.4           | The Kuratowski–Ryll–Nardzewski selection theorem . . . . .  | 366        |
| 6.2.5           | Aumann and Filippov theorems . . . . .  | 378        |
| 6.2.6           | Hausdorff measurable multi-valued maps . . . . .  | 382        |
| 6.2.7           | Product-measurability and the Scorza–Dragoni property . . . . .                                   | 383        |
| 6.3             | Decomposable sets . . . . .   | 390        |
| 6.3.1           | The Bressan–Colombo–Fryszkowski selection theorem . . . . .                                       | 390        |
| 6.3.2           | Decomposability in $L^1(T, E)$ . . . . .  | 390        |
| 6.3.3           | Integration of multi-valued maps . . . . .  | 392        |
| 6.3.4           | Nemytskii operators . . . . .   | 393        |
| <b>Appendix</b> |   | <b>399</b> |
| A.1             | Axioms of the Čech homology theory . . . . .  | 399        |
| A.2             | The Bochner integral . . . . .  | 400        |
| A.3             | Absolutely continuous functions . . . . .   | 403        |
| A.4             | Compactness criteria in $C([a, b], E)$ , $C_b([0, \infty), E)$ , and<br>$PC([a, b], E)$ . . . . . | 405        |
| A.5             | Weak-compactness in $L^1$ . . . . .   | 408        |
| A.6             | Proper maps and vector fields . . . . .   | 410        |
| A.7             | Fundamental theorems in functional analysis . . . . .   | 411        |
| A.8             | $C_0$ -Semigroups . . . . .   | 412        |
| References      |   | 415        |
| Index           |   | 451        |