

## Mathematical Risk Analysis

Dependence, Risk Bounds, Optimal Allocations and Portfolios

Bearbeitet von  
Ludger Rüschendorf

1. Auflage 2013. Buch. xii, 408 S. Hardcover

ISBN 978 3 642 33589 1

Format (B x L): 15,5 x 23,5 cm

Gewicht: 789 g

[Weitere Fachgebiete > Mathematik > Operational Research > Finanz- und Versicherungsmathematik](#)

schnell und portofrei erhältlich bei

The logo for beck-shop.de features the text 'beck-shop.de' in a bold, red, sans-serif font. Above the 'i' in 'shop' are three red dots of increasing size. Below the main text, 'DIE FACHBUCHHANDLUNG' is written in a smaller, red, all-caps, sans-serif font.

**beck-shop.de**  
DIE FACHBUCHHANDLUNG

Die Online-Fachbuchhandlung [beck-shop.de](http://beck-shop.de) ist spezialisiert auf Fachbücher, insbesondere Recht, Steuern und Wirtschaft. Im Sortiment finden Sie alle Medien (Bücher, Zeitschriften, CDs, eBooks, etc.) aller Verlage. Ergänzt wird das Programm durch Services wie Neuerscheinungsdienst oder Zusammenstellungen von Büchern zu Sonderpreisen. Der Shop führt mehr als 8 Millionen Produkte.

# Contents

## Part I Stochastic Dependence and Extremal Risk

|          |                                                               |           |
|----------|---------------------------------------------------------------|-----------|
| <b>1</b> | <b>Copulas, Sklar's Theorem, and Distributional Transform</b> | <b>3</b>  |
| 1.1      | Sklar's Theorem and the Distributional Transform              | 3         |
| 1.2      | Copula Models and Copula Constructions                        | 7         |
| 1.2.1    | Some Classes of Copulas                                       | 8         |
| 1.2.2    | Copulas and $L^2$ -Projections                                | 11        |
| 1.3      | Multivariate Distributional and Quantile Transform            | 14        |
| 1.4      | Pair Copula Construction of Copula Models                     | 17        |
| 1.5      | Applications of the Distributional Transform                  | 21        |
| 1.5.1    | Application to Stochastic Ordering                            | 21        |
| 1.5.2    | Optimal Couplings                                             | 23        |
| 1.5.3    | Identification and Goodness of Fit Tests                      | 25        |
| 1.5.4    | Empirical Copula Process and Empirical Dependence Function    | 26        |
| 1.6      | Multivariate and Overlapping Marginals                        | 28        |
| 1.6.1    | Generalized Fréchet Class                                     | 28        |
| 1.6.2    | Copulas with Given Independence Structure                     | 31        |
| 1.6.3    | Copulas, Overlapping Marginals, and $L^2$ -Projections        | 33        |
| <b>2</b> | <b>Fréchet Classes, Risk Bounds, and Duality Theory</b>       | <b>35</b> |
| 2.1      | Dual Representation of Generalized Fréchet Bounds             | 37        |
| 2.2      | Fréchet Bounds Comonotonicity and Extremal Risk               | 45        |
| <b>3</b> | <b>Convex Order, Excess of Loss, and Comonotonicity</b>       | <b>53</b> |
| 3.1      | Convex Order and Comonotonicity                               | 53        |
| 3.2      | Schur Order and Rearrangements                                | 57        |
| 3.3      | Rearrangements and Excess of Loss                             | 63        |
| 3.4      | Integral Orders and $\prec_{\mathcal{F}}$ -Diffusions         | 66        |

|          |                                                                                            |     |
|----------|--------------------------------------------------------------------------------------------|-----|
| <b>4</b> | <b>Bounds for the Distribution Function and Value at Risk of the Joint Portfolio</b> ..... | 71  |
| 4.1      | Standard Bounds .....                                                                      | 72  |
| 4.2      | Conditional Moment Method .....                                                            | 79  |
| 4.3      | Dual Bounds .....                                                                          | 82  |
| <b>5</b> | <b>Restrictions on the Dependence Structure</b> .....                                      | 91  |
| 5.1      | Restriction to Positive Dependent Risk Vectors .....                                       | 91  |
| 5.2      | Higher Order Marginals.....                                                                | 95  |
| 5.2.1    | A Reduction Principle and Bonferroni Type Bounds .....                                     | 97  |
| 5.2.2    | The Conditioning Method .....                                                              | 103 |
| 5.2.3    | Reduction Bounds for the Joint Portfolio in<br>General Marginal Systems .....              | 107 |
| <b>6</b> | <b>Dependence Orderings of Risk Vectors<br/>and Portfolios</b> .....                       | 113 |
| 6.1      | Positive Orthant Dependence and Supermodular Ordering .....                                | 113 |
| 6.2      | Association, Conditional Increasing Vectors, and<br>Positive Supermodular Dependence ..... | 120 |
| 6.3      | Directionally Convex Order .....                                                           | 124 |
| 6.3.1    | Basic Properties of the Directionally Convex Order .....                                   | 124 |
| 6.3.2    | Further Criteria for $\leq_{\text{dex}}$ .....                                             | 126 |
| 6.3.3    | Directionally Convex Order in Functional Models .....                                      | 128 |
| 6.4      | Dependence Orderings in Models with Multivariate Marginals ...                             | 131 |

## Part II Risk Measures and Worst Case Portfolios

|          |                                                                                            |     |
|----------|--------------------------------------------------------------------------------------------|-----|
| <b>7</b> | <b>Risk Measures for Real Risks</b> .....                                                  | 141 |
| 7.1      | Some Classes of Risk Measures for Real Variables .....                                     | 142 |
| 7.1.1    | Basic Properties of Risk Measures.....                                                     | 142 |
| 7.1.2    | Examples of Risk Measures .....                                                            | 146 |
| 7.2      | Representation and Continuity Properties of Convex<br>Risk Measures on $L^p$ -Spaces ..... | 153 |
| 7.2.1    | Convex Duality and Continuity Results .....                                                | 154 |
| 7.2.2    | Representation of Coherent and Convex Risk<br>Measures on $L^p$ .....                      | 157 |
| 7.2.3    | Continuity Results for Risk Measures on $L^p$ .....                                        | 160 |
| <b>8</b> | <b>Risk Measures for Portfolio Vectors</b> .....                                           | 167 |
| 8.1      | Basic Properties of Portfolio Risk Measures.....                                           | 168 |
| 8.2      | Classes of Examples of Portfolio Risk Measures .....                                       | 174 |
| 8.2.1    | Aggregation Type Risk Measures.....                                                        | 174 |
| 8.2.2    | Multivariate Distortion and Quantile-Type<br>Risk Measures .....                           | 180 |

|          |                                                                                                       |     |
|----------|-------------------------------------------------------------------------------------------------------|-----|
| 8.3      | Representation and Continuity of Convex Risk Measures on $L_d^p$ .....                                | 184 |
| <b>9</b> | <b>Law Invariant Convex Risk Measures on <math>L_d^p</math> and Optimal Mass Transportation</b> ..... | 189 |
| 9.1      | Law Invariant Risk Measures and Optimal Mass Transportation .....                                     | 190 |
| 9.2      | Multivariate Comonotonicity and the $n$ -Coupling Problem .....                                       | 198 |
| 9.3      | Worst Case Portfolio Vectors and Diversification Effects .....                                        | 207 |
| 9.4      | Examples of Worst Case Risk Portfolios and Worst Case Diversification Effects .....                   | 214 |

## Part III Optimal Risk Allocation

|           |                                                                                                    |     |
|-----------|----------------------------------------------------------------------------------------------------|-----|
| <b>10</b> | <b>Optimal Allocations and Pareto Equilibrium</b> .....                                            | 227 |
| 10.1      | Pareto Equilibrium and Related Risk Measures in the Coherent Case .....                            | 227 |
| 10.2      | Optimal Allocations Under Admissibility Restrictions .....                                         | 235 |
| 10.3      | Pareto Equilibrium for Convex Risk Measures .....                                                  | 248 |
| 10.4      | Pareto Optimality, Comonotonicity, and Existence of Optimal Allocations .....                      | 256 |
| <b>11</b> | <b>Characterization and Examples of Optimal Risk Allocations for Convex Risk Functionals</b> ..... | 265 |
| 11.1      | Inf-Convolution and Convex Conjugates .....                                                        | 266 |
| 11.2      | Characterization of Optimal Allocations .....                                                      | 269 |
| 11.3      | Examples of Optimal Risk Allocations .....                                                         | 276 |
| 11.3.1    | Expected Risk Functionals .....                                                                    | 277 |
| 11.3.2    | Dilated Risk Functionals .....                                                                     | 278 |
| 11.3.3    | Average Value at Risk and Stop-Loss Contracts .....                                                | 279 |
| 11.3.4    | Mean Variance Versus Standard Deviation Risk Functionals .....                                     | 280 |
| 11.4      | Optimal Allocation of Risk Vectors .....                                                           | 283 |
| 11.4.1    | Characterization of Optimal Allocations .....                                                      | 284 |
| 11.4.2    | Law Invariant Risk Measures and Comonotonicity .....                                               | 289 |
| 11.4.3    | Existence of Minimal Risk Allocations .....                                                        | 293 |
| 11.4.4    | Uniqueness of Optimal Allocations .....                                                            | 299 |
| 11.4.5    | Examples of Optimal Allocations .....                                                              | 300 |
| 11.5      | The Capital Allocation Problem .....                                                               | 302 |
| <b>12</b> | <b>Optimal Contingent Claims and (Re)insurance Contracts</b> .....                                 | 305 |
| 12.1      | Optimal Contingent Claims .....                                                                    | 305 |
| 12.1.1    | Optimal Investment Problems .....                                                                  | 306 |
| 12.1.2    | Minimal Demand Problem .....                                                                       | 309 |
| 12.2      | Optimal (Re)insurance Contracts .....                                                              | 314 |
| 12.2.1    | Optimality of Stop-Loss Contracts .....                                                            | 314 |
| 12.2.2    | Optimal Worst Case (Re)insurance Contracts .....                                                   | 318 |

## Part IV Optimal Portfolios and Extreme Risks

|                                                                                                   |     |
|---------------------------------------------------------------------------------------------------|-----|
| <b>13 Optimal Portfolio Diversification w.r.t. Extreme Risks</b> .....                            | 325 |
| 13.1 Heavy-Tailed Portfolios and Multivariate Regular Variation .....                             | 325 |
| 13.2 Extreme Risk Index and Portfolio Diversification .....                                       | 328 |
| 13.3 Estimation of the Extreme Risk Index and the Optimal<br>Portfolio .....                      | 333 |
| 13.4 Asymptotic Normality of $\widehat{\gamma}_{\xi}$ .....                                       | 342 |
| 13.5 Application to Risk Minimization .....                                                       | 350 |
| <b>14 Ordering of Multivariate Risk Models with Respect to<br/>Extreme Portfolio Losses</b> ..... | 353 |
| 14.1 Asymptotic Portfolio Loss Ordering .....                                                     | 353 |
| 14.2 Characterization of $\preceq_{\text{apl}}$ in Multivariate Regularly<br>Varying Models ..... | 359 |
| 14.2.1 Multivariate Regular Variation .....                                                       | 359 |
| 14.2.2 Ordering of Canonical Spectral Measures .....                                              | 365 |
| 14.2.3 Unbalanced Tails .....                                                                     | 372 |
| 14.3 Relations to the Convex and Supermodular Order .....                                         | 374 |
| 14.4 Examples of apl-Ordering .....                                                               | 379 |
| <b>References</b> .....                                                                           | 385 |
| <b>List of Symbols</b> .....                                                                      | 399 |
| <b>Index</b> .....                                                                                | 401 |