

#### **Advanced Model Order Reduction Techniques in VLSI Design**

Model order reduction (MOR) techniques are important in reducing the complexity of nanometer VLSI designs, and consequently controlling "parasitic" electromagnetic effects, so that higher operating speeds and smaller feature sizes can be achieved. This book presents a systematic introduction to, and treatment of, the key MOR methods used in general linear circuits, using real-world examples to illustrate the advantages and disadvantages of each algorithm.

Starting with a review of traditional projection-based techniques and proofs of some fundamental theories, coverage progresses to advanced "state-of-the-art" MOR methods for VLSI design. These include HMOR, passive truncated balanced realization (TBR) methods, efficient inductance modeling via the VPEC model, general model optimization and passivity enforcement methods, passive model realization techniques, and structure-preserving MOR techniques. Numerical methods have been used throughout and, where possible, approached from the CAD engineer's perspective. This avoids complex mathematics, and allows the reader to take on real design problems and develop more effective tools.

With practical examples and over 100 illustrations, this book is suitable for researchers and graduate students of electrical and computer engineering, as well as for practitioners working in the VLSI design and design automation industries.

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## Contents

Contents

|   | Figu | ures  | viii |
|---|------|---|------|
|   | Tab  | les   | xiv  |
|   | Fore | eword   | xv   |
|   | Ack  | nowledgments  | xvii |
| 1 | Intr | oduction  | 1    |
|   | 1.1  | The need for compact modeling of interconnects            | 1    |
|   | 1.2  | Interconnect analysis and modeling methods in a nutshell  | 2    |
|   | 1.3  | Book outline  | 4    |
|   | 1.4  | Summary   | 7    |
| 2 | Pro  | jection-based model order reduction algorithms            | 8    |
|   | 2.1  | Moments and moment-matching methods                       | 8    |
|   | 2.2  | Moment computation in MNA formulation                     | 11   |
|   | 2.3  | Asymptotic waveform evaluation                            | 13   |
|   | 2.4  | Projection-based model order reduction methods            | 20   |
|   | 2.5  | Numerical examples  | 32   |
|   | 2.6  | Historical notes  | 32   |
|   | 2.7  | Summary   | 34   |
|   | 2.8  | Appendices  | 34   |
| 3 | Tru  | ncated balanced realization methods for MOR               | 37   |
|   | 3.1  | Introduction  | 37   |
|   | 3.2  | The singular value decomposition (SVD)                    | 38   |
|   | 3.3  | Proper orthogonal decomposition (POD)                     | 38   |
|   | 3.4  | Classic truncated balanced realization methods            | 39   |
|   | 3.5  | Passive-preserving truncated balanced realization methods | 43   |
|   | 3.6  | Hybrid TBR and combined TBR-Krylov subspace methods       | 45   |
|   | 3.7  | Empirical TBR and poor man's TBR                          | 45   |
|   | 3.8  | Computational complexities of TBR methods                 | 47   |
|   | 3.9  | Practical implementation and numerical issues             | 48   |
|   | 3.10 | Numerical examples  | 53   |
|   | 3.11 | Summary   | 54   |
|   |      |   |      |

V

page v



vi Contents

| 4 | Pas | sive balanced truncation of linear systems in descriptor form | 56  |
|---|-----|---|-----|
|   | 4.1 | Introduction  | 56  |
|   | 4.2 | The passive balanced truncation algorithm: PriTBR             | 57  |
|   | 4.3 | Structure-preserved balanced truncation                       | 60  |
|   | 4.4 | Numerical examples  | 62  |
|   | 4.5 | Summary   | 64  |
| 5 | Pas | sive hierarchical model order reduction                       | 67  |
|   | 5.1 | Overview of hierarchical MOR algorithm                        | 68  |
|   | 5.2 | DDD-based hierarchical decomposition                          | 70  |
|   | 5.3 | Hierarchical reduction versus moment-matching                 | 76  |
|   | 5.4 | Preservation of reciprocity                                   | 80  |
|   | 5.5 | Multi-point expansion hierarchical reduction                  | 81  |
|   | 5.6 | Numerical examples  | 84  |
|   | 5.7 | Summary   | 91  |
|   | 5.8 | Historical notes on node-elimination-based reduction methods  | 91  |
| 6 | Ter | minal reduction of linear dynamic circuits                    | 93  |
|   | 6.1 | Review of the SVDMOR method                                   | 95  |
|   | 6.2 | Input and output moment matrices                              | 96  |
|   | 6.3 | The extended-SVDMOR (ESVDMOR) method                          | 99  |
|   | 6.4 | Determination of cluster number by SVD                        | 102 |
|   | 6.5 | K-means clustering algorithm                                  | 104 |
|   | 6.6 | TermMerg algorithm  | 106 |
|   | 6.7 | Numerical examples  | 111 |
|   | 6.8 | Summary   | 116 |
| 7 | Vec | tor-potential equivalent circuit for inductance modeling      | 118 |
|   | 7.1 | Vector-potential equivalent circuit                           | 119 |
|   | 7.2 | VPEC via PEEC inversion                                       | 124 |
|   | 7.3 | Numerical examples  | 128 |
|   | 7.4 | Inductance models in hierarchical reduction                   | 131 |
|   | 7.5 | Summary   | 136 |
| 8 | Str | ucture-preserving model order reduction                       | 137 |
|   | 8.1 | Introduction  | 137 |
|   | 8.2 | Chapter overview  | 138 |
|   | 8.3 | Background  | 139 |
|   | 8.4 | Block-structure-preserving model reduction                    | 141 |
|   | 8.5 | TBS method  | 144 |
|   | 8.6 | Two-level analysis  | 149 |
|   | 8.7 | Numerical examples  | 151 |
|   | 8.8 | Summary   | 157 |
| 9 | Blo | ck structure-preserving reduction for RLCK circuits           | 158 |



| Contents   | vii   |
|--|-------|
|  |       |
| 9.1 Introduction   | 158   |
| 9.2 Block structure-preserving model reduction                       | 159   |
| 9.3 Structure preservation for admittance transfer-function matrices | 161   |
| 9.4 General block structure-preserving MOR method                    | 163   |
| 9.5 Numerical examples   | 167   |
| 9.6 Summary  | 169   |
| 9.7 Appendix   | 170   |
| 10 Model optimization and passivity enforcement                      | 172   |
| 10.1 Passivity enforcement   | 172   |
| 10.2 Model optimization for active circuits                          | 176   |
| 10.3 Optimization for magnitude and phase responses                  | 178   |
| 10.4 Numerical examples  | 181   |
| 10.5 Summary   | 185   |
| 11 General multi-port circuit realization                            | 187   |
| 11.1 Review of existing circuit-realization methods                  | 187   |
| 11.2 General multi-port network realization                          | 195   |
| 11.3 Multi-port non-reciprocal circuit realization                   | 197   |
| 11.4 Numerical examples  | 199   |
| 11.5 Summary   | 203   |
| 12 Reduction for multi-terminal interconnect circuits                | 204   |
| 12.1 Introduction  | 204   |
| 12.2 Problems of subspace projection-based MOR methods               | 205   |
| 12.3 Model order reduction for multiple-terminal circuits: MTermMO   | R 208 |
| 12.4 Numerical examples  | 212   |
| 12.5 Summary   | 214   |
| 13 Passive modeling by signal waveform shaping                       | 215   |
| 13.1 Introduction  | 215   |
| 13.2 Passivity and positive-realness                                 | 217   |
| 13.3 Conditional passivity and positive-realness                     | 218   |
| 13.4 Passivity enforcement by waveform shaping                       | 221   |
| 13.5 Numerical examples  | 225   |
| 13.6 Summary   | 226   |
| References   | 229   |
| Index  | 238   |



## **Figures**

| 2.1  | The network of an ideal delay of T.   | 9  |
|------|---|----|
| 2.2  | The unit impulse and unit step responses.   | 11 |
| 2.3  | Block diagram of $(2.54)$ .   | 21 |
| 2.4  | Arnoldi method based on modified Gram–Schmidt orthonormaliza-                                       |    |
|      | tion for SISO systems.  | 25 |
| 2.5  | Non-symmetric Lanczos method for SISO systems.  | 26 |
| 2.6  | Transient response of a non-passive circuit.  | 29 |
| 2.7  | Block Arnoldi method for MIMO systems.  | 31 |
| 2.8  | A two-port large lumped RCL circuit.  | 32 |
| 2.9  | Comparison of the magnitudes of $Y(11)$ for different reduction orders for the lumped RLC circuit.  | 33 |
| 2.10 | Comparison of the magnitudes of $Y(12)$ for different reduction orders for the lumped RLC circuit.  | 33 |
| 3.1  | Frequency responses of a reduced model and its original system.                                     | 54 |
| 3.2  | Frequency response of the input impedance of a reduced model and its original system.               | 55 |
| 4.1  | Frequency responses of TBR, PriTBR, and PRIMA reduced models and the original circuit.              | 63 |
| 4.2  | Nyquist plots of the TBR reduced model and the PriTBR reduced model.                                | 64 |
| 4.3  | Pole zero map of system before mapping.   | 65 |
| 4.4  | Frequency responses of PRIMA and combined PRIMA and PriTBR reduced models and the original circuit. | 65 |
| 4.5  | Frequency responses of SPRIM and SP-PriTBR reduced models and the original circuit.                 | 66 |
| 5.1  | A hierarchical circuit. Reprinted with permission from [126] (c) 2000 IEEE.                         | 68 |
| 5.2  | A simple RC circuit.  | 71 |
| 5.3  | A matrix determinant and its DDD.   | 71 |
| 0.0  | TI MOVIM GOVERNMENTO WHE TOO DEED.  | 11 |

viii



FIGURES ix

| 5.4  | Illustration of Theorem 5.1. Reprinted with permission from [122] (c) 2005 IEEE.   | 72   |
|------|--|------|
| 5.5  | A determinant and its YDDD. Reprinted with permission from [122] (c) 2005 IEEE.  | 73   |
| 5.6  | Y-expanded DDD construction. Reprinted with permission from [122] (c) 2005 IEEE.   | 74   |
| 5.7  | The general hierarchical model order algorithm flow.   | 76   |
| 5.8  | Frequency responses of $\mu A741$ circuit under different reduction or-  | •    |
| 0.0  | ders. Reprinted with permission from [94] (c) 2006 IEEE.   | 78   |
| 5.9  | Frequency responses of an RC tree circuit under different reduction orders. Reprinted with permission from [94] (c) 2006 IEEE.   | 79   |
| 5.10 | Responses of a typical $k_i/(s-p_i)$ . Reprinted with permission from [94] (c) 2006 IEEE.  | 83   |
| 5.11 | Frequency responses of the three-turn spiral inductor and its reduced  |      |
| 5.12 | model by using waveform matching and the common-pole method.<br>Reprinted with permission from [94] (c) 2006 IEEE.<br>Colpitts LC oscillator with spiral inductors. Reprinted with permis- | 85   |
| 0.12 | sion from [94] (c) 2006 IEEE.  | 86   |
| 5.13 | Time-domain comparison between original and synthesized models for a Colpitts LC oscillator with a three-turn spiral inductor. Reprinted   | 00   |
|      | with permission from [94] (c) 2006 IEEE.   | 86   |
| 5.14 | Frequency responses of $Y_{11}$ of a two-bit transmission line. Reprinted with permission from [94] (c) 2006 IEEE.   | 87   |
| 5.15 | Frequency responses of $Y_{12}$ of a two-bit transmission line. Reprinted with permission from [94] (c) 2006 IEEE.   | 88   |
| 5.16 | Transient responses of a two-bit transmission line. Reprinted with permission from [94] (c) 2006 IEEE.   | 89   |
| 5.17 | Frequency responses of a two-bit transmission line at two ports. Reprinted with permission from [94] (c) 2006 IEEE.  | 89   |
| 6.1  | Terminal reduction versus traditional model order reduction.   | 94   |
| 6.2  | Frequency responses from SVDMOR and ESVDMOR for $net27$ circuit  | .102 |
| 6.3  | Frequency response from SVDMOR and ESVDMOR with different  |      |
|      | terminals for $net27$ circuit.   | 103  |
| 6.4  | K-means clustering algorithm. Reprinted with permission from [75] (c) 2005 IEEE.   | 105  |
| 6.5  | The reduction flow of combined terminal and model order reductions.  | 107  |
| 6.6  | Simple interface circuit.  | 108  |
| 6.7  | Frequency impedance responses from the SVDMOR method for<br>net1026 circuit.   | 112  |
| 6.8  | Output terminal distribution for each cluster for $net1026$ circuit.<br>Reprinted with permission from [75] (c) 2005 IEEE.   | 113  |
| 6.9  | Step responses of representative output terminals. Reprinted with permission from [75] (c) 2005 IEEE.  | 114  |



#### x FIGURES

| 6.10 | Comparison of 50% delay time among representative output terminals. Reprinted with permission from [75] (c) 2005 IEEE.  | 114        |
|------|---|------------|
| 6.11 | Step responses of representative output terminals and two suppressed outputs. Reprinted with permission from [75] (c) 2005 IEEE.  | 115        |
| 6.12 | Comparison of $50\%$ delay time among representative output terminals and two suppressed outputs. Reprinted with permission from [75] (c) 2005 IEEE.  | 115        |
| 6.13 | Output terminal distribution for each cluster for $net27$ circuit. Reprinted with permission from [75] (c) 2005 IEEE.   | 116        |
| 6.14 | Input terminal distribution for each cluster for circuit $net38$ .  | 117        |
| 6.15 | Output terminal distribution for each cluster for circuit $net38$ .   | 117        |
| 7.1  | (a) Electronic current-controlled vector-potential current source; (b) The Kirchoff current law for vector potential circuit. An invoking vector potential current source is employed at $a_i$ , and the responding vector potential at $a_j$ is $A_j^k$ , determined by the full effective resistance network. Reprinted with permission from [135] (c) 2005 IEEE.             | 121        |
| 7.2  | Vector potential equivalent circuit model for three filaments.  Reprinted with permission from [135] (c) 2005 IEEE.   | 123        |
| 7.3  | For five-bit bus, (a) a 1-V step voltage with 10 ps rising time and (b) a 1-V ac voltage are applied to the first bit and all other bits are quiet. The responses of the PEEC model, full VPEC model, and localized VPEC model are measured at the far end of the second bit.   | 120        |
| 7.4  | Reprinted with permission from [135] (c) 2005 IEEE.  For 128-bit bus by numerical truncation, a 1-V step voltage with 10 ps rising time is applied to the first bit, and all other bits are quiet. The responses of the PEEC model, the full VPEC model, and the tVPEC model are measured at the far end of the second bit. Reprinted with permission from [135] (c) 2005 IEEE. | 129<br>130 |
| 7.5  | Example of a coupled two-bit RLCM circuit under the PEEC model. Reprinted with permission from [135] (c) 2005 IEEE.   | 132        |
| 7.6  | Example of a coupled two-bit RLCM circuit under the nodal susceptance model. Reprinted with permission from [135] (c) 2005 IEEE.  | 133        |
| 7.7  | Frequency responses of PEEC model in SPICE, susceptance under NA and VPEC models for the two-bit bus. Reprinted with permission from [135] (c) 2005 IEEE.   | 133        |
| 7.8  | Stamp of the second-order admittance in the NA matrix, where (a), (b) and (c) represent for $G$ , $\Gamma$ , $C$ and $B$ . $G$ (rank=4) and $\Gamma$ (rank=4) are both singular for $6 \times 6$ matrices. Reprinted with permission from [135] (c) 2005 IEEE.  | 134        |
| 7.9  | Example of a coupled two-bit RLCM circuit under the VPEC model. Reprinted with permission from [135] (c) 2005 IEEE.   | 135        |



FIGURES xi

| 8.1  | Pole matching comparison: $mq$ poles matched by TBS and BSMOR, and $q$ poles matched by HiPRIME. Reprinted with permission from [138] (c) 2006 ACM.   | 150 |
|------|---|-----|
| 8.2  | Non-zero (nz) pattern of conductance matrices: (a) original system, (b) triangular system, (c) reduced system by TBS. (a)–(c) have different dimensions, but (b)–(c) have the same triangular structure and the same diagonal block structure. Reprinted with permission from [138] (c) 2006 ACM. | 151 |
| 8.3  | Comparison of time-domain responses between HiPRIME, BSMOR, [139], TBS and the original. TBS is identical to the original. Reprinted with permission from [138] (c) 2006 ACM.   | 152 |
| 8.4  | Comparison of frequency-domain responses between HiPRIME, BSMOR, TBS, and the original. TBS is identical to the original. Reprinted with permission from [138] (c) 2006 ACM.  | 153 |
| 8.5  | Comparison of runtime under similar accuracy. (a) macro-model building time (log scale) comparison; (b) macro-model time-domain simulation time (log scale) comparison. Reprinted with permission from [138] (c) 2006 ACM.  | 154 |
| 8.6  | A P/G voltage bounce map without decoupling capacitor allocations.<br>Reprinted with permission from [138] (c) 2006 ACM.  | 155 |
| 8.7  | A P/G voltage bounce map with decoupling capacitors allocated at the centers of four blocks. Reprinted with permission from [138] (c) 2006 ACM.   | 156 |
| 9.1  | Comparison between SPRIM, PRIMA, and BSPRIM for impedance form.   | 167 |
| 9.2  | Sparsity preservation of BSPRIM.  | 168 |
| 9.3  | Comparison between PRIMA and structure-preserving algorithm (BSPRIM) for admittance form.   | 168 |
| 9.4  | Comparison between PRIMA and BSPRIM with and without re-orthonormalization for admittance form.   | 169 |
| 10.1 | Admittance $Y_{21}$ response of the $\mu A725$ opamp without considering phase. Reprinted with permission from [73] (c) 2005 IEEE.  | 178 |
| 10.2 | Frequency response of $Y_{12}$ of opamp model. Reprinted with permission from [73] (c) 2005 IEEE.   | 182 |
| 10.3 | Active Sallen–Key topology low-pass filter. Reprinted with permission from [73] (c) 2005 IEEE.  | 182 |
| 10.4 | Frequency response of $Y_{21}$ of the Sallen–Key topology low-pass filter.<br>Reprinted with permission from [73] (c) 2005 IEEE.  | 183 |
| 10.5 | Frequency response of $Y_{21}$ of the Sallen–Key topology low-pass filter without considering phase. Reprinted with permission from [73] (c) 2005 IEEE.   | 183 |



#### xii FIGURES

| 10.6  | Frequency response of the transfer function of the Sallen–Key topology low-pass filter. Reprinted with permission from [73] (c) 2005 IEEE    | E.184 |
|-------|--|-------|
| 10.7  | Transient response of the Sallen–Key topology low-pass filter with different excitations. Reprinted with permission from [73] (c) 2005 IEEE. | 184   |
| 10.8  | Active low-pass FDNR filter. Reprinted with permission from [73] (c) 2005 IEEE.  | 185   |
| 10.9  | Frequency response of the transfer function of the low-pass FDNR filter. Reprinted with permission from [73] (c) 2005 IEEE.                  | 185   |
| 10.10 | Transient response of the FDNR filter with different excitations. Reprinted with permission from [73] (c) 2005 IEEE.                         | 186   |
| 11.1  | Realization of $Z(s)$ in (11.2) and $Y(s)$ in (11.3).  | 189   |
| 11.2  | Realization of $Z(s)$ in (11.4) and $Y(s)$ in (11.5).  | 190   |
| 11.3  | Realization of $Z(s)$ in (11.6).   | 190   |
| 11.4  | Real-part responses of $Z(s)$ and the remainder $Z_1(s) = Z(s) - R_{\min}$ .   | 191   |
| 11.5  | Brune's driving point synthesis by multiple-stage RLCM ladders   |       |
|       | (Brune's cycle).   | 193   |
| 11.6  | Brune's multiple level ladder macromodel synthesis.  | 193   |
| 11.7  | Example of Brune's synthesis with passivity-preserved transformation: the non-passive T circuit is transformed to a passive coupled-         |       |
|       | inductor circuit.  | 194   |
| 11.8  | One-port Foster admittance realization. Reprinted with permission from [94] (c) 2006 IEEE.   | 196   |
| 11.9  | General two-port realization $\Pi$ model. Reprinted with permission from [94] (c) 2006 IEEE.   | 197   |
| 11.10 | Six-port realization based on Π-structure. Reprinted with permission from [94] (c) 2006 IEEE.  | 198   |
| 11.11 | General two-port non-reciprocal active realization. Reprinted with permission from [73] (c) 2005 IEEE.                                       | 199   |
| 11 19 |  | 133   |
| 11.12 | Comparison between the transfer function $Y_{1-port}(s)$ and its circuit realization.  | 200   |
| 11.13 | Comparison between the transfer function $Y_{12}(s)$ and its circuit realization.  | 202   |
| 11.14 | Comparison between the transfer function $Y_{22}(s)$ and its circuit realization.  | 202   |
| 12.1  | Frequency response of the three-input circuit. Reprinted with permission from [76] (c) 2006 IEEE.  | 207   |
| 12.2  | Frequency response of the three-input circuit with different approximation. Reprinted with permission from [76] (c) 2006 IEEE.               | 210   |
| 12.3  | Comparison of computation cost for admittance. Reprinted with permission from [76] (c) 2006 IEEE.  | 212   |



FIGURES xiii

| 12.4  | Frequency response comparison among the original circuit, PRIMA model, and MTermMOR model of the circuit <i>clktree50</i> . Reprinted with permission from [76] (c) 2006 IEEE. | 213 |
|-------|--|-----|
| 12.5  | Frequency response comparison among the original circuit, PRIMA model, and MTermMOR model of the circuit <i>sram1026</i> . Reprinted with permission from [76] (c) 2006 IEEE.  | 213 |
| 13.1  | Transient response of a non-passive circuit.   | 217 |
| 13.2  | Frequency responses of a reduced model and its original RC circuit.  | 218 |
| 13.3  | Transient responses of a reduced model and its original RC circuit for   |     |
|       | a 10 GHz input.  | 219 |
| 13.4  | Transient responses of a reduced model and its original RC circuit for   |     |
|       | a 60 Ghz input.  | 220 |
| 13.5  | Algorithm flow of FFT-IFFT-based waveform shaping.   | 221 |
| 13.6  | Algorithm of FFT-IFFT-based waveform shaping.  | 222 |
| 13.7  | Ramp signal shaped at different frequencies.   | 223 |
| 13.8  | Low-pass-filter-based waveform shaping.  | 224 |
| 13.9  | Group-delay characteristic and magnitude response for different order  |     |
|       | Bessel filters (normalized frequency).   | 224 |
| 13.10 | Comparison of responses of different models in time domain for the   |     |
|       | first example.   | 227 |
| 13.11 | Comparison in time domain between reduced models based on Bessel   |     |
|       | filters and ellipse filters.   | 228 |
| 13.12 | Comparison of responses of different models in time domain for second  |     |
|       | example.   | 228 |



## **Tables**

| 3.1 | The Hankel singular values for a six-port linear interconnect circuit.   | 53    |
|-----|--|-------|
| 5.1 | Simulation efficiency comparison between original and synthesized model (part I). Reprinted with permission from [94] (c) 2006 IEEE.   | 88    |
| 5.2 | Simulation efficiency comparison between original and synthesized model (part-II). Reprinted with permission from [94] (c) 2006 IEEE.  | 90    |
| 5.3 | Comparison of reduction CPU times. Reprinted with permission from [94] (c) 2006 IEEE.  | 90    |
| 6.1 | Singular values of DC moment, input moment matrix and output moment matrix of the circuit net27.   | 101   |
| 6.2 | Singular values of the DC admittance moment, 1st order admittance moment matrices of the circuit <i>net1026</i> when all the terminals are treated as bidirectional.         | 110   |
| 6.3 | Singular values of DC admittance moment, input moment matrix and output moment matrix of the circuit <i>net1026</i> .  | 112   |
| 6.4 | Output clustering results for the one-bit lines circuit <i>net1026</i> . Reprinted with permission from [75] (c) 2005 IEEE.  | 113   |
| 7.1 | Table of notations. Reprinted with permission from [135] (c) 2005 IEEE   | E.119 |
| 7.2 | Settings and results of geometrical $tVPEC$ models. Reprinted with permission from [135] (c) 2005 IEEE.  | 130   |
| 7.3 | Settings and results of numerical $t\mbox{VPEC}$ models. Reprinted with permission from [135] (c) 2005 IEEE.   | 131   |
| 8.1 | Time-domain waveform error of reduced models by HiPRIME, BSMOR, and TBS under the same order (number of matched moments). Reprinted with permission from [138] (c) 2006 ACM. | 155   |

xiv



### **Foreword**

Interconnect model reduction has emerged as one crucial operation for circuit analysis in the last decade as a result of the phenomenon of interconnect dominance of advanced VLSI technologies. Because interconnect contributes to a significant portion of the system performance, we have to take into account the coupling effects between subcircuit modules. However, the extraction of the coupling renders many small fragments of parasitics. While the values of the parasitics are small, the number of fragments is huge and this makes the accumulated effect non-negligible. If left untreated, the amount of parasitics can gobble up the memory capacity and consume long CPU time during circuit analysis.

Model reduction transforms a system into a circuit of much smaller size to approximate the behavior of the original description. Many researchers have contributed to the advancement of the techniques and demonstrated drastic reduction of the circuit sizes with satisfactory output responses in published reports. Many of these techniques have also been implemented in software tools for applications. However, it is important for the users to understand the techniques in order to use the package properly. To adopt these approaches, we need to inspect the following features.

- 1. Efficiency of the reduction: the complexity of the reduction algorithm determines the CPU time of the model reduction. The size of the reduced circuit affects the simulation time.
- 2. Reduction of both model order and terminals of circuits: reduction of terminals was investigated less in the past and combined terminal and model order reduction leads to more compact models.
- 3. Robustness of the algorithms: the numerical stability of the reduction algorithm ensures the robustness of the operation.
- 4. Structure of the reduced systems: the reduced systems may or may not preserve important characteristics like symmetry, reciprocity, etc. Those structure characteristics are important for reduction itself and for systems using the models.
- 5. Realizablility of the reduced system: the reduced system is realizable if it is passive and we can implement it using electrical elements with positive or negative values. We can simulate a realizable system with general simulation tools. Otherwise, we need to check if the reduced system satisfies the constraints of the simulation package.
  - 6. Passivity of the reduced circuits: the passivity ensures that the simulation



xvi Foreword

outputs are bounded for bounded inputs even if the reduced circuit is combined with other passive subcircuits.

7. Error bounds: The error bounds of the output responses provide users with confidence in the results.

In this book, Professors Sheldon X.-D. Tan and Lei He presented a comprehensive description of the reduction techniques. They have provided motivations for the approaches and insights into the algorithms as active researchers in the field. I found that the treatment of the subject is innovative and the general description is pleasant to read. The book covers the contemporary results and opens windows on future research directions in the field.

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xvii



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