

Contents

Preface

page xi

I

MATRIX THEORY

1

Matrix Algebra

3

1.1

Definitions and Notations

4

1.2

Fundamental Matrix Operations

6

1.3

Properties of Matrix Operations

18

1.4

Block Matrix Operations

30

1.5

Matrix Calculus

31

1.6

Sparse Matrices

39

1.7

Exercises

41

2

Solution of Multiple Equations

54

2.1

Gauss-Jordan Elimination

55

2.2

LU Decomposition

59

2.3

Direct Matrix Splitting

65

2.4

Iterative Solution Methods

66

2.5

Least-Squares Solution

71

2.6

QR Decomposition

77

2.7

Conjugate Gradient Method

78

2.8

GMRES

79

2.9

Newton’s Method

80

2.10

Enhanced Newton Methods via Line Search

82

2.11

Exercises

86

3

Matrix Analysis

99

3.1

Matrix Operators

100

3.2

Eigenvalues and Eigenvectors

107

3.3

Properties of Eigenvalues and Eigenvectors

113

3.4

Schur Triangularization and Normal Matrices

116

3.5

Diagonalization

117

3.6

Jordan Canonical Form

118

3.7

Functions of Square Matrices

120

3.8	Stability of Matrix Operators	124
3.9	Singular Value Decomposition	127
3.10	Polar Decomposition	132
3.11	Matrix Norms	135
3.12	Exercises	138
II VECTORS AND TENSORS		
4	Vector and Tensor Algebra and Calculus	149
4.1	Notations and Fundamental Operations	150
4.2	Vector Algebra Based on Orthonormal Basis Vectors	154
4.3	Tensor Algebra	157
4.4	Matrix Representation of Vectors and Tensors	162
4.5	Differential Operations for Vector Functions of One Variable	164
4.6	Application to Position Vectors	165
4.7	Differential Operations for Vector Fields	169
4.8	Curvilinear Coordinate System: Cylindrical and Spherical	184
4.9	Orthogonal Curvilinear Coordinates	189
4.10	Exercises	196
5	Vector Integral Theorems	204
5.1	Green’s Lemma	205
5.2	Divergence Theorem	208
5.3	Stokes’ Theorem and Path Independence	210
5.4	Applications	215
5.5	Leibnitz Derivative Formula	224
5.6	Exercises	225
III ORDINARY DIFFERENTIAL EQUATIONS		
6	Analytical Solutions of Ordinary Differential Equations	235
6.1	First-Order Ordinary Differential Equations	236
6.2	Separable Forms via Similarity Transformations	238
6.3	Exact Differential Equations via Integrating Factors	242
6.4	Second-Order Ordinary Differential Equations	245
6.5	Multiple Differential Equations	250
6.6	Decoupled System Descriptions via Diagonalization	258
6.7	Laplace Transform Methods	262
6.8	Exercises	263
7	Numerical Solution of Initial and Boundary Value Problems	273
7.1	Euler Methods	274
7.2	Runge Kutta Methods	276
7.3	Multistep Methods	282
7.4	Difference Equations and Stability	291
7.5	Boundary Value Problems	299
7.6	Differential Algebraic Equations	303
7.7	Exercises	305

8	Qualitative Analysis of Ordinary Differential Equations	311
8.1	Existence and Uniqueness	312
8.2	Autonomous Systems and Equilibrium Points	313
8.3	Integral Curves, Phase Space, Flows, and Trajectories	314
8.4	Lyapunov and Asymptotic Stability	317
8.5	Phase-Plane Analysis of Linear Second-Order Autonomous Systems	321
8.6	Linearization Around Equilibrium Points	327
8.7	Method of Lyapunov Functions	330
8.8	Limit Cycles	332
8.9	Bifurcation Analysis	340
8.10	Exercises	340
9	Series Solutions of Linear Ordinary Differential Equations	347
9.1	Power Series Solutions	347
9.2	Legendre Equations	358
9.3	Bessel Equations	363
9.4	Properties and Identities of Bessel Functions and Modified Bessel Functions	369
9.5	Exercises	371
IV	PARTIAL DIFFERENTIAL EQUATIONS	
10	First-Order Partial Differential Equations and the Method of Characteristics	379
10.1	The Method of Characteristics	380
10.2	Alternate Forms and General Solutions	387
10.3	The Lagrange-Charpit Method	389
10.4	Classification Based on Principal Parts	393
10.5	Hyperbolic Systems of Equations	397
10.6	Exercises	399
11	Linear Partial Differential Equations	405
11.1	Linear Partial Differential Operator	406
11.2	Reducible Linear Partial Differential Equations	408
11.3	Method of Separation of Variables	411
11.4	Nonhomogeneous Partial Differential Equations	431
11.5	Similarity Transformations	439
11.6	Exercises	443
12	Integral Transform Methods	450
12.1	General Integral Transforms	451
12.2	Fourier Transforms	452
12.3	Solution of PDEs Using Fourier Transforms	459
12.4	Laplace Transforms	464
12.5	Solution of PDEs Using Laplace Transforms	474
12.6	Method of Images	476
12.7	Exercises	477

13	Finite Difference Methods	483
13.1	Finite Difference Approximations	484
13.2	Time-Independent Equations	491
13.3	Time-Dependent Equations	504
13.4	Stability Analysis	512
13.5	Exercises	519
14	Method of Finite Elements	523
14.1	The Weak Form	524
14.2	Triangular Finite Elements	527
14.3	Assembly of Finite Elements	533
14.4	Mesh Generation	539
14.5	Summary of Finite Element Method	541
14.6	Axisymmetric Case	546
14.7	Time-Dependent Systems	547
14.8	Exercises	552
	<i>Bibliography</i>	B-1
	<i>Index</i>	I-1
A	Additional Details and Fortification for Chapter 1	561
A.1	Matrix Classes and Special Matrices	561
A.2	Motivation for Matrix Operations from Solution of Equations	568
A.3	Taylor Series Expansion	572
A.4	Proofs for Lemma and Theorems of Chapter 1	576
A.5	Positive Definite Matrices	586
B	Additional Details and Fortification for Chapter 2	589
B.1	Gauss Jordan Elimination Algorithm	589
B.2	SVD to Determine Gauss-Jordan Matrices Q and W	594
B.3	Boolean Matrices and Reducible Matrices	595
B.4	Reduction of Matrix Bandwidth	600
B.5	Block LU Decomposition	602
B.6	Matrix Splitting: Diakoptic Method and Schur Complement Method	605
B.7	Linear Vector Algebra: Fundamental Concepts	611
B.8	Determination of Linear Independence of Functions	614
B.9	Gram-Schmidt Orthogonalization	616
B.10	Proofs for Lemma and Theorems in Chapter 2	617
B.11	Conjugate Gradient Algorithm	620
B.12	GMRES Algorithm	629
B.13	Enhanced-Newton Using Double-Dogleg Method	635
B.14	Nonlinear Least Squares via Levenberg-Marquardt	639
C	Additional Details and Fortification for Chapter 3	644
C.1	Proofs of Lemmas and Theorems of Chapter 3	644
C.2	QR Method for Eigenvalue Calculations	649
C.3	Calculations for the Jordan Decomposition	655

C.4	Schur Triangularization and SVD	658
C.5	Sylvester’s Matrix Theorem	659
C.6	Danilevskii Method for Characteristic Polynomial	660
D	Additional Details and Fortification for Chapter 4	664
D.1	Proofs of Identities of Differential Operators	664
D.2	Derivation of Formulas in Cylindrical Coordinates	666
D.3	Derivation of Formulas in Spherical Coordinates	669
E	Additional Details and Fortification for Chapter 5	673
E.1	Line Integrals	673
E.2	Surface Integrals	678
E.3	Volume Integrals	684
E.4	Gauss-Legendre Quadrature	687
E.5	Proofs of Integral Theorems	691
F	Additional Details and Fortification for Chapter 6	700
F.1	Supplemental Methods for Solving First-Order ODEs	700
F.2	Singular Solutions	703
F.3	Finite Series Solution of $dx/dt = Ax + b(t)$	705
F.4	Proof for Lemmas and Theorems in Chapter 6	708
G	Additional Details and Fortification for Chapter 7	715
G.1	Differential Equation Solvers in MATLAB	715
G.2	Derivation of Fourth-Order Runge Kutta Method	718
G.3	Adams-Bashforth Parameters	722
G.4	Variable Step Sizes for BDF	723
G.5	Error Control by Varying Step Size	724
G.6	Proof of Solution of Difference Equation, Theorem 7.1	730
G.7	Nonlinear Boundary Value Problems	731
G.8	Ricatti Equation Method	734
H	Additional Details and Fortification for Chapter 8	738
H.1	Bifurcation Analysis	738
I	Additional Details and Fortification for Chapter 9	745
I.1	Details on Series Solution of Second-Order Systems	745
I.2	Method of Order Reduction	748
I.3	Examples of Solution of Regular Singular Points	750
I.4	Series Solution of Legendre Equations	753
I.5	Series Solution of Bessel Equations	757
I.6	Proofs for Lemmas and Theorems in Chapter 9	761
J	Additional Details and Fortification for Chapter 10	771
J.1	Shocks and Rarefaction	771
J.2	Classification of Second-Order Semilinear Equations: $n > 2$	781
J.3	Classification of High-Order Semilinear Equations	784

K Additional Details and Fortification for Chapter 11 786

 K.1 d’Alembert Solutions 786

 K.2 Proofs of Lemmas and Theorems in Chapter 11 791

L Additional Details and Fortification for Chapter 12 795

 L.1 The Fast Fourier Transform 795

 L.2 Integration of Complex Functions 799

 L.3 Dirichlet Conditions and the Fourier Integral Theorem 819

 L.4 Brief Introduction to Distribution Theory and Delta Distributions 820

 L.5 Tempered Distributions and Fourier Transforms 830

 L.6 Supplemental Lemmas, Theorems, and Proofs 836

 L.7 More Examples of Laplace Transform Solutions 840

 L.8 Proofs of Theorems Used in Distribution Theory 846

M Additional Details and Fortification for Chapter 13 851

 M.1 Method of Undetermined Coefficients for Finite
 Difference Approximation of Mixed Partial Derivative 851

 M.2 Finite Difference Formulas for 3D Cases 852

 M.3 Finite Difference Solutions of Linear Hyperbolic Equations 855

 M.4 Alternating Direction Implicit (ADI) Schemes 863

N Additional Details and Fortification for Chapter 14 867

 N.1 Convex Hull Algorithm 867

 N.2 Stabilization via Streamline-Upwind Petrov-Galerkin (SUPG) 870