

Structural and Failure Mechanics of Sandwich Composites

SOLID MECHANICS AND ITS APPLICATIONS

Volume 121

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 Department of Civil Engineering
 University of Waterloo
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Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

L.A. Carlsson • G.A. Kardomateas

Structural and Failure Mechanics of Sandwich Composites

L.A. Carlsson
Department of Mechanical Engineering
Florida Atlantic University
777 Glades Road
Boca Raton, Florida 33431
USA
carlsson@fau.edu

G.A. Kardomateas
School of Aerospace Engineering
Georgia Institute of Technology
270 Ferst Dr.
Atlanta, Georgia 30332
USA
george.kardomateas@aerospace.gatech.edu

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Preface

The need for a text focusing on the structural behavior and failure characteristics of sandwich materials and structures has emerged during recent years as a consequence of the increased acceptance of this structural concept in several weight critical applications. Much new research in this subject area has been reported and a new journal devoted to this specific topic has been successfully launched.¹ In addition to the more or less traditional topics, this book covers two topics of sandwich structures which seem not to have been systematically presented thus far in the existing book literature, namely the failure mechanisms (face/core debonding, core cracking, etc.) and the associated fracture mechanics treatment and the benchmark elasticity solutions for several classical mechanics problems (sandwich plate under transverse loading, sandwich shell under internal pressure, etc.).

The subject matter is laid out in the following way: Chapter 1 defines the concept of structural sandwich, highlights some important structural applications, and introduces some of the more common core concepts. By simple analysis of sandwich beams, important stiffnesses and strengths of the constituent faces and core are identified. Typical face and core material properties are listed. Chapter 2 describes test methods for determination of mechanical properties of face and core. Chapter 3 provides the framework for first-order shear deformation plate theory for analysis of stresses and displacement of flat sandwich panels and beams. This analysis is applied to determine bending deflections, buckling loads, and twisting response of sandwich beams and panels. In Chapter 4, analysis and description of the three-point test method for sandwich beams is outlined and examples are provided. Chapter 5 outlines elasticity theory for sandwich plates and shells, and presents application of this theory to cylindrical shell structures under

¹ *Journal of Sandwich Structures and Materials* (J.R. Vinson, Editor).

pressure and torsion loading. In Chapter 6, high-order core shear theories are presented. Chapters 7 and 8 are devoted to analysis and experimental test methods for some very important failure modes of sandwich panels, viz. global buckling and face wrinkling. Another important failure mode of sandwich panels is face/core separation, and analysis and testing related to this failure mode is presented in the four final chapters (Chapters 9 through 12). At the end of the book we have cited the literature which includes several already published texts on sandwich structures.

This book does not cover all aspects of sandwich structures. We have not considered dynamic loading of a sandwich body nor have we touched upon questions of plasticity and large deformations or the behavior at elevated temperatures. Further, important topics such as failure of sandwich structures under cyclic loads (fatigue), and localized failures due to concentrated static and dynamic loads, and failure of core joints are not addressed. We have rather focused on some topics that have received much attention in the research and engineering community. We would thus like to acknowledge at this point the need for attention to topics not included in this text.

This work would not have been possible without help from our friends and colleagues. Both of us very much value the long-term support and friendship of Dr. Yapa D.S. Rajapakse of the Office of Naval Research (ONR). His vision and directions for research have been beneficial, not only for our research but also for stimulating a transition from metal structures to much more efficient composite structures for naval applications. The first author (LAC) would like to express his gratitude to the late Dr. Alf de Ruvo for introducing me to sandwich structures as a graduate student and the many years of fruitful cooperation on sandwich structures at SCA Research. Support from DIAB through Chris Kilbourn and Baltek through Kurt Feichtinger in the form of supply of core materials for experimental testing has been most beneficial. Several of our colleagues have provided encouragement and critique, and we would like to acknowledge Profs George Simitses, Victor Birman, Charles W. Bert, Shuki Frostig, Dan Zenkert, Karl-Axel Olsson, Hassan Mahfuz, and Christian Berggreen. In particular, Professor Frostig provided valuable input on Chapter 6 (High-Order Theories). Moreover, our graduate students have made invaluable contributions. We owe thanks to Francis Aviles, Catherine Ferrie, Haiying Huang, Valeria LaSaponara, Andrew Layne, Renfu Li, Xiaoming Li, Tomas Nordstrand, Poorvi Patel, Srinivas Prasad, Mina Pelegri, Catherine Phan, Felipe Ramirez, Magnus Renman, Scott Sendlein, Vinod Vadakke, and Gilmer Viana. The 2004 summer visit at NAVSEA in Carderock by the first author enabled cooperation in research on sandwich specimens and panels with Douglas Loup, Diane

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We hope that this book will enable the specialists who encounter questions in the structural and failure mechanics of sandwich composites to have the basic material of this topic at their fingertips, including the recent advances, and to utilize it in their research or engineering practice.

Leif A. Carlsson, Boca Raton, Florida

George A. Kardomateas, Atlanta, Georgia

