
CONTENTS

Preface	viii
1. Finite-dimensional inverse theory	1
1.1 Introduction	1
1.2 The β -spiral	1
1.3 Wunsch's method: a first analysis	7
1.4 The general problem of linear algebra: the Moore-Penrose inverse and its singular-value decomposition	13
1.5 Wunsch's method revisited	18
1.6 Remarks	23
1.7 Water mass analysis	27
1.8 Steady tracers	37
1.9 A smoothed inversion	44
2. The smoothing of observations	47
2.1 Introduction	47
2.2 Gauss-Markov smoothing	47
2.3 Smoothing splines	56
2.4 Wunsch's method as smoothing	65
3. Data assimilation	67
3.1 Introduction	67
3.2 Initialization: theory and examples	69
3.3 The Kalman filter	76
3.4 Remarks and exercises	80
3.5 Examples of Kalman filters	84
4. The spatial structure of the Kalman filter	90
4.1 Spatial structure	90
4.2 Measurement at a single position	94
4.3 Altimetry	104

5. Generalized inverses of dynamical models	112
5.1 Introduction	112
5.2 The field, the data, and the model	113
5.3 The generalized inverse: Euler-Lagrange equations	114
5.4 The representer solution	118
5.5 Geometrical formulation	121
5.6 Error estimates for the generalized inverse	124
5.7 The sweep algorithm	129
5.8 The method of Backus and Gilbert	135
5.9 Gradient descent	140
5.10 Remarks	145
5.11 The calculus of variations: a tutorial	148
6. Antenna analysis	151
6.1 Introduction	151
6.2 Canonical decomposition	152
6.3 Examples	157
7. Nonlinear quasi-geostrophic models	181
7.1 Introduction	181
7.2 A nonlinear quasi-geostrophic model	183
7.3 The generalized inverse	184
7.4 A naïve iteration scheme	187
7.5 A bounded iteration scheme	188
7.6 An illustration	193
8. Open-ocean modeling: quasi-geostrophy	203
8.1 Introduction	203
8.2 Classical boundary conditions of plane hydrodynamics	204
8.3 Charney-Fjörtoft-von Neumann boundary condition	207
8.4 Well-posedness	214
8.5 Discontinuities, wavefronts, the β -effect and characteristics	220
8.6 Fixes	230
8.7 Inverse formulation (I)	234
8.8 Inverse formulation (II)	239
8.9 Representer solutions	242
8.10 An atmospheric example	247
8.11 Transient tracers	255

Contents vii

9. Primitive-equation models	257
9.1 Introduction	257
9.2 Boundary conditions: shallow-water theory	259
9.3 Boundary conditions: stratified flow	267
9.4 Inverse formulation	275
10. Outstanding problems	283
10.1 The general circulation	283
10.2 Parameter estimation	291
10.3 Discontinuous fields and discontinuous dynamics	298
10.4 Computer architecture	304
10.5 Prior estimates	315
Bibliography	317
Subject index	341