

Contents

1	Introduction	1
1.1	Minicases and Exercises	1
2	The Linear Programming Problem	39
2.1	Exercises	40
3	Basic Concepts	47
3.1	Exercises	49
4	Five Preliminaries	55
4.1	Exercises	57
5	Simplex Algorithms	63
5.1	Exercises	68
6	Primal-Dual Pairs	93
6.1	Exercises	100
7	Analytical Geometry	125
7.1	Points, Lines, Subspaces	125
7.2	Polyhedra, Ideal Descriptions, Cones	127
7.2.1	Faces, Valid Equations, Affine Hulls	128
7.2.2	Facets, Minimal Complete Descriptions, Quasi-Uniqueness	129
7.2.3	Asymptotic Cones and Extreme Rays	130
7.2.4	Adjacency I, Extreme Rays of Polyhedra, Homogenization	130
7.3	Point Sets, Affine Transformations, Minimal Generators	131
7.3.1	Displaced Cones, Adjacency II, Images of Polyhedra	132
7.3.2	Carathéodory, Minkowski, Weyl	133
7.3.3	Minimal Generators, Canonical Generators, Quasi-Uniqueness	133
7.4	Double Description Algorithms	135
7.4.1	Correctness and Finiteness of the Algorithm	136
7.4.2	Geometry, Euclidean Reduction, Analysis	137
7.4.3	The Basis Algorithm and All-Integer Inversion	138
7.4.4	An All-Integer Algorithm for Double Description	139
7.5	Digital Sizes of Rational Polyhedra and Linear Optimization	140
7.5.1	Facet Complexity, Vertex Complexity, Complexity of Inversion	141
7.5.2	Polyhedra and Related Polytopes for Linear Optimization	142
7.5.3	Feasibility, Binary Search, Linear Optimization	142
7.5.4	Perturbation, Uniqueness, Separation	144
7.6	Geometry and Complexity of Simplex Algorithms	146
7.6.1	Pivot Column Choice, Simplex Paths, Big M Revisited	147
7.6.2	Gaussian Elimination, Fill-In, Scaling	148

7.6.3	Iterative Step I, Pivot Choice, Cholesky Factorization	149
7.6.4	Cross Multiplication, Iterative Step II, Integer Factorization	150
7.6.5	Division Free Gaussian Elimination and Cramer's Rule	151
7.7	Circles, Spheres, Ellipsoids	153
7.8	Exercises	156
8	Projective Algorithms	201
8.1	A Basic Algorithm	203
8.1.1	The Solution of the Approximate Problem	203
8.1.2	Convergence of the Approximate Iterates	205
8.1.3	Correctness, Finiteness, Initialization	206
8.2	Analysis, Algebra, Geometry	207
8.2.1	Solution to the Problem in the Original Space	207
8.2.2	The Solution in the Transformed Space	209
8.2.3	Geometric Interpretations and Properties	211
8.2.4	Extending the Exact Solution and Proofs	214
8.2.5	Examples of Projective Images	215
8.3	The Cross Ratio	215
8.4	Reflection on a Circle and Sandwiching	218
8.4.1	The Iterative Step	220
8.5	A Projective Algorithm	221
8.6	Centers, Barriers, Newton Steps	223
8.6.1	A Method of Centers	224
8.6.2	The Logarithmic Barrier Function	226
8.6.3	A Newtonian Algorithm	228
8.7	Exercises	230
9	Ellipsoid Algorithms	263
9.1	Matrix Norms, Approximate Inverses, Matrix Inequalities	265
9.2	Ellipsoid "Halving" in Approximate Arithmetic	266
9.3	Polynomial-Time Algorithms for Linear Programming	269
9.4	Deep Cuts, Sliding Objective, Large Steps, Line Search	272
9.4.1	Linear Programming the Ellipsoidal Way: Two Examples	274
9.4.2	Correctness and Finiteness of the DCS Ellipsoid Algorithm	277
9.5	Optimal Separators, Most Violated Separators, Separation	278
9.6	ε -Solidification of Flats, Polytopal Norms, Rounding	280
9.6.1	Rational Rounding and Continued Fractions	282
9.7	Optimization and Separation	285
9.7.1	ε -Optimal Sets and ε -Optimal Solutions	287
9.7.2	Finding Direction Vectors in the Asymptotic Cone	287
9.7.3	A CCS Ellipsoid Algorithm	288
9.7.4	Linear Optimization and Polyhedral Separation	289
9.8	Exercises	293

10 Combinatorial Optimization: An Introduction	323
10.1 The Berlin Airlift Model Revisited	323
10.2 Complete Formulations and Their Implications	327
10.3 Extremal Characterizations of Ideal Formulations	331
10.4 Polyhedra with the Integrality Property	334
10.5 Exercises	336
 Appendices	
A Short-Term Financial Management	359
A.1 Solution to the Cash Management Case	362
B Operations Management in a Refinery	371
B.1 Steam Production in a Refinery	371
B.2 The Optimization Problem	374
B.3 Technological Constraints, Profits and Costs	378
B.4 Formulation of the Problem	380
B.5 Solution to the Refinery Case	381
C Automatized Production: PCBs and Ulysses' Problem	399
C.1 Solutions to Ulysses' Problem	411
Bibliography	431
Index	445

