

Quantum Field Theory II: Quantum Electrodynamics

A Bridge between Mathematicians and Physicists

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