

Contents

Preface	x
I. Review of Relevant Continuum Mechanics	1
<i>Anne M. Robertson</i>	
1. Kinematics	3
1.1. Description of motion of material points in a body	3
1.2. Referential and spatial descriptions	4
1.3. Deformation gradient and measures of stretch and strain	5
1.4. Velocity gradient and the rate of deformation tensor	7
1.5. Special motions	8
2. Governing equations	9
2.1. The transport theorem	9
2.2. Conservation of mass	10
2.3. Balance of linear momentum	11
2.4. Balance of angular momentum	13
2.5. Mechanical energy equation	14
2.6. Balance of energy	14
2.7. Restrictions on constitutive equations	15
3. Nonlinear viscous fluids	18
3.1. Restrictions due to invariance requirements	19
3.2. Restrictions on the Reiner-Rivlin equation due to behavior of real fluids	19
3.3. Restrictions on generalized Newtonian fluids due to thermodynamic considerations	20
3.4. Examples of generalized Newtonian fluids	20
4. Yield stress “fluids”	21
4.1. Bingham model	22
4.2. Modified Bingham model	24
4.3. Herschel-Bulkley model	24
4.4. Casson model	25
5. Viscoelastic fluids	25
5.1. Simple fluids	26
5.2. Approximations for simple fluids with fading memory	31
5.3. Finite viscoelastic models	34
6. Thixotropic fluids	36
7. Rheometrical flows	38
7.1. Viscometric flows	38
7.2. Periodic flows	45
7.3. Non-periodic unsteady flows	51
8. Rheometers	53
8.1. Couette rheometer	54
8.2. Cone and plate rheometer	54

8.3. Capillary rheometer	55
9. Nonlinear elastic solids	55
9.1. Introduction to hyperelastic materials	56
9.2. Invariance restrictions	57
9.3. Example: incompressible, isotropic hyperelastic materials	58
References	59
II. Hemorheology	63
<i>Anne M. Robertson, Adélia Sequeira and Marina V. Kameneva</i>	
1. Blood components	66
1.1. Plasma	67
1.2. Red blood cells (Erythrocytes)	67
1.3. White blood cells (Leukocytes)	68
1.4. Platelets (Thrombocytes)	69
2. Relevant parameters for flow in the human cardiovascular system	69
3. Multiphase behavior of blood in shear flows	72
3.1. Low shear behavior: aggregation and disaggregation of erythrocytes	73
3.2. High shear rate behavior: deformation, tumbling and realignment of erythrocytes	75
3.3. Spatial distribution of erythrocytes in shear flows	76
3.4. Outcomes of non-homogeneous distribution of erythrocytes	77
4. Platelet activation and blood coagulation	78
4.1. Vasoconstriction	78
4.2. Primary hemostasis	78
4.3. Secondary hemostasis or clot formation	79
4.4. Wall repair and clot dissolution	79
4.5. Models of activation and blood coagulation	79
5. Special considerations in rheometry of blood	80
5.1. Inhomogeneous distribution of particles	80
5.2. Thixotropy	82
5.3. Biochemical effects	82
5.4. Other considerations	82
6. Viscosity of whole blood	83
6.1. Nomenclature for blood viscosity	83
6.2. Experimental data for whole blood viscosity	84
6.3. Significance of shear thinning in the circulatory system	86
6.4. Viscosity models for blood	89
6.5. Dependence of blood viscosity on factors other than shear rate	90
7. Yield stress behavior of blood	96
7.1. Yield stress data for blood	97
7.2. Yield stress constitutive models for blood	99

8.	Viscoelasticity of blood	100
8.1.	Measurements of blood viscoelasticity	100
8.2.	Dependence of viscoelasticity on other factors	105
8.3.	Significance of viscoelasticity in the circulatory system	107
8.4.	Viscoelastic constitutive models	107
9.	Disease states and mechanical properties of blood	108
9.1.	Increased hematocrit	108
9.2.	Plasma hyperviscosity	109
9.3.	Hyperaggregation of red blood cells	109
9.4.	Decreased RBC deformability	109
10.	Gender and the mechanical properties of blood	110
	References	112

III. Mathematical Problems in Classical and Non-Newtonian

Fluid Mechanics

121

Giovanni P. Galdi

1.	Problems in the pipe flow of a Navier–Stokes liquid	125
1.1.	Fully developed flows	125
1.2.	The entry flow problem	142
1.3.	Mathematical modeling of a piping system. Unbounded domain approach	148
1.4.	Mathematical modeling of a piping system. Bounded domain approach with “do-nothing” boundary conditions	171
2.	Problems in non-Newtonian fluid mechanics	193
2.1.	Why non-Newtonian models?	193
2.2.	Problems related to generalized Newtonian models	196
2.3.	Problems related to viscoelastic liquid models	220
2.4.	Some results in the mathematical theory of second-order fluids	226
2.5.	Some results in the mathematical theory of Oldroyd-B fluids and related models	232
3.	Problems in liquid-particle interaction	236
3.1.	Sedimentation of symmetric particles in viscoelastic liquid	238
3.2.	Shape-tilting phenomenon	245
3.3.	Motion of a disk in the shear flow of a liquid in a horizontal channel	247
	References	266

IV. Methods for Numerical Flow Simulation

275

Rolf Rannacher

1.	Finite-element methods for the simulation of viscous flow	276
1.1.	The Navier–Stokes equations	276
1.2.	Discretization of space	278
1.3.	The stationary algebraic problems	284
1.4.	Discretization of time	284

1.5.	The quasi-stationary algebraic problems	288
2.	Numerical simulation of pipe flow	289
2.1.	Variational ‘open’ boundary conditions	289
2.2.	Problems with the ‘do-nothing’ boundary condition	292
2.3.	The problem of well-posedness	293
2.4.	The closure problem	294
3.	Mesh adaptation and model calibration	295
3.1.	Principles of a-posteriori error estimation	296
3.2.	The dual weighted residual (DWR) method	298
3.3.	Model problems and practical aspects	304
3.4.	Hyperbolic model case: transport problem	310
3.5.	Application to flow models	315
3.6.	Application in optimal flow control	320
3.7.	Application in hydrodynamic stability analysis	321
3.8.	Calibration of flow models	323
3.9.	Current work and further development	328
	References	328

V. Numerics of Fluid-Structure Interaction **333**

Sebastian Bönnisch, Thomas Dunne and Rolf Rannacher

1.	Fluid-‘single rigid body’ interaction	334
1.1.	Model setup in the body frame	335
1.2.	The stationary free-fall problem	337
1.3.	Numerical approximation	339
1.4.	The issue of domain truncation	343
1.5.	Toward economical meshes	344
1.6.	Hydrodynamic stability	350
1.7.	Dynamics of non-stationary free fall	350
1.8.	Open problems and further development	351
2.	Fluid-‘many rigid bodies-wall’ interaction	352
2.1.	The stress-DLM method	352
2.2.	The fractional-step scheme	353
2.3.	Open problems and further development	356
3.	Fluid-‘elastic structure’ interaction	357
3.1.	Solution methods for FSI problems	357
3.2.	Variational formulation	358
3.3.	Numerical approximation	363
3.4.	Mesh adaptation	365
3.5.	Stationary test case ‘Elastic Flow Cavity’	369
3.6.	Non-stationary test case ‘Vibrating Thin Plate’	371
3.7.	Open problems and further development	373
	References	375

VI. Numerical Techniques for Multiphase Flow with Liquid-Solid Interaction	379
<i>Stefan Turek and Jaroslav Horn</i>	
1. Numerical methods for incompressible flow	380
1.1. Introduction	380
1.2. Discretization of the Navier–Stokes equations in time	382
1.3. Discretization of the Navier–Stokes equations in space	387
1.4. Pressure Schur complement solvers	394
1.5. Global MPSC approach	397
1.6. Local MPSC approach	401
1.7. Multilevel solution strategy	403
1.8. Coupling with scalar equations	405
1.9. Coupling with $k - \varepsilon$ turbulence model	408
1.10. Adaptive time-step control	413
1.11. Some numerical examples	415
1.12. Application to more complex flow models	418
1.13. Conclusions	429
2. FSI for fluid – elastic solid configurations	431
2.1. Overview	431
2.2. Continuum description	433
2.3. Fluid structure interaction problem formulation	438
2.4. Applications	447
2.5. FSI benchmark	451
2.6. Conclusion	454
3. Numerical techniques for fluid-rigid solid configurations	457
3.1. Introduction	457
3.2. Description of the physical models	460
3.3. Moving-mesh method	462
3.4. Numerical method	464
3.5. Numerical results	471
3.6. Conclusions	485
References	492