

Springer Finance

Interest Rate Models - Theory and Practice

With Smile, Inflation and Credit

Bearbeitet von
Damiano Brigo, Fabio Mercurio

Neuausgabe 2007. Buch. LVI, 982 S. Hardcover

ISBN 978 3 540 22149 4

Format (B x L): 15,5 x 23,5 cm

Gewicht: 920 g

[Weitere Fachgebiete > Mathematik > Stochastik > Wahrscheinlichkeitsrechnung](#)

Zu [Leseprobe](#)

schnell und portofrei erhältlich bei


DIE FACHBUCHHANDLUNG

Die Online-Fachbuchhandlung beck-shop.de ist spezialisiert auf Fachbücher, insbesondere Recht, Steuern und Wirtschaft. Im Sortiment finden Sie alle Medien (Bücher, Zeitschriften, CDs, eBooks, etc.) aller Verlage. Ergänzt wird das Programm durch Services wie Neuerscheinungsdienst oder Zusammenstellungen von Büchern zu Sonderpreisen. Der Shop führt mehr als 8 Millionen Produkte.

Contents

Preface	VII
Motivation	VII
Aims, Readership and Book Structure	XII
Final Word and Acknowledgments	XIV
Description of Contents by Chapter	XIX
Abbreviations and Notation	XXXV

Part I. BASIC DEFINITIONS AND NO ARBITRAGE

1. Definitions and Notation	1
1.1 The Bank Account and the Short Rate	2
1.2 Zero-Coupon Bonds and Spot Interest Rates	4
1.3 Fundamental Interest-Rate Curves	9
1.4 Forward Rates	11
1.5 Interest-Rate Swaps and Forward Swap Rates	13
1.6 Interest-Rate Caps/Floors and Swaptions	16
2. No-Arbitrage Pricing and Numeraire Change	23
2.1 No-Arbitrage in Continuous Time	24
2.2 The Change-of-Numeraire Technique	26
2.3 A Change of Numeraire Toolkit (Brigo & Mercurio 2001c)	28
2.3.1 A helpful notation: “DC”	35
2.4 The Choice of a Convenient Numeraire	37
2.5 The Forward Measure	38
2.6 The Fundamental Pricing Formulas	39
2.6.1 The Pricing of Caps and Floors	40
2.7 Pricing Claims with Deferred Payoffs	42
2.8 Pricing Claims with Multiple Payoffs	42
2.9 Foreign Markets and Numeraire Change	44

Part II. FROM SHORT RATE MODELS TO HJM

3. One-factor short-rate models	51
3.1 Introduction and Guided Tour	51
3.2 Classical Time-Homogeneous Short-Rate Models	57
3.2.1 The Vasicek Model	58
3.2.2 The Dothan Model	62
3.2.3 The Cox, Ingersoll and Ross (CIR) Model	64
3.2.4 Affine Term-Structure Models	68
3.2.5 The Exponential-Vasicek (EV) Model	70
3.3 The Hull-White Extended Vasicek Model	71
3.3.1 The Short-Rate Dynamics	72
3.3.2 Bond and Option Pricing	75
3.3.3 The Construction of a Trinomial Tree	78
3.4 Possible Extensions of the CIR Model	80
3.5 The Black-Karasinski Model	82
3.5.1 The Short-Rate Dynamics	83
3.5.2 The Construction of a Trinomial Tree	85
3.6 Volatility Structures in One-Factor Short-Rate Models	86
3.7 Humped-Volatility Short-Rate Models	92
3.8 A General Deterministic-Shift Extension	95
3.8.1 The Basic Assumptions	96
3.8.2 Fitting the Initial Term Structure of Interest Rates ...	97
3.8.3 Explicit Formulas for European Options	99
3.8.4 The Vasicek Case	100
3.9 The CIR++ Model	102
3.9.1 The Construction of a Trinomial Tree	105
3.9.2 Early Exercise Pricing via Dynamic Programming	106
3.9.3 The Positivity of Rates and Fitting Quality	106
3.9.4 Monte Carlo Simulation	109
3.9.5 Jump Diffusion CIR and CIR++ models (JCIR, JCIR++)	109
3.10 Deterministic-Shift Extension of Lognormal Models	110
3.11 Some Further Remarks on Derivatives Pricing	112
3.11.1 Pricing European Options on a Coupon-Bearing Bond	112
3.11.2 The Monte Carlo Simulation	114
3.11.3 Pricing Early-Exercise Derivatives with a Tree	116
3.11.4 A Fundamental Case of Early Exercise: Bermudan- Style Swaptions.	121
3.12 Implied Cap Volatility Curves	124
3.12.1 The Black and Karasinski Model	125
3.12.2 The CIR++ Model	126
3.12.3 The Extended Exponential-Vasicek Model	128
3.13 Implied Swaption Volatility Surfaces	129
3.13.1 The Black and Karasinski Model	130

3.13.2	The Extended Exponential-Vasicek Model	131
3.14	An Example of Calibration to Real-Market Data	132
4.	Two-Factor Short-Rate Models	137
4.1	Introduction and Motivation	137
4.2	The Two-Additive-Factor Gaussian Model G2++	142
4.2.1	The Short-Rate Dynamics	143
4.2.2	The Pricing of a Zero-Coupon Bond	144
4.2.3	Volatility and Correlation Structures in Two-Factor Models	148
4.2.4	The Pricing of a European Option on a Zero-Coupon Bond	153
4.2.5	The Analogy with the Hull-White Two-Factor Model .	159
4.2.6	The Construction of an Approximating Binomial Tree.	162
4.2.7	Examples of Calibration to Real-Market Data	166
4.3	The Two-Additive-Factor Extended CIR/LS Model CIR2++	175
4.3.1	The Basic Two-Factor CIR2 Model	176
4.3.2	Relationship with the Longstaff and Schwartz Model (LS)	177
4.3.3	Forward-Measure Dynamics and Option Pricing for CIR2	178
4.3.4	The CIR2++ Model and Option Pricing	179
5.	The Heath-Jarrow-Morton (HJM) Framework	183
5.1	The HJM Forward-Rate Dynamics	185
5.2	Markovianity of the Short-Rate Process	186
5.3	The Ritchken and Sankarasubramanian Framework	187
5.4	The Mercurio and Moraleda Model	191

Part III. MARKET MODELS

6.	The LIBOR and Swap Market Models (LFM and LSM) . .	195
6.1	Introduction	195
6.2	Market Models: a Guided Tour	196
6.3	The Lognormal Forward-LIBOR Model (LFM)	207
6.3.1	Some Specifications of the Instantaneous Volatility of Forward Rates	210
6.3.2	Forward-Rate Dynamics under Different Numeraires . .	213
6.4	Calibration of the LFM to Caps and Floors Prices	220
6.4.1	Piecewise-Constant Instantaneous-Volatility Structures	223
6.4.2	Parametric Volatility Structures	224
6.4.3	Cap Quotes in the Market	225
6.5	The Term Structure of Volatility	226
6.5.1	Piecewise-Constant Instantaneous Volatility Structures	228

6.5.2	Parametric Volatility Structures	231
6.6	Instantaneous Correlation and Terminal Correlation	234
6.7	Swaptions and the Lognormal Forward-Swap Model (LSM)	237
6.7.1	Swaptions Hedging	241
6.7.2	Cash-Settled Swaptions	243
6.8	Incompatibility between the LFM and the LSM	244
6.9	The Structure of Instantaneous Correlations	246
6.9.1	Some convenient full rank parameterizations	248
6.9.2	Reduced-rank formulations: Rebonato's angles and eigenvalues zeroing	250
6.9.3	Reducing the angles	259
6.10	Monte Carlo Pricing of Swaptions with the LFM	264
6.11	Monte Carlo Standard Error	266
6.12	Monte Carlo Variance Reduction: Control Variate Estimator	269
6.13	Rank-One Analytical Swaption Prices	271
6.14	Rank- r Analytical Swaption Prices	277
6.15	A Simpler LFM Formula for Swaptions Volatilities	281
6.16	A Formula for Terminal Correlations of Forward Rates	284
6.17	Calibration to Swaptions Prices	287
6.18	Instantaneous Correlations: Inputs (Historical Estimation) or Outputs (Fitting Parameters)?	290
6.19	The exogenous correlation matrix	291
6.19.1	Historical Estimation	292
6.19.2	Pivot matrices	295
6.20	Connecting Caplet and $S \times 1$ -Swaption Volatilities	300
6.21	Forward and Spot Rates over Non-Standard Periods	307
6.21.1	Drift Interpolation	308
6.21.2	The Bridging Technique	310
7.	Cases of Calibration of the LIBOR Market Model	313
7.1	Inputs for the First Cases	315
7.2	Joint Calibration with Piecewise-Constant Volatilities as in TABLE 5	315
7.3	Joint Calibration with Parameterized Volatilities as in Formulation 7	319
7.4	Exact Swaptions "Cascade" Calibration with Volatilities as in TABLE 1	322
7.4.1	Some Numerical Results	330
7.5	A Pause for Thought	337
7.5.1	First summary	337
7.5.2	An automatic fast analytical calibration of LFM to swaptions. Motivations and plan	338
7.6	Further Numerical Studies on the Cascade Calibration Algorithm	340

7.6.1	Cascade Calibration under Various Correlations and Ranks	342
7.6.2	Cascade Calibration Diagnostics: Terminal Correlation and Evolution of Volatilities	346
7.6.3	The interpolation for the swaption matrix and its impact on the CCA	349
7.7	Empirically efficient Cascade Calibration	351
7.7.1	CCA with Endogenous Interpolation and Based Only on Pure Market Data	352
7.7.2	Financial Diagnostics of the RCCAEI test results	359
7.7.3	Endogenous Cascade Interpolation for missing swaptions volatilities quotes	364
7.7.4	A first partial check on the calibrated σ parameters stability	364
7.8	Reliability: Monte Carlo tests	366
7.9	Cascade Calibration and the cap market	369
7.10	Cascade Calibration: Conclusions	372
8.	Monte Carlo Tests for LFM Analytical Approximations	377
8.1	First Part. Tests Based on the Kullback Leibler Information (KLI)	378
8.1.1	Distance between distributions: The Kullback Leibler information	378
8.1.2	Distance of the LFM swap rate from the lognormal family of distributions	381
8.1.3	Monte Carlo tests for measuring KLI	384
8.1.4	Conclusions on the KLI-based approach	391
8.2	Second Part: Classical Tests	392
8.3	The “Testing Plan” for Volatilities	392
8.4	Test Results for Volatilities	396
8.4.1	Case (1): Constant Instantaneous Volatilities	396
8.4.2	Case (2): Volatilities as Functions of Time to Maturity	401
8.4.3	Case (3): Humped and Maturity-Adjusted Instantaneous Volatilities Depending only on Time to Maturity	410
8.5	The “Testing Plan” for Terminal Correlations	421
8.6	Test Results for Terminal Correlations	427
8.6.1	Case (i): Humped and Maturity-Adjusted Instantaneous Volatilities Depending only on Time to Maturity, Typical Rank-Two Correlations	427
8.6.2	Case (ii): Constant Instantaneous Volatilities, Typical Rank-Two Correlations	430
8.6.3	Case (iii): Humped and Maturity-Adjusted Instantaneous Volatilities Depending only on Time to Maturity, Some Negative Rank-Two Correlations	432

8.6.4	Case (iv): Constant Instantaneous Volatilities, Some Negative Rank-Two Correlations.....	438
8.6.5	Case (v): Constant Instantaneous Volatilities, Perfect Correlations, Upwardly Shifted Φ 's	439
8.7	Test Results: Stylized Conclusions	442

Part IV. THE VOLATILITY SMILE

9.	Including the Smile in the LFM	447
9.1	A Mini-tour on the Smile Problem.....	447
9.2	Modeling the Smile	450
10.	Local-Volatility Models	453
10.1	The Shifted-Lognormal Model	454
10.2	The Constant Elasticity of Variance Model	456
10.3	A Class of Analytically-Tractable Models	459
10.4	A Lognormal-Mixture (LM) Model	463
10.5	Forward Rates Dynamics under Different Measures	467
10.5.1	Decorrelation Between Underlying and Volatility	469
10.6	Shifting the LM Dynamics.....	469
10.7	A Lognormal-Mixture with Different Means (LMDM)	471
10.8	The Case of Hyperbolic-Sine Processes	473
10.9	Testing the Above Mixture-Models on Market Data	475
10.10	A Second General Class	478
10.11	A Particular Case: a Mixture of GBM's	483
10.12	An Extension of the GBM Mixture Model Allowing for Implied Volatility Skews	486
10.13	A General Dynamics à la Dupire (1994)	489
11.	Stochastic-Volatility Models	495
11.1	The Andersen and Brotherton-Ratcliffe (2001) Model	497
11.2	The Wu and Zhang (2002) Model.....	501
11.3	The Piterbarg (2003) Model	504
11.4	The Hagan, Kumar, Lesniewski and Woodward (2002) Model	508
11.5	The Joshi and Rebonato (2003) Model	513
12.	Uncertain-Parameter Models	517
12.1	The Shifted-Lognormal Model with Uncertain Parameters (SLMUP)	519
12.1.1	Relationship with the Lognormal-Mixture LVM	520
12.2	Calibration to Caplets	520
12.3	Swaption Pricing	522
12.4	Monte-Carlo Swaption Pricing	524
12.5	Calibration to Swaptions	526

12.6	Calibration to Market Data	528
12.7	Testing the Approximation for Swaptions Prices	530
12.8	Further Model Implications	535
12.9	Joint Calibration to Caps and Swaptions	539

Part V. EXAMPLES OF MARKET PAYOFFS

13.	Pricing Derivatives on a Single Interest-Rate Curve	547
13.1	In-Arrears Swaps	548
13.2	In-Arrears Caps	550
13.2.1	A First Analytical Formula (LFM)	550
13.2.2	A Second Analytical Formula (G2++)	551
13.3	Autocaps	551
13.4	Caps with Deferred Caplets	552
13.4.1	A First Analytical Formula (LFM)	553
13.4.2	A Second Analytical Formula (G2++)	553
13.5	Ratchet Caps and Floors	554
13.5.1	Analytical Approximation for Ratchet Caps with the LFM	555
13.6	Ratchets (One-Way Floaters)	556
13.7	Constant-Maturity Swaps (CMS)	557
13.7.1	CMS with the LFM	557
13.7.2	CMS with the G2++ Model	559
13.8	The Convexity Adjustment and Applications to CMS	559
13.8.1	Natural and Unnatural Time Lags	559
13.8.2	The Convexity-Adjustment Technique	561
13.8.3	Deducing a Simple Lognormal Dynamics from the Adjustment	565
13.8.4	Application to CMS	565
13.8.5	Forward Rate Resetting Unnaturally and Average-Rate Swaps	566
13.9	Average Rate Caps	568
13.10	Captions and Floortions	570
13.11	Zero-Coupon Swaptions	571
13.12	Eurodollar Futures	575
13.12.1	The Shifted Two-Factor Vasicek G2++ Model	576
13.12.2	Eurodollar Futures with the LFM	577
13.13	LFM Pricing with “In-Between” Spot Rates	578
13.13.1	Accrual Swaps	579
13.13.2	Trigger Swaps	582
13.14	LFM Pricing with Early Exercise and Possible Path Dependence	584
13.15	LFM: Pricing Bermudan Swaptions	588
13.15.1	Least Squared Monte Carlo Approach	589
13.15.2	Carr and Yang’s Approach	591

13.15.3 Andersen's Approach	592
13.15.4 Numerical Example	595
13.16 New Generation of Contracts	601
13.16.1 Target Redemption Notes	602
13.16.2 CMS Spread Options	603
14. Pricing Derivatives on Two Interest-Rate Curves	607
14.1 The Attractive Features of G2++ for Multi-Curve Payoffs ...	608
14.1.1 The Model	608
14.1.2 Interaction Between Models of the Two Curves "1" and "2"	610
14.1.3 The Two-Models Dynamics under a Unique Conve- nient Forward Measure	611
14.2 Quanto Constant-Maturity Swaps	613
14.2.1 Quanto CMS: The Contract	613
14.2.2 Quanto CMS: The G2++ Model	615
14.2.3 Quanto CMS: Quanto Adjustment	621
14.3 Differential Swaps	623
14.3.1 The Contract	623
14.3.2 Differential Swaps with the G2++ Model	624
14.3.3 A Market-Like Formula	626
14.4 Market Formulas for Basic Quanto Derivatives	626
14.4.1 The Pricing of Quanto Caplets/Floorlets	627
14.4.2 The Pricing of Quanto Caps/Floors	628
14.4.3 The Pricing of Differential Swaps	629
14.4.4 The Pricing of Quanto Swaptions	630
14.5 Pricing of Options on two Currency LIBOR Rates	633
14.5.1 Spread Options	635
14.5.2 Options on the Product	637
14.5.3 Trigger Swaps	638
14.5.4 Dealing with Multiple Dates	639

Part VI. INFLATION

15. Pricing of Inflation-Indexed Derivatives	643
15.1 The Foreign-Currency Analogy	644
15.2 Definitions and Notation	645
15.3 The JY Model	646
16. Inflation-Indexed Swaps	649
16.1 Pricing of a ZCIIS	649
16.2 Pricing of a YYIIS	651
16.3 Pricing of a YYIIS with the JY Model	652
16.4 Pricing of a YYIIS with a First Market Model	654

16.5 Pricing of a YYIS with a Second Market Model	657
17. Inflation-Indexed Caplets/Floorlets	661
17.1 Pricing with the JY Model	661
17.2 Pricing with the Second Market Model	663
17.3 Inflation-Indexed Caps	665
18. Calibration to market data	669
19. Introducing Stochastic Volatility	673
19.1 Modeling Forward CPI's with Stochastic Volatility	674
19.2 Pricing Formulae	676
19.2.1 Exact Solution for the Uncorrelated Case	677
19.2.2 Approximated Dynamics for Non-zero Correlations	680
19.3 Example of Calibration	681
20. Pricing Hybrids with an Inflation Component	689
20.1 A Simple Hybrid Payoff	689

Part VII. CREDIT

21. Introduction and Pricing under Counterparty Risk	695
21.1 Introduction and Guided Tour	696
21.1.1 Reduced form (Intensity) models	697
21.1.2 CDS Options Market Models	699
21.1.3 Firm Value (or Structural) Models	702
21.1.4 Further Models	704
21.1.5 The Multi-name picture: FtD, CDO and Copula Functions	705
21.1.6 First to Default (FtD) Basket	705
21.1.7 Collateralized Debt Obligation (CDO) Tranches	707
21.1.8 Where can we introduce dependence?	708
21.1.9 Copula Functions	710
21.1.10 Dynamic Loss models	718
21.1.11 What data are available in the market?	719
21.2 Defaultable (corporate) zero coupon bonds	723
21.2.1 Defaultable (corporate) coupon bonds	724
21.3 Credit Default Swaps and Defaultable Floaters	724
21.3.1 CDS payoffs: Different Formulations	725
21.3.2 CDS pricing formulas	727
21.3.3 Changing filtration: \mathcal{F}_t without default VS complete \mathcal{G}_t	728
21.3.4 CDS forward rates: The first definition	730

21.3.5	Market quotes, model independent implied survival probabilities and implied hazard functions	731
21.3.6	A simpler formula for calibrating intensity to a single CDS	735
21.3.7	Different Definitions of CDS Forward Rates and Analogies with the LIBOR and SWAP rates	737
21.3.8	Defaultable Floater and CDS	739
21.4	CDS Options and Callable Defaultable Floaters	743
21.5	Constant Maturity CDS	744
21.5.1	Some interesting Financial features of CMCDs	745
21.6	Interest-Rate Payoffs with Counterparty Risk	747
21.6.1	General Valuation of Counterparty Risk	748
21.6.2	Counterparty Risk in single Interest Rate Swaps (IRS)	750
22.	Intensity Models	757
22.1	Introduction and Chapter Description	757
22.2	Poisson processes	759
22.2.1	Time homogeneous Poisson processes	760
22.2.2	Time inhomogeneous Poisson Processes	761
22.2.3	Cox Processes	763
22.3	CDS Calibration and Implied Hazard Rates/ Intensities	764
22.4	Inducing dependence between Interest-rates and the default event	776
22.5	The Filtration Switching Formula: Pricing under partial information	777
22.6	Default Simulation in reduced form models	778
22.6.1	Standard error	781
22.6.2	Variance Reduction with Control Variate	783
22.7	Stochastic Intensity: The SSRD model	785
22.7.1	A two-factor shifted square-root diffusion model for intensity and interest rates (Brigo and Alfonsi (2003))	786
22.7.2	Calibrating the joint stochastic model to CDS: Separability	789
22.7.3	Discretization schemes for simulating (λ, r)	797
22.7.4	Study of the convergence of the discretization schemes for simulating CIR processes (Alfonsi (2005))	801
22.7.5	Gaussian dependence mapping: A tractable approximated SSRD	812
22.7.6	Numerical Tests: Gaussian Mapping and Correlation Impact	815
22.7.7	The impact of correlation on a few “test payoffs”	817
22.7.8	A pricing example: A Cancellable Structure	818
22.7.9	CDS Options and Jamshidian’s Decomposition	820
22.7.10	Bermudan CDS Options	830

22.8	Stochastic diffusion intensity is not enough: Adding jumps.	
	The JCIR(++) Model	830
22.8.1	The jump-diffusion CIR model (JCIR)	831
22.8.2	Bond (or Survival Probability) Formula.	832
22.8.3	Exact calibration of CDS: The JCIR++ model	833
22.8.4	Simulation.	833
22.8.5	Jamshidian's Decomposition.	834
22.8.6	Attaining high levels of CDS implied volatility	836
22.8.7	JCIR(++) models as a multi-name possibility.	837
22.9	Conclusions and further research	838
23.	CDS Options Market Models	841
23.1	CDS Options and Callable Defaultable Floaters	844
23.1.1	Once-callable defaultable floaters	846
23.2	A market formula for CDS options and callable defaultable floaters	847
23.2.1	Market formulas for CDS Options	847
23.2.2	Market Formula for callable DFRN	849
23.2.3	Examples of Implied Volatilities from the Market	852
23.3	Towards a Completely Specified Market Model	854
23.3.1	First Choice. One-period and two-period rates	855
23.3.2	Second Choice: Co-terminal and one-period CDS rates market model	860
23.3.3	Third choice. Approximation: One-period CDS rates dynamics	861
23.4	Hints at Smile Modeling.	863
23.5	Constant Maturity Credit Default Swaps (CMCDS) with the market model	864
23.5.1	CDS and Constant Maturity CDS	864
23.5.2	Proof of the main result	867
23.5.3	A few numerical examples	869

Part VIII. APPENDICES

A.	Other Interest-Rate Models	877
A.1	Brennan and Schwartz's Model.	877
A.2	Balduzzi, Das, Foresi and Sundaram's Model.	878
A.3	Flesaker and Hughston's Model	879
A.4	Rogers's Potential Approach	881
A.5	Markov Functional Models.	881

B. Pricing Equity Derivatives under Stochastic Rates	883
B.1 The Short Rate and Asset-Price Dynamics	883
B.1.1 The Dynamics under the Forward Measure	886
B.2 The Pricing of a European Option on the Given Asset	888
B.3 A More General Model	889
B.3.1 The Construction of an Approximating Tree for r	890
B.3.2 The Approximating Tree for S	892
B.3.3 The Two-Dimensional Tree	893
C. A Crash Intro to Stochastic Differential Equations and Poisson Processes	897
C.1 From Deterministic to Stochastic Differential Equations	897
C.2 Ito's Formula	904
C.3 Discretizing SDEs for Monte Carlo: Euler and Milstein Schemes	906
C.4 Examples	908
C.5 Two Important Theorems	910
C.6 A Crash Intro to Poisson Processes	913
C.6.1 Time inhomogeneous Poisson Processes	915
C.6.2 Doubly Stochastic Poisson Processes (or Cox Processes)	916
C.6.3 Compound Poisson processes	917
C.6.4 Jump-diffusion Processes	918
D. A Useful Calculation	919
E. A Second Useful Calculation	921
F. Approximating Diffusions with Trees	925
G. Trivia and Frequently Asked Questions	931
H. Talking to the Traders	935
References	951
Index	967