
Contents

Preface	V
Introduction	XI
1 Estimates for analytic functions bounded with respect to their real part	1
1.1 Introduction	1
1.2 Different proofs of the real-part theorem	2
1.3 Extremal values of the real part of the rotated Schwarz kernel	6
1.4 Upper estimate of $\Re\{e^{i\alpha}\Delta f\}$ by the supremum of $\Re\Delta f$	8
1.5 Two-sided estimates of $\Re\{e^{i\alpha}\Delta f\}$ by upper and lower bounds of $\Re\Delta f$	11
1.6 Inequalities for the modulus, real and imaginary parts	12
1.7 Variants and extensions	14
2 Estimates for analytic functions with respect to the L_p-norm of $\Re\Delta f$ on the circle	17
2.1 Introduction	17
2.2 Estimate of $ \Re\{e^{i\alpha}\Delta f\} $ by the L_p -norm of $\Re\Delta f$ on the circle. General case	19
2.3 The cases $p = 1$ and $p = 2$	23
2.4 The case $p = \infty$	25
2.5 Generalization of the Carathéodory and Plemelj inequality	30
2.6 Variants and extensions	33
3 Estimates for analytic functions by the best L_p-approximation of $\Re f$ on the circle	37
3.1 Introduction	37
3.2 Estimate of $ \Re\{e^{i\alpha}\Delta f\} $ by the L_p -norm of $\Re f - c$ on the circle. General case	39

3.3	The cases $p = 1$ and $p = 2$	42
3.4	The case $p = \infty$	43
3.5	Inequalities for the real and imaginary parts	49
3.6	Estimate for the oscillation of $\Re\{e^{i\alpha}f\}$ and its corollaries	51
3.7	Variants and extensions	53
4	Estimates for directional derivatives of harmonic functions	57
4.1	Introduction	57
4.2	Interior estimates for derivatives in a domain	58
4.3	Estimates for directional derivatives with constant direction...	61
4.4	Estimates for directional derivatives with varying direction ...	63
5	Estimates for derivatives of analytic functions	69
5.1	Introduction	69
5.2	Estimate for $ f^{(n)}(z) $ by $\ \Re\{f - \mathcal{P}_m\}\ _p$. General case	73
5.3	Estimate for $ f^{(n)}(0) $ by $\ \Re\{f - \mathcal{P}_m\}\ _p$	75
5.4	The case $p = 1$ and its corollaries	77
5.4.1	Explicit estimate in the case $p = 1$	77
5.4.2	Hadamard's real-part theorem for derivatives	78
5.4.3	Landau type inequality	81
5.4.4	Generalization of the Landau inequality	84
5.4.5	Generalization of the Carathéodory inequality	86
5.5	The case $p = 2$	87
5.6	The case $p = \infty$	92
5.7	Variants and extensions	94
6	Bohr's type real part estimates	99
6.1	Introduction	99
6.2	Estimate for the l_q -norm of the Taylor series remainder by $\ \Re f\ _1$	101
6.3	Other estimates for the l_q -norm of the Taylor series remainder	103
6.4	Bohr's type theorems	109
6.5	Variants and extensions	111
7	Estimates for the increment of derivatives of analytic functions	115
7.1	Introduction	115
7.2	Estimate for $ \Delta f^{(n)}(z) $ by $\ \Re\{f - \mathcal{P}_m\}\ _p$. General case	116
7.3	The case $p = 1$ and its corollaries	118
7.3.1	Explicit estimate in the case $p = 1$	118
7.3.2	Hadamard-Borel-Carathéodory type inequality for derivatives	119

7.3.3	Landau type inequalities	121
7.3.4	Carathéodory type inequality	124
7.4	The cases $p = 2$ and $p = \infty$	125
References		129
Index		135
List of Symbols		139

