

Extremal Combinatorics

With Applications in Computer Science

Bearbeitet von
Stasys Jukna

1. Auflage 2001. Buch. XVII, 375 S. Hardcover

ISBN 978 3 540 66313 3

Format (B x L): 15,5 x 23,5 cm

Gewicht: 706 g

[Weitere Fachgebiete > Mathematik > Stochastik > Wahrscheinlichkeitsrechnung](#)

Zu [Leseprobe](#)

schnell und portofrei erhältlich bei


DIE FACHBUCHHANDLUNG

Die Online-Fachbuchhandlung beck-shop.de ist spezialisiert auf Fachbücher, insbesondere Recht, Steuern und Wirtschaft. Im Sortiment finden Sie alle Medien (Bücher, Zeitschriften, CDs, eBooks, etc.) aller Verlage. Ergänzt wird das Programm durch Services wie Neuerscheinungsdienst oder Zusammenstellungen von Büchern zu Sonderpreisen. Der Shop führt mehr als 8 Millionen Produkte.

Contents

Preface	VII
Prolog: What this Book Is About	1
Notation	5
Part I. The Classics	
1. Counting	11
1.1 The binomial theorem	11
1.2 Selection with repetitions	13
1.3 Partitions	14
1.4 Double counting	14
1.5 The averaging principle	16
Exercises	19
2. Advanced Counting	23
2.1 Bounds on intersection size	23
2.2 Zarankiewicz's problem	25
2.3 Density of 0-1 matrices	27
Exercises	29
3. The Principle of Inclusion and Exclusion	32
3.1 The principle	32
3.2 The number of derangements	34
Exercises	35
4. The Pigeonhole Principle	37
4.1 Some quickies	37
4.2 The Erdős–Szekeres theorem	38
4.3 Mantel's theorem	40
4.4 Turán's theorem	41
4.5 Dirichlet's theorem	42
4.6 Swell-colored graphs	43

4.7	The weight shifting argument	45
4.8	Pigeonhole and resolution	47
4.8.1	Resolution refutation proofs	47
4.8.2	Haken's lower bound	48
	Exercises	51
5.	Systems of Distinct Representatives	55
5.1	The marriage theorem	55
5.2	Two applications	57
5.2.1	Latin rectangles	57
5.2.2	Decomposition of doubly stochastic matrices	58
5.3	Min-max theorems	59
5.4	Matchings in bipartite graphs	60
	Exercises	63
6.	Colorings	65
6.1	Property B	65
6.2	The averaging argument	67
6.2.1	Almost good colorings	67
6.2.2	The number of mixed triangles	68
6.3	Coloring the cube: the algorithmic aspect	70
	Exercises	71
Part II. Extremal Set Theory		
7.	Sunflowers	77
7.1	The sunflower lemma	77
7.2	Modifications	79
7.2.1	Relaxed core	79
7.2.2	Relaxed disjointness	80
7.3	Applications	81
7.3.1	The number of minterms	81
7.3.2	Small depth formulas	82
	Exercises	84
8.	Intersecting Families	87
8.1	The Erdős–Ko–Rado theorem	87
8.2	Finite ultrafilters	88
8.3	Maximal intersecting families	89
8.4	A Helly-type result	91
8.5	Intersecting systems	91
	Exercises	93

9. Chains and Antichains	95
9.1 Decomposition of posets	95
9.1.1 Symmetric chains	97
9.1.2 Application: the memory allocation problem	98
9.2 Antichains	99
9.2.1 Sperner's theorem	99
9.2.2 Bollobás's theorem	100
9.2.3 Strong systems of distinct representatives	103
9.2.4 Union-free families	104
Exercises	105
10. Blocking Sets and the Duality	107
10.1 Duality	107
10.2 The blocking number	109
10.3 Generalized Helly theorems	110
10.4 Decision trees	112
10.4.1 Depth versus certificate complexity	113
10.4.2 Block sensitivity	114
10.5 The switching lemma	115
10.6 Monotone circuits	119
10.6.1 The lower bounds criterion	120
10.6.2 Explicit lower bounds	123
Exercises	128
11. Density and Universality	131
11.1 Dense sets	131
11.2 Hereditary sets	132
11.3 Universal sets	134
11.3.1 Isolated neighbor condition	135
11.3.2 Paley graphs	136
11.4 Full graphs	138
Exercises	139
12. Witness Sets and Isolation	141
12.1 Bondy's theorem	141
12.2 Average witnesses	142
12.3 The isolation lemma	145
12.4 Isolation in politics: the dictator paradox	148
Exercises	150
13. Designs	151
13.1 Regularity	151
13.2 Finite linear spaces	153
13.3 Difference sets	154
13.4 Projective planes	155

13.4.1	The construction	156
13.4.2	Bruen's theorem	157
13.5	Resolvable designs	159
13.5.1	Affine planes	160
13.5.2	Blocking sets in affine planes	161
	Exercises	163

Part III. The Linear Algebra Method

14.	The Basic Method	167
14.1	The linear algebra background	167
14.2	Spaces of incidence vectors	170
14.2.1	Fisher's inequality	170
14.2.2	Inclusion matrices	171
14.2.3	Disjointness matrices	173
14.3	Spaces of polynomials	174
14.3.1	Two-distance sets	175
14.3.2	Sets with few intersection sizes	176
14.3.3	Constructive Ramsey graphs	177
14.3.4	Bollobás theorem – another proof	178
14.4	Combinatorics of linear spaces	179
14.4.1	Universal sets from linear codes	180
14.4.2	Short linear combinations	180
14.5	The flipping cards game	182
	Exercises	184
15.	Orthogonality and Rank Arguments	189
15.1	Orthogonality	189
15.1.1	Orthogonal coding	189
15.1.2	A bribery party	190
15.1.3	Hadamard matrices	192
15.2	Rank arguments	194
15.2.1	Balanced families	194
15.2.2	Lower bounds for boolean formulas	195
	Exercises	201
16.	Span Programs	203
16.1	The model	203
16.2	Span programs and switching networks	204
16.3	Monotone span programs	204
16.3.1	Threshold functions	205
16.3.2	Non-bipartite graphs	206
16.3.3	Odd factors	206
16.3.4	A lower bound for threshold functions	209

16.4 A general lower bound	210
16.5 Explicit self-avoiding families	212
Exercises	214

Part IV. The Probabilistic Method

17. Basic Tools	219
17.1 Probabilistic preliminaries	219
17.2 Elementary tools	222
17.3 Advanced tools	223
Exercises	225
18. Counting Sieve	227
18.1 Ramsey numbers	227
18.2 Van der Waerden's theorem	228
18.3 Tournaments	229
18.4 Property B revised	229
18.5 The existence of small universal sets	230
18.6 Cross-intersecting families	231
Exercises	234
19. The Lovász Sieve	235
19.1 The local lemma	235
19.2 Counting sieve for almost independence	237
19.3 Applications	238
19.3.1 Colorings	238
19.3.2 Hashing functions	241
Exercises	242
20. Linearity of Expectation	243
20.1 Hamilton paths in tournaments	243
20.2 Sum-free sets	244
20.3 Dominating sets	245
20.4 The independence number	245
20.5 Low degree polynomials	246
20.6 Maximum satisfiability	248
20.7 Hashing functions	249
20.8 Submodular complexity measures	251
20.9 Discrepancy	254
20.9.1 Example: matrix multiplication	257
Exercises	258

21. The Deletion Method	261
21.1 Ramsey numbers	261
21.2 Independent sets	262
21.3 Coloring large-girth graphs	263
21.4 Point sets without obtuse triangles	264
21.5 Covering designs	266
21.6 Affine cubes of integers	267
Exercises	270
22. The Second Moment Method	271
22.1 The method	271
22.2 Separators	272
22.3 Threshold for cliques	274
Exercises	276
23. The Entropy Function	277
23.1 Basic properties	277
23.2 Subadditivity	278
23.3 Combinatorial applications	280
Exercises	283
24. Random Walks	284
24.1 Satisfying assignments for 2-CNF	284
24.2 The best bet for simpletons	286
24.3 Small formulas for complicated functions	288
24.4 Random walks and search problems	292
24.4.1 Long words over a small alphabet	293
24.4.2 Short words over a large alphabet	294
Exercises	296
25. Randomized Algorithms	297
25.1 Zeroes of multivariate polynomials	297
25.2 Verifying the equality of long strings	300
25.3 The equivalence of branching programs	300
25.4 A min-cut algorithm	302
Exercises	304
26. Derandomization	305
26.1 The method of conditional probabilities	305
26.1.1 A general frame	306
26.1.2 Splitting graphs	307
26.1.3 Maximum satisfiability: the algorithmic aspect	308
26.2 The method of small sample spaces	310
26.3 Sum-free sets: the algorithmic aspect	314
Exercises	315

Part V. Fragments of Ramsey Theory

27. Ramsey's Theorem	319
27.1 Colorings and Ramsey numbers	319
27.2 Ramsey's theorem for graphs	320
27.3 Ramsey's theorem for sets	322
27.4 Schur's theorem	324
27.5 Geometric application: convex polygons	325
Exercises	325
28. Ramseyan Theorems for Numbers	327
28.1 Sum-free sets	327
28.2 Zero-sum sets	330
28.3 Szemerédi's cube lemma	332
Exercises	334
29. The Hales–Jewett Theorem	335
29.1 The theorem and its consequences	335
29.1.1 Van der Waerden's theorem	336
29.1.2 Gallai–Witt's Theorem	337
29.2 Shelah's proof of HJT	338
29.3 Application: multi-party games	341
29.3.1 Few players: the hyperplane problem	342
29.3.2 Many players: the matrix product problem	346
Exercises	347
Epilog: What Next	349
References	351
Name Index	365
Subject Index	369