

Contents

1	Introduction	1
1.1	Notation	1
1.2	Relations	2
1.3	Functions	5
1.4	Fuzzy Subsets	6
1.5	Semigroups	7
1.6	Codes	12
1.7	Finite-State Machines	20
1.8	Finite-State Automata	21
1.9	Languages and Grammars	25
1.10	Nondeterministic Finite-State Automata	30
1.11	Relationships Between Languages and Automata	34
2	Fuzzy Ideals	39
2.1	Introduction	39
2.2	Ideals in Semigroups	40
2.3	Fuzzy Ideals in Semigroups	40
2.4	Fuzzy Bi-ideals in Semigroups	43
2.5	Fuzzy Interior Ideals in Semigroups	45
2.6	Fuzzy Quasi-ideals in Semigroups	48
2.7	Fuzzy Generalized Bi-ideals in Semigroups	52
2.8	Fuzzy Ideals Generated by Fuzzy Subsets of Semigroups	53

3	Regular Semigroups	59
3.1	Regular Semigroups	59
3.2	Completely Regular Semigroups	76
3.3	Intra-regular Semigroups	79
3.4	Semisimple Semigroups	87
3.5	On Fuzzy Regular Subsemigroups of a Semigroup	92
3.6	Fuzzy Weakly Regular Subsemigroups	95
3.7	Fuzzy Completely Regular and Weakly Completely Regular Subsemigroups	97
3.8	Weakly Regular Semigroups	99
4	Semilattices of Groups	101
4.1	A Semilattice of Left (Right) Simple Semigroups	101
4.2	A Semilattice of Left (Right) Groups	105
4.3	A Semilattices of Groups	113
4.4	Fuzzy Normal Semigroups	119
4.5	Convexity and Green's Relations	124
4.6	The Compact Convex Set of Fuzzy Ideals	125
4.7	Fuzzy Ideals and Green's Relations	126
5	Fuzzy Congruences on Semigroups	131
5.1	Fuzzy Congruences on a Semigroup	131
5.2	Fuzzy Congruences on a Group	133
5.3	Fuzzy Factor Semigroups	139
5.4	Homomorphism Theorems	142
5.5	Idempotent-separating Fuzzy Congruences	144
5.6	Group Fuzzy Congruences	146
5.7	The Lattice of Fuzzy Congruence Relations on a Semigroup	147
5.8	Fuzzy Congruence Pairs of Inverse Semigroups	153
5.9	Fuzzy Rees Congruences on Semigroups	159
5.10	Additional Fuzzy Congruences on Semigroups	167
6	Fuzzy Congruences on T^*-pure Semigroups	169
6.1	T^* -pure Semigroups	169
6.2	Semilattice Fuzzy Congruences	175
6.3	Group Fuzzy Congruences	178
7	Prime Fuzzy Ideals	183
7.1	Preliminaries	183
7.2	Prime Fuzzy Ideals	185
7.3	Weakly Prime Fuzzy Ideals	188
7.4	Completely Prime and Weakly Completely Prime Fuzzy Ideals	189
7.5	Relationships	190
7.6	Types of Prime Fuzzy Left Ideals	191
7.7	Prime Fuzzy Left Ideals	193

7.8	Fuzzy m -systems and Quasi-prime Fuzzy Left Ideals	194
7.9	Weakly Quasi-prime Fuzzy Left Ideals	195
7.10	Fuzzy Ideals $i(f)$ and $I(f)$	198
7.11	Strongly Semisimple Semigroups	200
7.12	Fuzzy Multiplication Semigroups	203
7.13	Properties of Fuzzy Multiplication Semigroups	204
7.14	Fuzzy Ideal Extensions	209
7.15	Prime Fuzzy Ideals	214
8	Fuzzy Codes on Free Monoids	219
8.1	Fuzzy Codes	219
8.2	Prefix Codes	222
8.3	Maximal Fuzzy Prefix Codes	226
8.4	Algebraic Properties of Fuzzy Prefix Codes on a Free Monoid	229
8.5	Fuzzy Prefix Codes Related to Fuzzy Factor Theorems . . .	236
8.6	Equivalent Depictions of Fuzzy Codes	241
8.7	Fuzzy Codes and Fuzzy Submonoids	247
8.8	An Algorithm of test for Fuzzy Codes	250
8.9	Measure of a Fuzzy Code	254
8.10	Code Theory and Fuzzy Subsemigroups	257
8.11	Construction of Examples by Closure Systems	259
8.12	Examples by $*$ -morphisms	261
9	Generalized State Machines	265
9.1	T -generalized State Machines	265
9.2	T -generalized Transformation Semigroups	267
9.3	Coverings	270
9.4	Direct Products	271
9.5	Decompositions of T -generalized Transformation Semigroups	276
9.6	On Proper Fuzzification of Finite State Machines	282
9.7	Generalized Fuzzy Finite State Machines	283
9.8	Fuzzy Relations and Fuzzy Finite State Machines	284
9.9	Completion of Fuzzy Finite State Machines	286
9.10	Generalized State Machines and Homomorphisms	288
10	Regular Fuzzy Expressions	291
10.1	Regular Fuzzy Expressions	291
10.2	Codes Over Languages	296
10.3	Regulated Codes and Fuzzy Grammars	297
	References	303
	Index	314