

The Mathematics of Minkowski Space-Time

With an Introduction to Commutative Hypercomplex Numbers

Bearbeitet von

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