

Contents

1	Linear Anisotropic Elastic Materials	1
1.1	Theory of Elasticity for Anisotropic Bodies	1
1.1.1	State of Stress	1
1.1.2	Deformation	3
1.1.3	Constitutive Laws	5
1.1.4	Boundary Conditions	5
1.2	Three-Dimensional Constitutive Relations	7
1.2.1	Generalized Hooke's Law	7
1.2.2	Material Symmetry	9
1.2.3	Engineering Constants	12
1.3	Two-Dimensional Constitutive Relations	14
1.3.1	Isotropic Materials	14
1.3.2	Anisotropic Materials	16
1.3.3	Monoclinic Materials	18
1.3.4	Orthotropic Materials	18
1.4	Laminate Constitutive Relations	19
1.4.1	Specially Orthotropic Lamina	20
1.4.2	Generally Orthotropic Lamina	21
1.4.3	Classical Lamination Theory	23
2	Lekhnitskii Formalism	29
2.1	Governing Differential Equations	29
2.2	General Solutions	34
2.3	Boundary Conditions	37
2.3.1	Lateral Surface Conditions	37
2.3.2	End Conditions	39
2.4	Special Cases	43
2.4.1	Generalized Plane Deformation	43
2.4.2	Plane Deformation	44
2.4.3	Generalized Plane Stress	47
2.4.4	Anisotropic Rod by Bending and Twisting	47
2.5	Anisotropic Cantilever Under Transverse Force	52

3	Stroh Formalism	53
3.1	General Solutions	53
3.2	Boundary Conditions	57
3.3	Material Eigenrelation	60
3.3.1	Sextic Eigenrelation	61
3.3.2	Generalized Sextic Eigenrelation	64
3.3.3	The Matrix Differential Equation	67
3.4	Some Identities	68
3.4.1	Explicit Expression of Fundamental Elasticity Matrix N	68
3.4.2	Explicit Expressions of Barnett–Lothe Tensors S , H , and L	69
3.4.3	Identities Relating N , $N(\theta)$, S , H , L	71
3.4.4	Identities Converting Complex Form to Real Form	77
3.5	Degenerate Materials	84
4	Infinite Space, Half-Space, and Bimaterials	87
4.1	Infinite Space	87
4.1.1	Uniform Loading	87
4.1.2	Pure In-Plane Bending	91
4.1.3	Concentrated Forces	92
4.1.4	Couple Moments	95
4.1.5	Dislocations	99
4.2	Half-Space	100
4.2.1	Green's Function	100
4.2.2	Surface Green's Function	105
4.2.3	Distributed Load Along the Half-Space Surface	106
4.2.4	Couple Moments	107
4.2.5	Dislocations	107
4.3	Bimaterials	108
4.3.1	Green's Function	108
4.3.2	Interfacial Green's Function	111
5	Wedges and Interface Corners	115
5.1	Uniform Traction on the Wedge Sides	115
5.1.1	Non-critical Wedge Angles	117
5.1.2	Critical Wedge Angles	122
5.1.3	Summary	123
5.2	Forces at the Wedge Apex	124
5.2.1	A Single Wedge – Under a Concentrated Force	124
5.2.2	A Single Wedge – Under a Concentrated Couple	125
5.2.3	Multi-material Wedge Spaces	128
5.2.4	Multi-material Wedges	130
5.3	Stress Singularities	131
5.3.1	Multi-material Wedge Spaces	134
5.3.2	Multi-material Wedges	135

5.3.3	Eigenfunctions	136
5.3.4	Special Cases	137
5.4	Stress Intensity Factors of Interface Corners	139
5.4.1	Near-Tip Field Solutions	141
5.4.2	A Unified Definition	144
5.4.3	H-Integral for Two-Dimensional Interface Corners	147
5.4.4	H-Integral for Three-Dimensional Interface Corners	152
5.4.5	Numerical Examples	155
6	Holes	159
6.1	Elliptical Holes	159
6.1.1	Uniform Loading at Infinity	161
6.1.2	In-Plane Bending at Infinity	165
6.1.3	Arbitrary Loading Along the Hole Boundary	168
6.1.4	Concentrated Force at Arbitrary Location	173
6.1.5	Dislocation at Arbitrary Location	176
6.2	Polygon-Like Holes	177
6.2.1	Transformation Function	178
6.2.2	Uniform Loading at Infinity	182
6.2.3	Pure In-Plane Bending at Infinity	184
6.2.4	Discussions	186
7	Cracks	187
7.1	Singular Characteristics of Cracks	187
7.1.1	Cracks in Homogeneous Materials	188
7.1.2	Interfacial Cracks	190
7.1.3	Cracks Terminating at the Interfaces	191
7.2	A Finite Straight Crack	192
7.2.1	Uniform Loading at Infinity	192
7.2.2	In-plane Bending at Infinity	193
7.2.3	Arbitrary Loading on the Crack Surfaces	194
7.2.4	Concentrated Force at Arbitrary Location	194
7.2.5	Dislocation at Arbitrary Location	195
7.3	Collinear Cracks	195
7.3.1	General Solutions	196
7.3.2	Two Collinear Cracks	198
7.3.3	Collinear Periodic Cracks	200
7.3.4	Fracture Parameters	201
7.4	Collinear Interface Cracks	206
7.4.1	General Solutions	207
7.4.2	A Semi-infinite Interface Crack	210
7.4.3	A Finite Interface Crack	211
7.4.4	Two Collinear Interface Cracks	214
7.4.5	Fracture Parameters	216

7.5	Delamination Fracture Criteria	225
7.5.1	Stress Intensity Factors and Energy Release Rates . .	226
7.5.2	Experimental Details	226
7.5.3	Delamination Fracture Toughness	229
7.5.4	Mixed-Mode Fracture Criteria	232
7.5.5	Prediction of Delamination Fracture	234
8	Inclusions	239
8.1	Elliptical Elastic Inclusions	239
8.1.1	Uniform Loading at Infinity	245
8.1.2	Concentrated Forces at the Matrix	246
8.2	Rigid Inclusions	248
8.2.1	Elliptical Rigid Inclusions	250
8.2.2	Rigid Line Inclusions	254
8.2.3	Polygon-Like Rigid Inclusions	255
8.3	Interactions Between Inclusions and Dislocations	256
8.3.1	Dislocations Outside the Inclusions	257
8.3.2	Dislocations Inside the Inclusions	258
8.3.3	Dislocations on the Interfaces	263
8.3.4	Interaction Energy	266
8.4	Interactions Between Inclusions and Cracks	268
8.4.1	Cracks Outside the Inclusions	268
8.4.2	Cracks Inside the Inclusions	271
8.4.3	Cracks Penetrating the Inclusions	272
8.4.4	Curvilinear Cracks Lying Along the Interfaces	273
9	Contact Problems	277
9.1	Rigid Punches on a Half-Plane	278
9.1.1	General Solution	278
9.1.2	Indentation by a Flat-Ended Punch	283
9.1.3	A Flat-Ended Punch Tilted by a Couple	285
9.1.4	Indentation by a Parabolic Punch	287
9.1.5	Analogy with the Interface Crack Problems	288
9.2	Rigid Stamp Indentation on a Curvilinear Hole Boundary . . .	290
9.2.1	General Solution	290
9.2.2	Elliptical Hole Boundaries	293
9.2.3	Polygonal Hole Boundaries	295
9.2.4	Numerical Calculation	297
9.3	Rigid Punches on a Perturbed Surface	299
9.3.1	Straight Boundary Perturbation	300
9.3.2	Elliptical Boundary Perturbation	302
9.3.3	Illustrative Examples	306
9.4	Sliding Punches With or Without Friction	308
9.4.1	General Solution	309
9.4.2	A Sliding Wedge-Shaped Punch	312

9.4.3	A Sliding Parabolic Punch	314
9.4.4	Two Sliding Flat-Ended Punches	318
9.5	Contact Between Two Elastic Bodies	321
9.5.1	Contact in the Presence of Friction	326
9.5.2	Contact in the Absence of Friction	328
9.5.3	Contact in Complete Adhesion	331
10	Thermoelastic Problems	333
10.1	Extended Stroh Formalism	333
10.2	Holes and Cracks	337
10.2.1	Elliptical Holes Under Uniform Heat Flow	338
10.2.2	Cracks Under Uniform Heat Flow	343
10.3	Rigid Inclusions	344
10.3.1	Elliptical Rigid Inclusions Under Uniform Heat Flow	345
10.3.2	Rigid Line Inclusions Under Uniform Heat Flow	347
10.4	Collinear Interface Cracks	348
10.4.1	General Solutions	349
10.4.2	Uniform Heat Flow	353
10.5	Multi-material Wedges	356
10.5.1	Stress and Heat Flux Singularity	356
10.5.2	Near-Tip Solutions	364
10.5.3	Special Cases	365
11	Piezoelectric Materials	369
11.1	Constitutive Laws	370
11.1.1	Three-Dimensional State	370
11.1.2	Two-Dimensional State	372
11.2	Expanded Stroh Formalism	374
11.2.1	General Solutions	374
11.2.2	Boundary Conditions	377
11.3	Explicit Expressions	379
11.3.1	Fundamental Matrix N	379
11.3.2	Material Eigenvector Matrices A and B	380
11.3.3	Barnett–Lothe Tensors S , H , and L	389
11.3.4	Bimaterial Matrices D and W	392
11.4	Multi-material Wedges	393
11.4.1	Orders of Stress/Electric Singularity	394
11.4.2	Near-Tip Solutions	394
11.4.3	Stress/Electric Intensity Factors	396
11.4.4	Corner Opening Displacement/Electric Potential	398
11.5	Singular Characteristics of Cracks	399
11.5.1	Cracks in Homogeneous Piezoelectric Materials	399
11.5.2	Interface Cracks Between Two Dissimilar Piezoelectric Materials	401

11.6	Some Crack Problems	404
11.6.1	Cracks	405
11.6.2	Interface Cracks	406
12	Plate Bending Analysis	411
12.1	Bending Theory of Anisotropic Plates	412
12.2	Lekhnitskii Bending Formalism	415
12.2.1	General Solutions	415
12.2.2	Boundary Conditions	416
12.2.3	Degenerate Materials	420
12.3	Stroh-Like Bending Formalism	420
12.3.1	General Solutions	421
12.3.2	Material Eigenrelation and Its Explicit Expressions . .	424
12.3.3	Explicit Expressions of S , H , and L	426
12.4	Holes/Inclusions/Cracks	428
12.4.1	Elliptical Holes	428
12.4.2	Elliptical Rigid Inclusions	431
12.4.3	Cracks	433
13	Coupled Stretching–Bending Analysis	435
13.1	Coupled Stretching–Bending Theory of Laminates	436
13.2	Complex Variable Formulation	440
13.2.1	Displacement Formalism	440
13.2.2	Mixed Formalism	446
13.2.3	Explicit Expressions of N , A , and B	451
13.2.4	Reduction to Symmetric Laminates	454
13.2.5	Comparison and Discussion	457
13.3	Stroh-Like Formalism	458
13.3.1	General Solutions	458
13.3.2	Material Eigenrelation	459
13.3.3	Stress Functions	461
13.3.4	Explicit Expressions of N	465
13.3.5	Explicit Expressions of A and B	467
13.3.6	Explicit Expressions of $N(\omega)$	469
13.3.7	Explicit Expressions of S , H , and L	472
13.4	Hygrothermal Stresses	474
13.4.1	Basic Equations	474
13.4.2	Extended Stroh-Like Formalism	476
13.5	Electro-elastic Composite Laminates	481
13.5.1	Basic Equations	481
13.5.2	Expanded Stroh-Like Formalism	484
14	Holes/Cracks/Inclusions in Laminates	493
14.1	Holes in Laminates Under Uniform Stretching and Bending Moments	494

14.1.1	Field Solutions	496
14.1.2	Stress Resultants and Moments Along the Hole Boundary	497
14.2	Holes in Laminates Under Uniform Heat Flow and Moisture Transfer	498
14.2.1	Uniform Heat Flow and Moisture Transfer in x_1x_2 -Plane	498
14.2.2	Uniform Heat Flow and Moisture Transfer in x_3 -Direction	501
14.3	Holes in Electro-Elastic Laminates	503
14.4	Green's Functions for Laminates	505
14.4.1	Concentrated In-Plane Forces and Out-of-Plane Moments ($\hat{f}_1, \hat{f}_2, \hat{m}_1, \hat{m}_2$)	506
14.4.2	Concentrated Transverse Force ($\hat{f}_3$)	508
14.4.3	Concentrated In-Plane Torsion ($\hat{m}_3$)	510
14.4.4	Explicit Real-Form Solutions	512
14.5	Green's Functions for Laminates with Holes/Cracks	515
14.5.1	Field Solutions	516
14.5.2	Stress Resultants and Moments Along the Hole Boundary	522
14.5.3	Verification and Discussions	523
14.5.4	Cracks	524
14.6	Green's Functions for Laminates with Elastic Inclusions	526
14.6.1	Concentrated Forces/Moments Outside the Inclusions	529
14.6.2	Concentrated Forces/Moments Inside the Inclusions	534
14.6.3	Verification and Discussions	539
15	Boundary Element Analysis	545
15.1	Two-Dimensional Elastic Analysis	545
15.1.1	Boundary Integral Equations	545
15.1.2	Fundamental Solutions	546
15.1.3	Boundary Element Formulation	552
15.1.4	Stresses and Displacements at Internal Points	554
15.1.5	Stress Intensity Factors for Crack Problems	555
15.1.6	Subregion Technique	557
15.1.7	Numerical Implementation	559
15.2	Two-Dimensional Electro-Elastic Analysis	560
15.2.1	Boundary Element Formulation	560
15.2.2	Numerical Examples	561
15.3	Coupled Stretching–Bending Analysis	565
15.3.1	Boundary Integral Equations – Internal Points	565
15.3.2	Fundamental Solutions	570
15.3.3	Boundary Integral Equations – Boundary Points	575
15.3.4	Free-Term Coefficients	580

Appendix A: Symbols, Sign Convention and Units	589
A.1 Common Symbols	589
A.2 Extended Symbols	594
A.3 Sign Convention	599
A.4 Units	600
Appendix B: Hilbert Problem	607
B.1 Solution to the Hilbert Problem in Scalar Form	607
B.2 Solution to the Hilbert Problem in Matrix form	608
B.3 Evaluation of a Line Integral in Scalar Form	610
B.4 Evaluation of a Line Integral in Matrix Form	612
Appendix C: Summary of Stroh formalism	615
C.1 Two-dimensional Problems	615
C.2 Coupled Stretching-Bending Problems	618
C.3 Dimensions of Matrices used in Stroh Formalism	624
Appendix D: Collection of Problem Solutions	625
References	655
Author Index	663
Subject Index	667