

# Partial Differential Equations in Mechanics 1

Fundamentals, Laplace's Equation, Diffusion Equation, Wave Equation

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