

Positive Polynomials

From Hilbert's 17th Problem to Real Algebra

Bearbeitet von
Alexander Prestel, Charles Delzell

1st Edition. Softcover version of original hardcover edition 2001. Taschenbuch. viii, 269 S. Paperback

ISBN 978 3 642 07445 5

Format (B x L): 15,5 x 23,5 cm

Gewicht: 433 g

[Weitere Fachgebiete > Mathematik > Algebra](#)

schnell und portofrei erhältlich bei

beck-shop.de
DIE FACHBUCHHANDLUNG

Die Online-Fachbuchhandlung beck-shop.de ist spezialisiert auf Fachbücher, insbesondere Recht, Steuern und Wirtschaft. Im Sortiment finden Sie alle Medien (Bücher, Zeitschriften, CDs, eBooks, etc.) aller Verlage. Ergänzt wird das Programm durch Services wie Neuerscheinungsdienst oder Zusammenstellungen von Büchern zu Sonderpreisen. Der Shop führt mehr als 8 Millionen Produkte.

Contents

| | |
|--|-----|
| Introduction | 1 |
| 1. Real Fields | 7 |
| 1.1 Ordered Fields | 7 |
| 1.2 Extensions of Orderings | 12 |
| 1.3 The Real Closure | 16 |
| 1.4 Exercises | 24 |
| 1.5 Bibliographical and Historical Comments | 28 |
| 2. Semialgebraic Sets | 31 |
| 2.1 Semialgebraic Sets | 31 |
| 2.2 Ultraproducts | 36 |
| 2.3 Elimination of Quantifiers | 41 |
| 2.4 The “Finiteness Theorem” | 45 |
| 2.5 Exercises | 47 |
| 2.6 Bibliographical and Historical Comments | 48 |
| 3. Quadratic Forms over Real Fields | 53 |
| 3.1 Witt Decomposition | 53 |
| 3.2 The Witt Ring of a Field | 59 |
| 3.3 Signatures | 62 |
| 3.4 Quadratic Forms Over Real Function Fields | 68 |
| 3.5 Generalization of Hilbert’s 17th Problem | 74 |
| 3.6 Exercises | 77 |
| 3.7 Bibliographical and Historical Comments | 79 |
| 4. Real Rings | 81 |
| 4.1 The Real Spectrum of a Commutative Ring | 81 |
| 4.2 The Positivstellensatz | 86 |
| 4.3 “Continuous” Representation of Polynomials | 91 |
| 4.4 η_α -Fields | 94 |
| 4.5 The Real Spectrum of $\mathbb{R}[X_1, \dots, X_n]$ | 101 |
| 4.6 Exercises | 107 |
| 4.7 Bibliographical and Historical Comments | 109 |

VIII Contents

| | |
|--|-----|
| 5. Archimedean Rings | 113 |
| 5.1 Quadratic Modules and Semiorderings | 113 |
| 5.2 Rings with Archimedean Preorderings | 119 |
| 5.3 Rings with Archimedean Quadratic Modules | 124 |
| 5.4 Rings with Archimedean Preprimes | 130 |
| 5.5 Exercises | 134 |
| 5.6 Bibliographical and Historical Comments | 136 |
| 6. Positive Polynomials on Semialgebraic Sets | 139 |
| 6.1 Semiorderings and Weak Isotropy | 139 |
| 6.2 Archimedean Quadratic Modules on $\mathbb{R}[X_1, \dots, X_n]$ | 142 |
| 6.3 Distinguished Representations of Positive Polynomials | 145 |
| 6.4 Applications to the Moment Problem | 152 |
| 6.5 Exercises | 157 |
| 6.6 Bibliographical and Historical Comments | 158 |
| 7. Sums of $2m$th Powers | 161 |
| 7.1 Preorderings and Semiorderings of Level $2m$ | 161 |
| 7.2 Semiorderings of Level $2m$ on Fields | 166 |
| 7.3 Archimedean Modules of Level $2m$ | 169 |
| 7.4 Exercises | 176 |
| 7.5 Bibliographical and Historical Comments | 177 |
| 8. Bounds | 179 |
| 8.1 Length of Sums of Squares | 179 |
| 8.2 Existence of Degree Bounds | 183 |
| 8.3 Positive Polynomials over Non-Archimedean Fields | 189 |
| 8.4 Distinguished Representations in the Non-Archimedean Case . | 196 |
| 8.5 Exercise | 201 |
| 8.6 Bibliographical and Historical Comments | 201 |
| Appendix: Valued Fields | 203 |
| A.1 Valuations | 203 |
| A.2 Algebraic Extensions | 207 |
| A.3 Henselian Fields | 213 |
| A.4 Complete Fields | 223 |
| A.5 Dependence and Composition of Valuations | 230 |
| A.6 Transcendental Extensions | 236 |
| A.7 Exercises | 242 |
| A.8 Bibliographical Comments | 245 |
| References | 247 |
| Glossary of Notations | 255 |
| Index | 259 |



<http://www.springer.com/978-3-540-41215-1>

Positive Polynomials

From Hilbert's 17th Problem to Real Algebra

Prestel, A.; Delzell, C.N.

2001, VIII, 268 p., Hardcover

ISBN: 978-3-540-41215-1