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978-1-107-00217-3 - Quantum Computation and Quantum Information, 10th Anniversary Edition

Michael A. Nielsen and Isaac L. Chuang

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