

Contents

1	Introduction	1
1.1	Preamble	1
1.2	Lyapunov Stability of Dynamical Systems	3
1.3	Hamiltonian Dynamical Systems	7
1.4	Complex Hamiltonian Dynamics	10
2	Hamiltonian Systems of Few Degrees of Freedom	13
2.1	The Case of $N = 1$ Degree of Freedom	13
2.2	The Case of $N = 2$ Degrees of Freedom	19
2.2.1	Coordinate Transformations and Solution by Quadratures	20
2.2.2	Integrability and Solvability of the Equations of Motion	24
2.3	Non-autonomous One Degree of Freedom Hamiltonian Systems	30
2.3.1	The Duffing Oscillator with Quadratic Nonlinearity	31
2.3.2	The Duffing Oscillator with Cubic Nonlinearity	34
	Exercises	37
	Problems	39
3	Local and Global Stability of Motion	41
3.1	Equilibrium Points, Periodic Orbits and Local Stability	41
3.1.1	Equilibrium Points	41
3.1.2	Periodic Orbits	44
3.2	Linear Stability Analysis	46
3.2.1	An Analytical Criterion for “Weak” Chaos	51
3.3	Lyapunov Characteristic Exponents and “Strong” Chaos	53
3.3.1	Lyapunov Spectra and Their Convergence	53
3.3.1.1	Lyapunov Spectra and the Thermodynamic Limit	55
3.4	Distinguishing Order from Chaos	56
3.4.1	The SALI Method	58
3.4.2	The GALI Method	59
	Exercises	60
	Problems	61

4	Normal Modes, Symmetries and Stability	63
4.1	Normal Modes of Linear One-Dimensional Hamiltonian Lattices ...	63
4.2	Nonlinear Normal Modes (NNMs) and the Problem of Continuation	65
4.3	Periodic Boundary Conditions and Discrete Symmetries	67
4.3.1	NNMs as One-Dimensional Bushes	67
4.3.2	Higher-Dimensional Bushes and Quasiperiodic Orbits	70
4.4	A Group Theoretical Study of Bushes	70
4.4.1	Subgroups of the Parent Group and Bushes of NNMs	72
4.4.2	Bushes in Modal Space and Stability Analysis	74
4.5	Applications to Solid State Physics	80
4.5.1	Bushes of NNMs for a Square Molecule	80
4.5.2	Bushes of NNMs for a Simple Octahedral Molecule	84
	Exercises	87
	Problems	88
5	Efficient Indicators of Ordered and Chaotic Motion	91
5.1	Variational Equations and Tangent Map	91
5.2	The SALI Method	94
5.3	The GALI Method	102
5.3.1	Theoretical Results for the Time Evolution of GALI	104
5.3.1.1	Exponential Decay of GALI for Chaotic Orbits	104
5.3.1.2	The Evaluation of GALI for Ordered Orbits	107
5.3.2	Numerical Verification and Applications	111
5.3.2.1	Low-Dimensional Hamiltonian Systems	111
5.3.2.2	High-Dimensional Hamiltonian Systems	117
5.3.2.3	Symplectic Maps	120
5.3.2.4	Motion on Low-Dimensional Tori	122
	Appendix A: Wedge product	124
	Appendix B: Example algorithms for the computation of the SALI and GALI chaos indicators	126
	Exercises	130
	Problems	131
6	FPU Recurrences and the Transition from Weak to Strong Chaos	133
6.1	The Fermi Pasta Ulam Problem	133
6.1.1	Historical Remarks	133
6.1.2	The Concept of q -breathers	136
6.1.3	The Concept of q -tori	138
6.2	Existence and Stability of q -tori	139
6.2.1	Construction of q -tori by Poincaré-Linstedt Series	140
6.2.2	Profile of the Energy Localization	147
6.3	A Numerical Study of FPU Trajectories	152
6.3.1	Long Time Stability near q -tori	155

6.4	Diffusion and the Breakdown of FPU Recurrences	157
6.4.1	Two Stages of the Diffusion Process	158
6.4.2	Rate of Diffusion of Energy to the Tail Modes	160
6.4.3	Time Interval for Energy Equipartition	162
	Exercises	163
	Problems	164
7	Localization and Diffusion in Nonlinear One-Dimensional Lattices ...	165
7.1	Introduction and Historical Remarks	165
7.1.1	Localization in Configuration Space	166
7.2	Discrete Breathers and Homoclinic Dynamics	168
7.2.1	How to Construct Homoclinic Orbits	171
7.3	A Method for Constructing Discrete Breathers	174
7.3.1	Stabilizing Discrete Breathers by a Control Method	175
7.4	Disordered Lattices	180
7.4.1	Anderson Localization in Disordered Linear Media	180
7.4.2	Diffusion in Disordered Nonlinear Chains	183
7.4.2.1	Two Basic Models	183
7.4.2.2	Regimes of Wave Packet Spreading	185
7.4.2.3	Numerical Results	186
	Exercises	188
	Problems	189
8	The Statistical Mechanics of Quasi-stationary States	191
8.1	From Deterministic Dynamics to Statistical Mechanics	191
8.1.1	Nonextensive Statistical Mechanics and q -Gaussian pdfs	193
8.2	Statistical Distributions of Chaotic QSS and Their Computation	195
8.3	FPU π -Mode Under Periodic Boundary Conditions	197
8.3.1	Chaotic Breathers and the FPU π -Mode	200
8.4	FPU SPO1 and SPO2 Modes Under Fixed Boundary Conditions	203
8.5	q -Gaussian Distributions for a Small Microplasma System	208
8.6	Chaotic Quasi-stationary States in Area-Preserving Maps	212
8.6.1	Time-Evolving Statistics of pdfs in Area- Preserving Maps	213
8.6.2	The Perturbed MacMillan Map	214
8.6.2.1	The $\varepsilon = 0.9$, $\mu = 1.6$ Class of Examples	215
8.6.2.2	The $\varepsilon = 1.2$, $\mu = 1.6$ Class of Examples	217
	Exercises	218
	Problems	219
9	Conclusions, Open Problems and Future Outlook	221
9.1	Conclusions	221
9.2	Open Problems	224
9.2.1	Singularity Analysis: Where Mathematics Meets Physics	224

9.2.2	Nonlinear Normal Modes, Quasiperiodicity and Localization	226
9.2.3	Diffusion, Quasi-stationary States and Complex Statistics ...	229
9.3	Future Outlook	231
9.3.1	Anomalous Heat Conduction and Control of Heat Flow	231
9.3.2	Complex Soliton Dynamics in Nonlinear Photonic Structures.....	232
9.3.3	Kinetic Theory of Hamiltonian Systems and Applications to Plasma Physics	235
References		239
Index		253